

BOLD VISIONS IN EDUCATIONAL RESEARCH

Key Works in Radical Constructivism

(edited by Marie Larochelle)

Ernst von Glasersfeld



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Key Works in Radical Constructivism

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Keyworks

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Key Works in Radical Constructivism

Ernst von Glasersfeld

Edited by:

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Comments by:

Edith Ackermann

Gérard Fourez

Jacques Désautels

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Postscript by:

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PREFACE

Only the first of the twenty-two chapters in this volume deals explicitly with education. The others discuss language, theory of knowledge, and the formation of concepts. These are three topics in which traditional educational research has rarely shown an interest. I am suggesting that this disregard can to a large extent be blamed for the dismal state of education today. By and large the notion that language, if used properly, is an efficient conveyer of knowledge was unquestioningly accepted. Any detailed analysis of a lesson and its effect on the students shows that this is an illusion. A closer look at some aspects of the actual workings of language may therefore not be a waste of time.

Western philosophers have with few exceptions perpetuated the belief that true knowledge exists apart from the knower. Although they have been unable to show how it might enter the human head, they have staunchly condemned as ‘genetic fallacy’ any investigation of how the knower might generate it. The essays in the section on the theory of knowledge are intended to provide teachers, parents, and perhaps students themselves with a more fruitful philosophical background.

The concepts analyzed in the third section are but a sample that needs to be greatly expanded. But they constitute a scaffolding for the construction of others. They provide the teacher with a model of how it could be done—and how concepts can be built up is surely something teachers should be able to show their students.

The list of the sources of the collected papers shows that they were written for diverse occasions and diverse audiences. Although I have always tried to take this into account, the essential elements of my position could not be changed. I regret that the reader of the present book, therefore, has to bear with a number of repetitions.

I thank Ken Tobin for his Postscript and Edith Ackermann, Jacques Désautels, Gérard Fourez, and Leslie P. Steffe for commentaries, which greatly enhance the value of this book.

To Marie Larochelle I am immensely grateful for many suggestions and the painstaking editorial work she has done; and I appreciate the willingness of Sense Publications to publish a book that may cause some ripples.

E.v.G.
November 2006

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Kenneth Tobin is Presidential Professor of Urban Education at the Graduate Center of The City University of New York. His research focuses on the teaching and learning of science in urban schools, using mixed methods including ethnography, conversation analysis, and autobiography supported with theoretical frameworks from cultural sociology and the sociology of emotions.

Leslie P. Steffe is a Distinguished Research Professor of Mathematics Education at the University of Georgia. He collaborated with Ernst von Glasersfeld on the project 'Interdisciplinary Research on Number' (IRON) at the University of Georgia. The influence of radical constructivism originated in that work in IRON and has spread through the work of many investigators.

ACKNOWLEDGEMENT OF SOURCES

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INTRODUCTION

ERNST VON GLASERSFELD'S WAY OF WORLDMAKING

In three short stories said to have inaugurated the genre of detective fiction, Edgar Allan Poe brought to life a character, Auguste C. Dupin, whose comments in many ways bring to mind the comments and thoughts of that other well-known character Ernst von Glasersfeld, the pioneering thinker of radical constructivism whose principal texts in education are the subject of this book.¹ Auguste C. Dupin and Ernst von Glasersfeld are alike in holding the view that we are always arriving too late on the scene to be able to behold a pure, as-yet un-interpreted world. Rather, the world that we are seeing and experiencing is one that has been configured according to both the notions that we entertain about it and the distinctions with which we have laden it; further, such notions and distinctions constitute practical means of our own invention, devised to co-ordinate and manage our experience of the world (Glasersfeld, 1993). Ultimately, whenever we claim to describe the world-in-itself (or the 'ontologically preexisting world'), we in fact are describing the product of the mapping process that has enabled us to make our way in this world and to actualize our projects within it (inclusive even of the 'dud' roadmaps—that is, the cognitive itineraries that have proved non-viable or indeed fatal to our assumptions and views). In short, we are describing what can be done in the world and not, to paraphrase Geertz (1988), seeing the world as it really is when only God is looking! Thus, from this perspective, knowledge is viewed as “a search for fitting ways of behaving and thinking” (Glasersfeld, 1988, p. 41), and thus said to be operative, as it allows us to operate, act and anticipate, just as it can, obviously, lead us into dead-ends, as is shown in one of the cases narrated by detective Dupin.

In *'The Purloined Letter'*, published circa 1845, Dupin comments on the failure of the Paris Prefect of Police to locate a letter of paramount importance, tying this inability to the police chief's habits of comprehending the world and figuring the capacities of others—in this instance, the thief, Minister D—, who also happened to be a poet. As Dupin informs us, in the Prefect's view a poet is by definition a fool and a scatterbrain—therefore, the kind of person who would think to hide the letter nowhere else than in some unlikely spot or other. On the basis of this assumption, the Prefect and his men painstakingly searched the thief's apartment, ripping up the inlaid pieces of the parquet floor, scrutinizing the bindings of his entire book collection beneath a microscope, peering inside the hollows of the chair legs and sinking long needles into the chair cushions—all to no avail. Further,

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throughout their searches, they remained completely oblivious to the letter that had been placed prominently on display atop a fireplace mantle!

All of which goes to show—and on this point Dupin the detective and Glasersfeld the epistemologist again think alike—the importance of developing a reflexive understanding of the world; in other words: an understanding that is conscious of its assumptions and that, as a result, is conscious of being *one* manner of understanding or *one* ‘take’ among other possible manners of understanding or ‘takes’. By the same token, this does not mean that all takes or intellectual constructions are equal or interchangeable. Indeed, the Prefect’s failed efforts at finding the letter offers a telling illustration of how this is not so. On the other hand, if he and his men had previously developed the habit of thinking reflexively, they might have been able to vary their investigative approaches somewhat. In addition, they might well have been able to work up not one but several composite drawings of the thief and, as a result, would have multiplied their potentialities for action, as Glasersfeld would say.

* * *

For more than 40 years now, Ernst von Glasersfeld has invited us to take up detective work and reflection on the question of ‘how we know what we know’. This he has done not only at numerous seminars and workshops, but also through hundreds of written contributions in a broad range of fields (such as psychology, philosophy, linguistics, cybernetics and, of course, education) across an equally wide spectrum of heavyweight themes (such as the notions of truth, objectivity and the transparency of language), and with a style of writing that knowingly and knowledgeably grapples with the epistemological genre framing the debate at hand. Regardless of the subject contemplated (such as poetic wisdom, the conceptual construction of time, or scientific fallibility) or the field involved, ongoing inquiry into radical constructivism has been his main, steadfast concern, which has been shaped by his acute sense that the things that are said are said from within a perspective that cannot claim to be all-knowing and all-encompassing.

In short, with Glasersfeld’s work, it is a line of thought we are contemplating—or rather, an oeuvre. All the more so as, to borrow from Foucault (1975), Glasersfeld is more than simply the author of texts, for he has indeed made possible the formation of other texts and other discourses. He has, by the same token, inaugurated *a way of viewing*. In other words, as is testified to by the numerous publications and curriculum designs drawing on constructivism, Ernst von Glasersfeld has opened up both intellectual and institutional space for revisiting the issues of cognition and learning (and in the process, moreover, has managed to chip away at certain privileges).²

Accordingly, the idea for this book—to which Kenneth Tobin and Joe Kincheloe, editors of the “Bold Visions in Educational Research” collection gave the initial impetus and to which Glasersfeld personally contributed thereafter—grew out of a desire to make his ideas on all these different issues more accessible and thus to highlight not only the originality, fecundity and consistency of his line

of thought but also the breaks with 'realist metaphysics' that it necessarily entails. Initially, we wished to confine ourselves to texts on education (according to the generally accepted meaning of this term). Before long, however, we discarded this plan, which would have had the effect of only partially illustrating the reversal of perspective implied in radical constructivism, and of ignoring the principles and larger implications of this reversal. We thus decided to take the approach outlined below, which brings together a number of interpretations and conceptual analyses through which Glasersfeld takes up the different issues in question, while also expanding the purview of inquiry beyond the 'school form' of these issues, nonetheless.

The first three parts of this book contain texts by Glasersfeld that are intended as a window onto the constructivist way of configuring the issues of cognition and learning and on the attendant—and frequently drastic—reconceptualization of closely related themes. Thus, Part I, which opens with a text dedicated to the set of issues surrounding teaching and learning, is concerned with reconceptualizing the concepts of knowledge and fact (chapters 2 and 3), as well as the concepts of communication, interpretation and meaning (chapters 4, 5 and 6). It is also concerned with reinterpreting Piagetian epistemology and bringing out its family ties to radical constructivism (chapter 7).

Part II outlines various theoretical frameworks that have shaped constructivism (such as cybernetics, chapter 15) and presents the major thinkers and researchers who have contributed to the development of this line of thought, such as Vico, Berkeley, Piaget (chapter 8), Ceccato (chapter 15) and Foerster (chapter 13). This part also proposes the outline of a pragmatic conception of science-making (chapter 14), as well as a discussion of the staple notions underpinning realist discourse, such as objectivity and truth (chapters 9, 11 and 13). Finally, it is also a question of the ways and means (rational or other) used to bring order to one's experience (chapters 10 and 12).

Part III is a collection of conceptual analyses bearing, among other things, on processes of change, abstraction, deduction and anticipation (chapters 16, 17, 18 and 21), on the construction of time and 'universals' (chapters 20 and 22), and on the experiential—as opposed to the ontological—nature of the foundations of mathematics (chapter 19).

Part IV, differing somewhat in form the preceding portions, consists of commentaries penned by colleagues and friends of Glasersfeld. Their common objective is to point out the avenues of reflection that he has been instrumental in opening up, and to indicate which among these paths, today, warrant undertaking new efforts to delineate them better or to reassess their import more fully. In other cases, the goal should be to suggest linking routes worth exploring between the main well-travelled avenues.

In the first commentary (chapter 23), Edith Ackermann proposes a set of reflections on our interactions with artifacts, which she views as offering a valuable opportunity for offsetting constructivism's tendency to focus on the subject's assimilative activity. In particular, she highlights the process of accommodation that is a necessary part of the effort to co-ordinate objects that cannot be bent to

one's will. So doing, she grounds the subject in a world requiring him or her to deal with objects that come laden with affordances (that is, guidelines and indeed injunctions concerning the way such objects are to be used) and invites us to examine how the subject will go about appropriating it.

G rard Fourez, who signs the second commentary (chapter 24), advocates re-examining the concept of representation and, at the same time, urges rehabilitating the latter—not in its iconic form, obviously, but instead as a model, a map or chart of our possibilities for action and, hence, as a tool with which to develop our world and to engage in deliberations about it. He suggests taking the same approach with the concept of fact, noting that the standardized, convention-ruled character of facts argues powerfully for developing a kind of constructivism that fully accounts for the sociopolitical dimension of the construction and legitimization of knowledge.

In the third commentary (chapter 25), Jacques D sautels also pleads for a variety of radical constructivism that opens onto the social and the political. He shows that through the concepts of intersubjectivity and recursive processes alone, constructivism already provides a basis for calling into question the classic division between individual and society. What is more, however, by conceiving of society and individual from the perspective of interdependence, it is possible, he argues, to take this challenge one step further. Indeed, by drawing on work in science studies, it is possible to seriously re-examine the other 'Great divide' that frequently dominates the landscape of scientific literacy in schools—namely, the separation of questions and issues pertaining to science from those pertaining to society.

The last commentary (chapter 26), by Leslie P. Steffe, urges us to pay greater attention to the context of reception for radical constructivism, and particularly in the field of mathematics education. In this setting, constructivism often appears to take form according to a logic reminiscent of that which characterized the Paris Prefect of Police and that prompts us to underestimate the competencies of others—and of students in particular. In other words, it would seem as if we continued to labour under the assumptions of the cognitive deficit model that we first adopted to reflect on learning/teaching relationships; it would seem as if we were shying away from entering the world of 'co-inspiration' and the 'interweaving (in French, *enroulement*) of perspectives' that is promoted by constructivism.³

Finally, in his postscript to this book, Kenneth Tobin recalls, from his privileged vantage point, several of the episodes and events that signalled the emergence of constructivism in the field of science education. His account serves to bring out the revolutionary quality of this perspective at a time when the field was still largely under the sway of a realist ontology. Furthermore, it relates how his own field work and research led him to frame constructivism within the terms of culture and, in the process, contribute to the development of this line of thinking by adopting a sociocultural approach to issues of cognition and education. Kenneth Tobin has thus crafted the conclusion to a book which, in an approach recalling of the notion of * quilibr tion majorante*⁴ so dear to Piaget, bears witness to the unceasing reflexive elaboration to which constructivism is submitted—not only in the work of

Glaserfeld but also in the texts of all those who have chosen to work from within this perspective.

* * *

It goes without saying that this edition of Glaserfeld's texts, which was brought out in part through funding from the Social Sciences and Humanities Research Council of Canada, has benefited from the close collaboration and valued contributions of a number of individuals. I would like to thank my colleagues Suzanne Vincent and Serge Desgagné for their suggestions, Bernard Jobin for his assistance in assembling the bibliography, and Sylvie Côté for her work creating the tables. I would also like to express my gratitude to my Faculty's administration for its financial contribution toward translation costs. Finally, I wish to thank Donald Kellough who, in connection with several contributions, has once again tackled the challenge of translating (i.e., re-mapping) into English a 'Francophone way of worldmaking'.

Marie Larochelle, Editor
 Université Laval, Québec

NOTES

- ¹ The first paragraphs of this introduction are based on excerpts from the address I gave on June 18, 2006, on the occasion of the Université Laval (Québec, Quebec) graduation ceremony at which Ernst von Glaserfeld was awarded the degree of doctor of education *honoris causa*. A slightly modified version of these passages can also be found elsewhere (Larochelle & Désautels, 2007). It was only shortly after having written this address, while re-reading Ernst von Glaserfeld's book entitled *The Construction of Knowledge* (published in 1987 by Intersystems Publications) that I realized that Heinz von Foerster, in his preface to this publication, also noted a similarity between the work of Glaserfeld and that of Dupin, particularly on the question of language. It had been some 20 years since I last read this preface, with the result that in June 2006, I genuinely imagined that I was working from an association that had heretofore gone unnoticed!
- ² By holding that there is not *one* but *many* 'ways of worldmaking', to borrow Goodman's (1992) felicitous expression, radical constructivism undertakes more than the promotion of a pragmatic conception of knowledge. He de-centers what Bourdieu (1980) called the 'racism of intelligence', which consists in ascribing the ability to produce valid bodies of learning and knowledge to certain groups only. In other words, students, for example, may not know what we would like them to know, but, following radical constructivism, they are not lacking in knowledge. Through their learning paths and interactions, they too have elaborated 'authentic knowledge', to use Bensaude-Vincent's (2000) phrase—they have, in other words, engaged with their experiences of the world and circumstances.
- ³ I borrow the concept of 'co-inspiration' from Maturana (1988) and that of the 'interweaving (*enroulement*) of perspectives' from Rocher (2005).
- ⁴ That is, as Glaserfeld has pointed out (1981), "the incremental equilibration that proceeds in spiral [form], incorporating more and more items and events in the developing organism's experience" (p. 93).

PART I

**LEARNING, LANGUAGE AND THE RADICAL
THEORY**

LEARNING AS CONSTRUCTIVE ACTIVITY

The general topic I was given for this chapter is ‘Research in Mathematics Education from an Epistemological Perspective’. That sounds no more dangerous than so many other academic topics. But don’t let the prosaic surface deceive you. To introduce epistemological considerations into a discussion of education has always been dynamite. Socrates did it, and he was promptly given hemlock. Giambattista Vico did it in the 18th century, and the philosophical establishment could not bury him fast enough. In our own time there was Jean Piaget. He really wanted to stay out of education but allowed himself to be drawn in—and we know what has happened to his epistemology at the hands of interpreters and translators. It seems that to discuss education from an epistemological point of view was a sure way of committing intellectual suicide. Recently, however, the world of education may have begun to change. At least the particular discipline that is represented in this meeting, the discipline that is concerned with numbers, with arithmetic, and ultimately with mathematics, is manifesting symptoms that indicate the will to change.

The rapid shifts in the methods of mathematics education that have taken place in the last few decades—from simplistic associationism to ‘New Math’ and ‘Back to Basics’—did not work the miracles that were expected of them. Their failure has created a mood that no longer fosters enthusiasm for new gimmicks. Today, I think it is fair to say, there is a more or less general disillusionment. This disillusionment is healthy and propitious because it pushes us closer to the point where we might be ready to review some of the fundamental presuppositions of the traditional theories of education. Among these presuppositions are our conception of teaching and learning and, most fundamental of all, the conception of what it is ‘to know’.

Ten or 15 years ago, it would have been all but inconceivable to subject educators or educational researchers to a talk that purported to deal with a theory of knowledge. Educators were concerned with getting knowledge into the heads of their students, and educational researchers were concerned with finding better ways of doing it. There was, then, little if any uncertainty as to what the knowledge was that students should acquire, and there was no doubt at all that, in one way or another, knowledge could be transferred from a teacher to a student. The only question was, which might be the best way to implement that transfer—and educational researchers, with their criterion-referenced tests and their sophisticated statistical methods, were going to provide the definitive answer.

Something, apparently, went wrong. Things did not work out as expected. Now there is disappointment, and this disappointment—I want to emphasize this—is not restricted to mathematics education but has come to involve teaching and the didactic methods in virtually all disciplines. To my knowledge, there is only one

exception that forms a remarkable contrast: the teaching of physical and, especially, athletic skills. There is no cause for disappointment in that area. In those same 10 or 15 years in which the teaching of intellectual matters has somehow foundered, the teaching of skills such as tennis and skiing, pole jumping and javelin throwing, has advanced quite literally by leaps and bounds. The contrast is not only spectacular but it is also revealing. I shall return to this phenomenon at a later point when, I hope, we will be able to consider an analogy which, at this moment, might seem utterly absurd.

If educational efforts are, indeed, failing, the presuppositions on which, implicitly or explicitly, these efforts have been founded must be questioned and it seems eminently reasonable to suggest, as did those who formulated the topic for this discussion, that we begin by inspecting the commodity that education claims to deal in, and that is 'knowledge'.

This chapter is an attempt to do three things. First, I shall go back to what I consider the origin of the troubles we have had with the traditional conception of knowledge. This historical review will not only be sketchy, but it will also be quite biased, because I have rather strong views on the subject. However, considering the mess in which the theory of knowledge has been during the last 50 years in the 'hard' sciences, my attempt will, I hope, not be deemed unjustified.

Second, I shall propose a conceptualization of 'knowledge' that does not run into the same problem and that, moreover, provides another benefit in that it throws helpful light on the process of communication. As teachers, I said a moment ago, we are intent upon generating knowledge in students. That, after all, is what we are being paid for, and since the guided acquisition of knowledge, no matter how we look at it, seems predicated on a process of communication, we should take some interest in how that process might work. In my experience, this is an aspect that has not been given much thought. Educators have spent and are rightly spending much time and effort on curriculum. That is, they do their best to work out what to teach and the sequence in which it should be taught. The underlying process of linguistic communication, however, the process on which their teaching relies, is usually simply taken for granted. There has been a naive confidence in language and its efficacy. Although it does not take a good teacher very long to discover that saying things is not enough to 'get them across', there is little if any theoretical insight into why linguistic communication does not do all it is supposed to do. The theory of knowledge which I am proposing, though it certainly does not solve all problems, makes this particular problem very clear.

Lastly, having provided what I would like to call a model of 'knowing' that incorporates a specific view of the process of imparting knowledge, I shall briefly explore a way to apply that model to the one thing all of us here are interested in: how to introduce children to the art, the mystery, and the marvelous satisfaction of numerical operations.

THE INSTRUMENTALIST ANSWER TO THE SCEPTICS' ATTACK

The nature of knowledge was a hotly debated problem as far back as the 6th century B.C. The debate has been more or less continuous, and while in many ways it has been colorful, it has been remarkably monotonous in one respect. The central problem has remained unsolved throughout, and the arguments that created the major difficulty at the beginning are the very same that today still preclude any settlement of the question.

The story begins with the first documents on epistemology that have come down to us, the so-called 'fragments' of the pre-Socratics. The ideas these men struggled with and tried to clarify must have arisen some time before them, but since we have no earlier written records, that background is extremely hazy. The pre-Socratics, at any rate, exhibit a degree of sophistication that is unlikely to have been acquired in one or two generations. Fragmentary though they are, their pronouncements leave no doubt that, towards the close of the 5th century B.C., the process of knowing had been conceptually framed in a relatively stable general scenario. By and large, the thinkers who concerned themselves with the cognizing activity tacitly accepted the scenario in which the knower and the things of which, or about which, he or she comes to know are, from the outset, separate and independent entities.

I want to stress that this dichotomy does not coincide with the split between the knowing subject and the subject's knowledge. That second dichotomy appears whenever an actor becomes aware of his or her own activity or when a thinker begins to think about his or her own thinking. That second problem of self-consciousness is not identical with the problem of cognition. Though the two are related in that they interact (e.g., in an analysis of reflective thought, which will enter our discussion later), I here want to deal only with the first. The pre-Socratics, in any case, took for granted the human ability to be aware of knowing. What they began to wonder about was how it was possible that one could come to know the world. It is in this quest that the cognitive scenario they accepted and that has been perpetuated by almost all epistemologists after them, is of decisive importance. Once it was chosen as the basis for the construction of a theory of knowledge, that construction was saddled with a paradox. The paradox is inescapable and it has haunted philosophers incessantly in the 2,500 years since then.

The reason why that particular cognitive scenario was adopted is very simple. It reflects the situation as it initially appears to any experiencer. The question, how it comes about that we know anything, is not likely to be asked at the beginning of a prospective knower's development. A six-year-old who bicycles home from school would be a very peculiar six-year-old if she suddenly asked herself, 'How on earth do I manage to find my way home?' or 'What exactly happened when I learned to ride my bicycle?'

I am not suggesting that these are questions a six-year-old or, indeed, anyone should ask. I am merely saying that if we ever do ask them, it will be at a somewhat later age. The same goes for the question, 'How is it that I can know

what I do know?’ Those who have felt such epistemological curiosity probably formulated their first relevant question in their middle teens or later. That is to say, they began to question their knowledge at a point in their cognitive career when they had already acquired an enormous amount of know-how and learning. Inevitably, nearly all they knew at that point was tacitly assumed to be knowledge about the environment, about the world in which they found themselves living. It is not surprising that this should be the case. Once one has learned to manage things, there is no reason to suspect that they might not be what they seem.

If a person whose knowledge has been growing and expanding over the years then raises questions about how one comes to have all that knowledge, it seems reasonable to postulate at the beginning an inexperienced and totally ignorant knower, who comes into the world, much as an explorer might come into a terra incognita, with both the need and the will to discover what that world is like. The first if not the only tools that seem to be available for such a task are obviously the senses. Therefore, the senses are at once categorized as organs, or channels, through which the experiencer receives messages from the environment. On the basis of these messages, the experiencer then must, and apparently can, build up a ‘picture’ of the world. In our contemporary jargon, this is often expressed by saying that the senses convey information which enables the experiencing subject to form a representation of the world. Usually this seems to work quite well. Occasionally, of course, the senses turn out to be somewhat deceptive, but by and large they work well enough for us to build up a *modus vivendi*. Provided we remain patient and flexible, we will continue to make adjustments, and as long as things work moderately well, there will be no need to question the over-all validity of whatever picture of the world we have built up.

The pre-Socratics started out in this thoroughly normal way, but because there were some highly original thinkers among them, they came up with mutually incompatible pictures of the world.¹ Obviously, that was felt to be a problem and it led to two closely connected questions: One, how could anyone compose a picture of the world out of sensory messages and, two, how could one be certain that a particular picture of the world was ‘true’? Attempts to answer these questions soon ran into troubles, some of them so serious that they have not yet been overcome.

Here I want to focus on the second problem because it is inherent and unavoidable in the discoverer’s scenario. If experience is the only contact a knower can have with the world, there is no way of comparing the products of experience with the reality from which whatever messages we receive are supposed to emanate. The question, how veridical the acquired knowledge might be, can therefore not be answered. To answer it, one would have to compare what one knows with what exists in the ‘real’ world—and to do that, one would have to know what ‘exists’. The paradox, then, is this: to assess the truth of your knowledge you would have to know what you come to know before you come to know it.

The argument that the likeness or trustworthiness of a picture can be assessed only by looking at both the picture and what it is supposed to depict, was brought forth already at the time of the pre-Socratics and it has been the mainstay of all

alternative scenario for the pursuit of scientific knowledge. In his preface to Copernicus' treatise *De revolutionibus*, Osiander (1627) suggested:

There is no need for these hypotheses to be true or even to be at all like the truth; rather, one thing is sufficient for them—that they yield calculations which agree with the observations. (in Popper, 1968, p. 98)

This introduces the notion of a second kind of knowledge, apart from faith and dogma, a knowledge that fits observations. It is knowledge that human reason derives from experience. It does not represent a picture of the 'real' world but provides structure and organization to experience. As such it has an all-important function: it enables us to solve experiential problems.

In Descartes' time, this instrumentalist theory of knowledge was formulated and developed by Mersenne and Gassendi.² It was then extended by Berkeley and Vico, given strong but unintended support by Hume and Kant; and at the end of the last century, it was applied to physics and science in general by Ernst Mach and to philosophy by Georg Simmel.³ It was not and still is not a theory popular with traditional philosophers. The idea that knowledge is good knowledge if and when it solves our problems is not acceptable as criterion to those who continue to hope that knowledge, ultimately, will at least approximate a true picture of the 'real' world.

Karl Popper (1968, chapter 3), who has given a lucid account of the beginnings of instrumentalism, has struggled hard to convince us that, though reasonable, it is an unsatisfactory theory. As he reiterates in his latest work:

What we are seeking in sciences are true theories—true statements, true descriptions of certain structural properties of the world we live in. These theories or systems of statements may have their instrumental use; yet what we are seeking in science is not so much usefulness as truth; approximations to truth; explanatory power, and the power of solving problems: and thus, understanding. (1982, p. 42)

This suggests that 'descriptions', 'explanations', and 'understanding' can indeed capture aspects of 'the world we live in'. Whether we can or cannot agree with this statement will depend on how we define 'the world we live in'. There is no doubt that Popper intended an objective world, that is, a ready-made world into which we are born and which, as explorers, we are supposed to get to know. This is the traditional realist view, and Popper does his best to defend it, in spite of all arguments one can hold against it. The realists and the sceptics are once more in the familiar deadlock.

Yet, there is another possibility. 'The world we live in' can be understood also as the world of our experience, the world as we see, hear, and feel it. This world does not consist of 'objective facts' or 'things-in-themselves' but of such invariants and constancies as we are able to compute on the basis of our individual experience. To adopt this reading, however, is tantamount to adopting a radically different scenario for the activity of knowing. From an explorer who is condemned to seek 'structural properties' of an inaccessible reality, the experiencing organism

now turns into a builder of cognitive structures intended to solve such problems as the organism perceives or conceives. Fifty years ago, Piaget (1937) characterized this scenario as neatly as one could wish: "Intelligence organizes the world by organizing itself" (p. 311). What determines the value of the conceptual structures is their experiential adequacy, their goodness of fit with experience, their viability as means for the solving of problems, among which is, of course, the never-ending problem of consistent organization that we call understanding.

The world we live in, from the vantage point of this new perspective, is always and necessarily the world as we conceptualize it. 'Facts', as Vico saw long ago, are made by us and our way of experiencing, rather than given by an independently existing objective world. But that does not mean that we can make them as we like. They are viable facts as long they do not clash with experience, as long as they remain tenable in the sense that they continue to do what we expect them to do.

This view of knowledge, clearly, has serious consequences for our conceptualization of teaching and learning. Above all, it will shift the emphasis from the student's 'correct' replication of what the teacher does, to the student's successful organization of his or her own experience. But before I expand on that I want to examine the widespread notion that knowledge is a commodity that can be communicated.

COMMUNICATION AND THE SUBJECTIVITY OF MEANING

The way we usually think of 'meaning' is conditioned by centuries of written language. We are inclined to think of the meaning of words in a text rather than of the meaning a speaker intends when he or she is uttering linguistic sounds. Written language and printed texts have a physical persistence. They lie on our desks or can be taken from shelves, they can be handled and read. When we understand what we read, we gain the impression that we have 'grasped' the meaning of the printed words, and we come to believe that this meaning was in the words and that we extracted it like kernels out of their shells. We may even say that a particular meaning is the 'content' of a word or of a text. This notion of words as containers in which the writer or speaker 'conveys' meaning to readers or listeners is extraordinarily strong and seems so natural that we are reluctant to question it. Yet, it is a misguided notion. To see this, we have to retrace our own steps and review how the meaning of words was acquired at the beginning of our linguistic career.

In order to attach any meaning to a word, a child must, first of all, learn to isolate that particular word as a recurrent sound pattern among the totality of available sensory signals. Next, she must isolate something else in her experiential field, something that recurs more or less regularly in conjunction with that sound pattern. Take an ordinary and relatively unproblematic word such as 'apple'. Let us assume that a child has come to recognize it as a recurrent item in her auditory experience. Let us further assume that the child already has a hunch that 'apple' is the kind of sound pattern that should be associated with some other experiential item. Adults interested in the child's linguistic progress can, of course, help in that process of association by specific actions and reactions, and they will consider their

'teaching' successful when the child has come to isolate in her experiential field something that enables her to respond in a way which they consider appropriate. When this has been achieved, when the appropriate association has been formed, there is yet another step the child must make before she can be said to have acquired the meaning of the word 'apple'. The child must learn to re-present to herself the designated compound of experiences whenever the word is uttered, even when none of the elements of that compound are actually present in her experiential field. That is to say, the child must acquire the ability to imagine or visualize, for instance, what she has associated with the word 'apple' whenever she hears the sound pattern of that word.⁴

This analysis, detailed though it may seem, is still nothing but a gross summary of certain indispensable steps in a long procedure of interactions. In the present context, however, it should suffice to justify the conclusion that the compound of experiential elements that constitutes the concept an individual has associated with a word cannot be anything but a compound of abstractions from that individual's own experience. For each one of us, then, the meaning of the word 'apple' is an abstraction which he or she has made individually from whatever apple-experiences he or she has had in the past. That is to say, it is subjective in origin and resides in a subject's head, not in the word which, because of an association, has the power to call up, in each of us, our own subjective representation.

If you grant this inherent subjectivity of concepts and, therefore, of meaning, you are immediately up against a serious problem. If the meanings of words are, indeed, our own subjective construction, how can we possibly communicate? How could anyone be confident that the representations called up in the mind of the listener are at all like the representations the speaker had in mind when he or she uttered the particular words? This question goes to the very heart of the problem of communication. Unfortunately, the general conception of communication was derived from and shaped by the notion of words as containers of meaning. If that notion is inadequate, so must be the general conception of communication.

The trouble stems from the mistaken assumption that, in order to communicate, the representations associated with the words that are used must be the same for all communicators. For communication to be considered satisfactory and to lead to what we call 'understanding', it is quite sufficient that the communicators' representations be compatible in the sense that they do not manifestly clash with the situational context or the speaker's expectations.

A simple example may help to make this clear. Let us assume that, for the first time, Jimmy hears the word 'mermaid'. He asks what it means and is told that a mermaid is a creature with a woman's head and torso and the tail of a fish. Jimmy need not have met such a creature in actual experience to imagine her. He can construct a representation out of familiar elements, provided he is somewhat familiar with and has established associations to 'woman', 'fish', and the other words used in the explanation. However, if Jimmy is not told that in mermaids the fish's tail replaces the woman's legs, he may construct a composite that is a fish-tailed biped and, therefore, rather unlike the intended creature of the seas.

Jimmy might then read stories about mermaids and take part in conversations about them for quite some time without having to adjust his image. In fact, his deviant notion of a mermaid's physique could be corrected only if he got into a situation where the image of a creature with legs as well as a fish's tail comes into explicit conflict with a picture or with what speakers of the language say about mermaids. That is, Jimmy would modify the concept that is subjective meaning of the word only if some context forced him to do so.

How, you may ask, can a context force one to modify one's concepts? The question must be answered not only in a theory of communication but also in a theory of knowledge. The answer I am proposing is essentially the same in both.

The basic assumption is one that is familiar to you. Organisms live in a world of constraints. In order to survive, they must be 'adapted' or, as I prefer to say, 'viable'. This means that they must be able to manage their living within the constraints of the world in which they live. This is a commonplace in the context of biology and evolution. In my view, the principle applies also to cognition—with one important difference. On the biological level, we are concerned with species, that is, with collections of organisms which, individually, cannot modify their biological make-up. But since they are not all the same, the species 'adapts' simply because all those individuals that are not viable are eliminated and do not reproduce. On the cognitive level, we are concerned with individuals and specifically with their 'knowledge' which, fortunately, is not immutable and only rarely fatal. The cognitive organism tries to make sense of experience in order better to avoid clashing with the world's constraints. It can actively modify ways and means to achieve greater viability.

'To make sense' is the same activity and involves the same presuppositions whether the stuff we want to make sense of is experience in general or the particular kind of experience we call communication. The procedure is the same but the motivation, the reason why we want to make sense, may be different.

Let me begin with ordinary experience. No matter how one characterizes cognizing organisms, one of their salient features is that they are capable of learning. Basically, to have 'learned' means to have drawn conclusions from experience and to act accordingly. To act accordingly, of course, implies that there are certain experiences which one would like to repeat rather than others which one would like to avoid. The expectation that any such control over experience can be gained must be founded on the assumptions that: (1) some regularities can be detected in the experiential sequence; and (2) future experience will, at least to some extent, conform to these regularities. These assumptions, as David Hume showed, are prerequisites for the inductive process and the knowledge that results from it.

In order to find regularities, we must segment our experience into separate pieces so that, after certain operations of recall and comparison, we may say of some of them that they recur. The segmenting and recalling, the assessing of similarities, and the decisions as to what is to be considered different, are all our doing. Yet, whenever some particular result of these activities turns out to be useful (in generating desirable or avoiding undesirable experiences), we quickly forget

that we could have segmented, considered, and assessed otherwise. When a scheme has worked several times, we come to believe, as Piaget has remarked, that it could not be otherwise and that we have discovered something about the real world. Actually we have merely found one viable way of organizing our experience. 'To make sense' of a given collection of experiences, then, means to have organized them in a way that permits us to make more or less reliable predictions. In fact, it is almost universally the case that we interpret experience either in view of expectations or with a view to making predictions about experiences that are to come.

In contrast, 'to make sense' of a piece of language does not usually involve the prediction of future non-linguistic experience. However, it does involve the forming of expectations concerning the remainder of the piece that we have not yet heard or read. These expectations concern words and concepts, not actions or other experiential events. The piece of language may, of course, be intended to express a prediction, for example, 'tomorrow it will rain', but the way in which that prediction is derived from the piece of language differs from the way in which it might be derived from, say, the observation of particular clouds in the sky. The difference comes out clearly when it is pointed out that, in order to make sense of the utterance 'tomorrow it will rain' it is quite irrelevant whether or not there is any belief in the likelihood of rain. To 'understand' the utterance it is sufficient that we come up with a conceptual structure which, given our past experience with words and the way they are combined, fits the piece of language in hand. The fact that, when tomorrow comes, it doesn't rain, in no way invalidates the interpretation of the utterance. On the other hand, if the prediction made from an observation of the sky is not confirmed by actual rain, we have to conclude that there was something wrong with our interpretation of the clouds.

In spite of this difference between the interpretation of experience and the interpretation of language, the two have one important feature in common. Both rely on the use of conceptual material which the interpreter must already have. 'Making sense', in both cases, means finding a way of fitting available conceptual elements into a pattern that is circumscribed by specific constraints. In the one case, the constraints are inherent in the way in which we have come to segment and organize experience; in the other, the constraints are inherent in the way in which we have learned to use language. In neither case is it a matter of matching an original. If our interpretation of experience allows us to achieve our purpose, we are quite satisfied that we 'know'; and if our interpretation of a communication is not countermanded by anything the communicator says or does, we are quite satisfied that we have 'understood'.

The process of understanding in the context of communication is analogous to the process of coming to know in the context of experience. In both cases, it is a matter of building up, out of available elements, conceptual structures that fit into such space as is left unencumbered by constraints. Though this is, of course, a spatial metaphor, it illuminates the essential character of the notion of viability and it brings out another aspect that differentiates that notion from the traditional one of 'truth'; having constructed a viable path of action, a viable solution to an

experiential problem, or a viable interpretation of a piece language, there is never any reason to believe that this construction is the only one possible.

THE CONSTRUCTION OF VIABLE KNOWLEDGE

When I began the section on communication by talking about the concept of meaning, it must have become apparent that I am not a behaviorist. For about half a century behaviorists have worked hard to do away with ‘mentalist’ notions such as meaning, representation, and thought. It is up to future historians to assess just how much damage this mindless fashion has wrought. Where education is concerned, the damage was formidable. Since behaviorism is by no means extinct, damage continues to be done, and it is done in many ways. One common root, however, is the presumption that all that matters—perhaps even all there is—are observable stimuli and observable responses. This presumption has been appallingly successful in wiping out the distinction between training and education.

As I hope to have shown in the preceding section, a child must learn more than just to respond ‘apple’ to instantiations of actual apple experiences. If that were all she could do, her linguistic proficiency would remain equivalent to that of a well-trained parrot. For the bird and its trainer to have come so far is a remarkable achievement. For a human child it is a starting point in the development of self-regulation, awareness, and rational control.

As mathematics educators, you know this better than most. To give correct answers to questions within the range of the multiplication table is no doubt a useful accomplishment, but it is, in itself, no demonstration of mathematical knowledge. Mathematical knowledge cannot be reduced to a stock of retrievable ‘facts’ but concerns the ability to compute new results. To use Piaget’s terms, it is operative rather than figurative. It is the product of reflection and while reflection as such is not observable, its products may be inferred from observable responses.

I am using ‘reflection’ in the sense in which it was originally introduced by Locke, that is, for the ability of the mind to observe its own operations. Operative knowledge, therefore, is not associative retrieval of a particular answer but rather knowledge of what to do in order to produce an answer. Operative knowledge is constructive and, consequently, is best demonstrated in situations where something new is generated, something that was not already available to the operator. The novelty that matters is, of course, novelty from the subject’s point of view. An observer, experimenter, or teacher can infer this subjective novelty, not from the correctness of a response but from the struggle that led to it. It is not the particular response that matters but the way in which it was arrive at.

In the preceding pages, I have several times used the term ‘interpretation’. I have done it deliberately, because it focuses attention on an activity that requires awareness and deliberate choice. Although all the material that is used in the process of interpreting may have been shaped and prepared by prior interaction with experiential things and with people, and although the validation of any particular interpretation does, as we have seen, require further interaction, the process of interpreting itself requires reflection. If an organism does no more than

act and react, it would be misusing the word to say that the organism is interpreting. Interpretation implies awareness of more than one possibility, deliberation, and rationally controlled choice.

A student's ability to carry out certain activities is never more than part of what we call 'competence'. The other part is the ability to monitor the activities. To do the right thing is not enough; to be competent, one must also know what one is doing and why it is right. This is perhaps the most stringent reason why longitudinal observation and Piaget's clinical method are indispensable if we want to find out anything about the reflective thought of children, about their operative knowledge, and about how to teach them to make progress towards competence.

At the beginning of this talk, I mentioned that a useful analogy might be found in the teaching of athletic skills. What I was alluding to are the recently developed methods that make it possible for athletes to see what they are doing. Some of these methods involve tachistoscopes and are very sophisticated, others are as simple as the slow-motion replay of movies and videotapes. Their purpose is to give performers of intricate actions an opportunity to observe themselves act. This visual feedback is a far more powerful didactic tool than instructions that refer to details of the action which, normally, are dimly or not at all perceived by the actors themselves.

The proficiency of good athletes springs, to a large extent, from the fact that they have, as it were, automated much of their action. As long as their way of acting is actually the most effective for the purpose, this automation is an advantage because it frees the conscious mind to focus on higher levels of control. When, however, something must be changed in the routine, this would be difficult, if not impossible, to achieve without awareness of the individual steps. Hence, the efficacy of visual feedback.

Although the speed of execution that comes with automation may be a characteristic of the expert calculator, the primary goal of mathematics instruction has to be the student's conscious understanding of what he or she is doing and why it is being done. This understanding is not unlike the self-awareness the athlete must acquire in order consciously to make an improvement in his physical routine. Unfortunately, we have no tachistoscope or camera that could capture the dynamics, the detailed progression of steps, of the mental operations that lead to the solution of a numerical problem. Yet, what the mathematics teacher is striving to instill into the student is ultimately the awareness of a dynamic program and its execution—and that awareness is in principle similar to what the athlete is able to glean from a slow-motion representation of his or her own performance. In the absence of any such technology to create self-reflection, the teacher must find other means to foster operative awareness. At the present state of the art, the method of the 'teaching experiment' developed by Steffe (1977) seems to be the most hopeful step in that direction.

The term 'teaching experiment' could easily be misunderstood. It is not intended to indicate an experiment in teaching an accepted way of operating, as for instance, the adult's way of adding and subtracting. Instead, it is primarily an exploratory tool, derived from Piaget's clinical interview and aimed at discovering what goes

child's head. In the one case as in the other, the best that can be achieved is a model that remains viable within the range of available experience.

The teaching experiment, as I suggested before, is, however, something more than a clinical interview. Whereas the interview aims at establishing 'where the child is', the experiment aims at ways and means of 'getting the child on'. Having generated a viable model of the child's present concepts and operations, the experimenter hypothesizes pathways to guide the child's conceptualizations towards adult competence. In order to formulate any such hypothetical path, let alone implement it, the experimenter/teacher must not only have a model of the student's present conceptual structures, but also an analytical model of the adult conceptualizations towards which his guidance is to lead.

The structure of mathematical concepts is still largely obscure.⁵ This may seem a strange complaint, given the amount of work that has been done in the last 100 years to clarify and articulate the foundations of mathematics. As a result of that work, there is no shortage of definitions, but these definitions, for the most part, are formal rather than conceptual. That is, they merely substitute other signs or symbols for the definiendum. Rarely, if ever, is there a hint, let alone an indication, of what one must do in order to build up the conceptual structures that are to be associated with the symbols. Yet, that is, of course, what a child has to find out if it is to acquire a new concept.

Let me give you an example. A current definition of number, in the sense of 'positive integer' says that it is "a symbol associated with a set and with all other sets which can be put into one-to-one correspondence with this set" (James & James, 1968, p. 193). The mention of 'put' makes it sound like an instruction to act, a directive for construction, which is what it ought to be. But, in order to begin that construction, the student would have to have a clear understanding of the 'set' and, more important still, of 'one'. Such understanding can be achieved only by reflecting on the operations of one's own mind and the realization that with these operations one can create units and sets anywhere and at any time, irrespective of any sensory signals. That means that, rather than speak of 'sets' and 'mathematical objects' as though they had an independent existence in some 'objective' reality, teachers would have to foster the students' reflective awareness of his or her mental operations, because it is only from these that the required concepts can be abstracted.

The teaching experiment, at any rate, presupposes an explicit model of adult functioning. The experimental part of the method then consists in a form of 'indirect guidance' aimed at modifying the child's present concepts and operations (which the experimenter 'knows' in terms of the model constructed on the basis of observing the particular child) towards the adult concepts and operations (which the experimenter 'knows' in terms of the model constructed on the basis of analyzing the adult procedures). Because the child necessarily interprets verbal instructions in terms of her own experience, the 'guidance' must take the form either of questions or of changes in the experiential field that lead the child into situations where her present way of operating runs into obstacles or contradictions. Analogous to the adult who organizes general experience, the child is unlikely to

modify a conceptual structure unless there is an experience of failure or, at least, surprise at something not working out in the expected fashion. Such failure or surprise, however, can be experienced only if there was an expectation—and that brings me to the last point I want to make.

If I have had any success at all in presenting the constructivist epistemology as a possible basis for education and educational research, this last point will be easy to make and its importance should become obvious.

The more abstract the concepts and operations that are to be constituted, the more reflective activity will be needed. Reflection, however, does not happen without effort. The concepts and operations involved in mathematics are not merely abstractions, but most of them are the product of several levels of abstraction. Hence, it is not just one act of reflection that is needed, but a succession of reflective efforts—and any succession of efforts requires solid motivation.

The need for motivation is certainly no news to anyone who has been teaching. How to foster motivation has been discussed for a long time. But here, again, I believe, the effect of behaviorism has been profoundly detrimental. The basic dogma of behaviorism merely says that behavior is determined by the consequences it has produced in the past (which is just another way of saying that organisms operate inductively). There is every reason to agree with that. The trouble arises from the usual interpretation of ‘reinforcement’, that is, of the consequence that is rewarding and thus strengthens specific behaviors and increases the probability of their recurrence.

There is the wide-spread misconception that reinforcement is the effect of certain well-known commodities, such as cookies, money, and social approval. It is a misconception, not because organisms will not work quite hard to obtain these commodities, but because it obscures the one thing that is often by far the most reinforcing for a cognitive organism: to achieve a satisfactory organization, a viable way of dealing with some sector of experience. This fact adds a different dimension to the conception of reinforcement because whatever constitutes the rewarding consequence in these cases is generated wholly within the organism’s own system.

Self-generated reinforcement has an enormous potential in cognitive, reflective organisms. (All of us, I am sure, have spent precious time and sweat on puzzles whose solution brought neither cookies, nor money, and negligible social approval.) But this potential has to be developed and realized.

When children begin to play with wooden blocks, they sooner or later place one upon another. Whatever satisfaction they derive from the resulting structure, it provides sufficient incentive for them to repeat the act and to improve on it. They may, for instance, implicitly or explicitly set themselves the goal of building a tower that comprises all the blocks. If they succeed, they are manifestly satisfied, irrespective of tangible rewards or an adult’s comment, for they build towers also in the absence of observers. The reward springs from the achievement, from the successful, deliberate imposition of an order that is inherent in their own way of organizing. To repeat the feat, the tower has to be knocked down. That, too, turns

out to be a source of satisfaction because it once more provides evidence of the experiencer's power over the structure of experience.

To some these observations may seem trivial. To me, they exemplify a basic feature of the model of the cognitive organism, a feature that must be taken into account if we want to educate.

From the constructivist point of view, it makes no sense to assume that any powerful cognitive satisfaction springs from simply being told that one has done something right, as long as 'rightness' is assessed by someone else. To become a source of real satisfaction, 'rightness' must be seen as the fit with an order one has established oneself. Teachers, as well as mathematicians, tend to assume that there exists in every particular case an objective problem and an objectively 'true' solution. Children and students of any age are, therefore, expected somehow to 'see' the problem, its solution, and the necessity that links the two. But the necessity is conceptual and it can spring from nothing but the awareness of the structures and operations involved in the thinking subject's conceptualization of the problem and its solution. Logical or mathematical necessity does not reside in any independent world—to see it and gain satisfaction from it, one must reflect on one's own constructs and the way in which one has put them together.

FINAL REMARKS

Educators share the goal of generating knowledge in their students. However, from the epistemological perspective I have outlined, it appears that knowledge is not a transferable commodity and communication not a conveyance.

If, then, we come to see knowledge and competence as products of the individual's conceptual organization of the individual's experience, the teacher's role will no longer be to dispense 'truth', but rather to help and guide the student in the conceptual organization of certain areas of experience. Two things are required for the teacher to do this: on the one hand, an adequate idea of where the student is and, on the other, an adequate idea of the destination. Neither is accessible to direct observation. What the student says and does can be interpreted in terms of a hypothetical model—and this is one area of educational research that every good teacher since Socrates has done intuitively. Today, we are a good deal closer to providing the teacher with a set of relatively reliable diagnostic tools.

As for the helping and guiding, good teachers have always found ways and means of doing it because, consciously or unconsciously, they realized that, while one can point the way with words and symbols, it is the student who has to do the conceptualizing and the operating.

That leaves the destination, the way of operating that would be considered 'right' from the expert's point of view. As I have mentioned earlier, a conceptual model of the formation of the structures and the operations that constitute mathematical competence is essential because it, alone, could indicate the direction in which the student is to be guided. The kind of analysis, however, that would yield a step by step path for the construction of mathematical concepts has barely begun. It is in this area that, in my view, research could make advances that would

immediately benefit educational practice. If the goal of the teacher's guidance is to generate understanding, rather than train specific performance, his task will clearly be greatly facilitated if that goal can be represented by an explicit model of the concepts and operations that we assume to be the operative source of mathematical competence. More important still, if students are to taste something of the mathematician's satisfaction in doing mathematics, they cannot be expected to find it in whatever rewards they might be given for their performance but only through becoming aware of the neatness of fit they have achieved in their own conceptual construction.

NOTES

- ¹ For one, Parmenides, the real world was an indivisible, static whole, for others, Leucippus and Democritus, the real world was a mass of constantly moving atoms.
- ² An excellent exposition can be found in Popkin (1979).
- ³ Berkeley has been interpreted in this way by Popper (1968, chapters 3 and 6). Vico's instrumentalism is explicit in his *De Antiquissima Italorum Sapientia* of 1710. Hume's analysis of induction is instrumentalist and Kant's program of 'transcendental inquiry' into the workings of reason in *Critique* supplied powerful ammunition to the modern instrumentalists; cf. Mach (1910) in Toulmin (1970) and Simmel (1910).
- ⁴ If this isolating of the named thing or 'referent' is a demanding task with relatively simple perceptual compounds, such as apple, it is obviously much more difficult when the meaning of the word is a concept that requires further abstraction from sensory experience or from mental operations. But since we want to maintain that words such as 'all' and 'some', 'mine' and 'yours', 'cause' and 'effect', 'space' and 'time', and scores of others have meaning, we must assume that these meanings, though they cannot be directly perceived, are nevertheless somehow isolated and made retrievable by every learner of the language.
- ⁵ For recent conceptual analyses see Steffe, Glaserfeld, Richards, and Cobb (1983).

RECONSTRUCTING THE CONCEPT OF KNOWLEDGE

Before I begin to pick apart and reconstruct the concept of knowledge, I want to express my profound gratitude for having been given this opportunity. As some of you know, my acquaintance with Piaget is relatively recent and was, to my lasting regret, never a personal one. Although I had worked with Silvio Ceccato for nearly two decades during which his name figured among the editorial committee of the *Études d'épistémologie génétique*, it was not until I moved to the United States that I began to read the Patron's works. The person who at last introduced me to the teachings of the Geneva School was Charles Smock. He died last year, and I want to remember him here and thank him for the gentleness and the wisdom with which he introduced me to a way of thinking which, though not alien in principle, opened up pathways I would never have found on my own. If I have been able to consolidate a constructivist theory of knowledge, it unquestionably owes a great deal to him and to his deep, intuitive understanding of Piaget.

Sinclair (this meeting) remarked that the number of people who call themselves 'constructivists' has swollen in recent years and that the meaning of the term seems to have been diluted. This is precisely the point with which I want to begin my presentation.

TRIVIAL CONSTRUCTIVISM

Once a cognizing subject posits itself in an independently 'existing' world and assumes the task of getting to 'know' that world, that subject must sooner or later embark on some 'theory of knowledge'. Given the irreconcilable dichotomy in the assumptions that constitute that starting point, any theory of knowledge that aspires to being more scientific than a myth, will have to come up with an explanation of the cognizing activity. Typically, that is what 'cognitive scientists' have been attempting for the last couple of decades. Because it seems clear that the world cannot enter into a cognitive organism's domain all in one piece, most cognitive scientists today would agree that knowledge has to be 'constructed'. By and large, therefore, cognitive scientists have set themselves apart from the behaviorist tradition by assuming an organism that actively constructs rather than one that passively receives everything ready-made from its environment (cf. Resnick, 1983).

A drastic step away from environmental determinism would have been a great step forward. As it happened, however, many who adopted the conception of an actively constructing organism did not and do not pursue the implications such a change of view has for the commodity that goes under the name of 'knowledge'. In fact, these new 'constructivists' do not push much further than John Locke who,

three hundred years ago, stressed the importance of the operations of a mind that produces complex ideas. But less conscientious than he, the present-day empiricists don't seem to acknowledge what the sceptics have been telling us ever since the pre-Socratics: namely that there is no logically feasible way of checking the 'objective truth' of knowledge if the object of that knowledge is accessible only through yet another act of knowing. From an epistemological point of view, therefore, much of what now claims to be 'constructivism' is trivial.

A genuinely constructivist theory of knowledge must deal not only with the process of cognitive construction but also with the relationship which the results of any such construction might have to the reality of the traditionally presumed ontological world. That relationship, I believe, is the very point in which radical constructivism is revolutionary.

Although professional philosophers have made many imaginative attempts to overcome the impasse uncovered by the sceptics, Western theories of knowledge are still more or less where they started. Descartes' sustained efforts were unsuccessful and ultimately shrivelled to the pious hope that God could not have been so malicious as to provide his favorite creation with untrustworthy sensory organs. And Popper (1982, p. 42), to cite a contemporary example, tells us that even if a sceptical, instrumentalist view of knowledge might make very good sense, it would not be 'right' if scientists adopted it, because the quest for objective truth must not be given up. Thus, for better or for worse, philosophers cling to the notion that knowledge should be considered *good* knowledge only if one can make oneself believe that it furnishes a reasonably 'true' picture of the world which their chosen ontology posits prior to any human act of knowing.

THE ROOTS OF THE RADICAL VIEW

There may be several reasons for the philosophers' persistence. The most important among them is, I think, a psychological one which the Genevan School has long ago defined as 'egocentricity'. It was, as Piaget pointed out, very difficult for mankind to relinquish the presumption that our planet should be the center of the universe. We may now add: giving up the notion that our knowledge will eventually enable us to 'see' the world as it is, requires an even greater effort of decentration.

It is surely no accident that the first suggestion to consider rational scientific knowledge a fallible instrument of prediction rather than a description of the 'real' world, was made by people who firmly believed that they had access to quite another source of infallible knowledge. Osiander, for instance, in his introduction to Copernicus' *De revolutionibus*, pointed out that scientific hypotheses did not have to be 'like the truth' all that may be required of them is that they allow us to calculate results which agree with observations (cf. Popper, 1968, p. 98).

The thing to remember in this context is that observations are made by an experiencing subject and therefore depend on that subject's ways of perceiving and conceiving. Observation isolates elements of experience, not elements of an objective world. We cannot alter this, even if we call these initial elements 'sense

data’—for whatever these data are, they are given only because a subject isolated them in its subjective experience.

In the same vein as Osiander, Cardinal Bellarmino, who was concerned about Galileo’s trial, suggested an eminently reasonable way to prevent science from becoming heresy. Galileo, he said, should take care to state his ideas in the form of hypotheses rather than as descriptions of God’s ‘real’ world:

To say that we give a better account of the appearances by supposing the earth to be moving, and the sun at rest, than we could if we used eccentricities and epicycles, is to speak properly: there is no danger in that, and it is all the mathematician requires. (in Popper, 1968, p. 98)

The ‘danger’ Bellarmino had in mind was, of course, the seemingly inevitable conflict of Galileo’s findings with the dogma of the Church which considered itself the repository of unquestionable truth. His suggestion was intended to safeguard the Vatican’s prerogative rather than the soundness of a physicist’s epistemological beliefs. A radical constructivist, on the other hand, being under no obligation to defend a Scripture or divine revelation, would make the same suggestion to any scientist, simply because what a scientist finds or concludes is under all circumstances determined by the way in which she sees, the way in which she observes, and the way in which she conceptually relates the elements she carves out of her experience.

When the wave of Pyrrhonist scepticism swept through France in the first half of the 17th century, Mersenne, Gassendi, and others elaborated the view that science deals with the phenomenal, the world of ‘appearances’, and that unquestionable truth could be attained only through faith and revelation.¹ Hand in hand with this relativization of rational, scientific knowledge came the notion that the practical, instrumental value of that knowledge was its sufficient justification.

VICO, THE FORGOTTEN PIONEER

The first to state quite bluntly that all rational knowledge was the result of the knower’s own construction was Giambattista Vico. At the very beginning of his widely ignored treatise on epistemology (1710), he announces what was to remain the guiding principle of his future work: ‘*Verum est ipsum factum*’ (to be true is one and the same as to be made).

Vico’s introductory paragraph contains two further statements of great relevance. The first expands the notion of ‘*verum/factum*’ to the notion of ‘science’ as the knowledge of how things originate, that is, of how they are put together or constructed. The second points out the limitations. Man cannot know the things that are in the world because their component elements lie outside man’s mind, and man, therefore has no access to them and cannot use them to build up true knowledge. This incapacity ‘to investigate the nature of things (of the world)’ is stressed a few pages later, in contrast to mathematics and geometry, whose objects man can investigate and know because all their components, as well as the

problems and theorems arising from them, are *operatione opus*, which is to say, they are the product of man's own mental activity.²

I cannot leave Vico's *De antiquissima* without quoting two of his most inspiring formulations. The one was used as motto by Silvio Ceccato, 30 years ago, when he wrote about his model of a perceiving machine (cf. Ceccato, 1962), and it crystalizes the view that, while God may know the world as it is, because He made it, man can know only what he constructs out of elements of his own experience: 'God is the artificer of Nature, man the god of artifacts' (1710, VII, §3). The other gem from Vico places him alongside today's most advanced philosophers of science: "Human science, thus, is no more and no less than an effort to bring things into pleasing relations with one another" (ibid.).

THE EVOLUTIONARY COMPONENT

To say that Vico's theory of knowledge was unpopular would be an understatement. It remained practically unnoticed. The 'decentering' it required was too forbidding. Besides, it left one of its flanks quite unsupported. While it would probably have been heresy to question *why* God should have decided to create the world, Vico's constructivism at once raised the more germane question why every human individual should strive to acquire and to perfect knowledge. The aesthetic notion of constructing pleasing relations among experiential elements and compositions did not and does not seem to provide sufficient motivation.

As far as I can see, it was Darwin's theory of evolution that made available the model of a mechanism that could successfully be applied to the generation of knowledge. That no one was eager to do that, is probably due to the fact that professional philosophers had for a long time condemned any attempt to consider the problems of knowledge from a developmental or evolutionary point of view. They had nothing but scorn for what they called the 'genetic fallacy'. A theory that proposed random variations and natural selection as an explanation of genesis must, therefore, have seemed far out of bounds for epistemologists.

Another stumbling block was the wide-spread misunderstanding of the concept of 'adaptation'. Given the fundamental realism that still pervaded scientific thinking, 'adaptation' was inevitably interpreted as a process of adjustment or adequation to the 'real' world. Thus, the protective coloring of certain living organisms was and still is often given as a prime example of that process, and it supports and strengthens the belief that 'adaptation' means to become more *like* the 'real' world of the environment.

A change of mind, initiated in Spencer and Baldwin, brings the concept back to what it was for Darwin (at least in some of his most revealing passages). To be 'adapted' now means no more and no less than that the organism or species in question has found *one* possible way of surviving and procreating within the constraints set by its present environment.

To my knowledge, Georg Simmel was the first who uncompromisingly applied this notion of adaptation to the theory of knowledge:

The dualism between the world as appearance, whose existence we posit logically and theoretically, and the world as the reality to which our practical acting responds, is eliminated once we realize that the forms of thinking that produce the world as an idea, are determined by the practical effects and counter effects which our mental constitution forms in a manner that is no different from the manner in which our physical constitution is formed by evolutionary necessities. (Simmel, 1895, p. 45)³

This is a tightly packed statement. To unravel it, one must first of all consider what Simmel intended when he spoke of ‘the forms of thinking that produce the world *as an idea*’. He used the German word *Vorstellung*, a word which, since English has no proper equivalent, is often translated as ‘representation’. ‘Representation’, however, is the proper translation of the German word *Darstellung*. Thus a confusion is created that happens to be of crucial importance in the context of epistemology. The second German term refers to an item that depicts something else, an item that has an iconic or isomorphic relation to an original. This is precisely what English speakers have in mind when they say ‘representation’, and it is very far from the meaning of *Vorstellung*. That word was used by German theorists of knowledge, such as Kant, Schopenhauer, and indeed Simmel. The item they were referring to, is not a replica of something else, but simply a product of the individual experienter’s imagination.

Die Welt als Vorstellung, therefore, is not the world as I depict it, but the world as I conceptualize it. The difference between the two expressions is enormous. The first is inextricably tied to realism and cannot but lead to an iconic notion of knowledge, such that ‘truth’ depends on the goodness of match with a presupposed ‘reality’ (a good representation, after all, must be a good ‘likeness’). The other term referring simply to one of many possible conceptions, does not imply that its value depends on the comparison with an original that must be replicated. Its value, instead, depends on its fit or viability. That is to say, what matters is whether or not the particular conceptualization functions satisfactorily in the context in which it arises.

Thus, Simmel says that our ‘forms of thinking that produce the world as an idea’ (i.e., the world as we imagine it to be) are no less the result of an evolutionary process than our physical constitution. That means that forms of thinking, like biological structures, are retained insofar as they turn out to be viable in the contexts of an organism’s activities.

This pushes instrumentalism a step further than it was with Bellarmino, Mersenne, or Vico. Not only ‘knowledge’, the result of cognitive activity, but ‘knowing’, the process of cognition itself, is now seen as instrumental. And that is not all. Instrumentalism could always be criticized for reducing the theory of knowledge to the lowly level of utilitarianism, a level too prosaic to be considered worthy of the true philosopher’s attention.⁴ Now, however, the instrumental theory of cognition was firmly embedded in the conceptual model of evolution and had thus become a matter of respectable science.

COGNITION AS AN ADAPTIVE FUNCTION

Although Piaget never tired of reiterating that cognition is an adaptive process, the deeper implication of his position has, I believe, been widely misunderstood. The reason is that most of his interpreters seemed to be unaware of how adaptation is supposed to work. Being realists of one kind or another, they understood 'adaptation' as the process of a structure becoming more and more *like* whatever it is adapting to. Applied to knowledge, this mistaken notion permits a comforting assimilation: adaptive cognition must lead to continuously improving representations of 'reality' and, consequently, there is no need to change one's conception of knowledge. Moreover, one can begin to think of this progressive improvement of representations as a kind of 'construction'—which constitutes the position that I have called *trivial* constructivism.

There are many places where Piaget indicated that his theory of cognition was different and that the results of cognitive activity could never be a replica of ontological structures. The most incisive is perhaps the passage in which he suggested that a cognizing organism could construct a relatively stable experiential world even if ontological reality were an everchanging flux:

... one sees no ground why it should be unreasonable to think that it is the ultimate nature of reality to be in permanent construction instead of consisting of an accumulation of ready-made structures. (Piaget, 1968, pp. 57-58)

Constructivism of the radical kind, then, does not require building blocks that are parts of ontological reality, but builds, as Vico suggested, with elements that are accessible to the knower, that is, elements found in, and belonging to, the knower's own experiential world. In that sense, the results of all cognitive construction are necessarily subjective. The moment this is said, the question arises, how, if at all, the notion of 'objectivity' could have meaning in a constructivist theory of knowledge.

Objective knowledge in the philosophers' sense was supposed to be knowledge of 'things-in-themselves', knowledge of what the world might be like before the subject has initiated its cognitive enterprise. That this constitutes an irreducible paradox has been repeated often enough by the sceptics. The meaning of 'objectivity' is, however, somewhat modified in the more general usage. An 'objective' view or judgement has come to characterize also a view or judgement of which one thinks that it is not biased by specific individual quirks or preferences and would therefore be shared by others under the same circumstances. The concept has thus been modified into something that could be called 'intersubjectivity'.

SUBJECT CREATES SUBJECTS

From the constructivist perspective, however, even the notion of intersubjectivity is problematic. The reason is simply that, given a theory of knowledge that claims the

subject's idea (*Vorstellung*) of the world to be the subject's own construction, it is not obvious how such an idea of the world should come to incorporate the notion of *Others* in the sense of *other cognizing organisms* who may construct their *own* 'idea of the world'.

Rather than circumvent this problem by the metaphysical assumption that 'other minds' *exist*, I propose to approach it by examining some of the steps in the cognizing subject's construction of his or her experiential reality. By 'experiential reality' I mean ordinary reality, which is, of course, the reality of relatively durable structures that we are able to establish, to maintain, and to *use* in the ordering and management of our actual experience.

It would be a monstrous presumption for me to think that I could tell *you*, here in Geneva, anything new about rhythms and regulations, circular reactions, and the construction of space, time, causality, and permanent objects. All this has been laid out in exemplary fashion by Piaget and his many brilliant collaborators. I shall gratefully take it for granted and start my crude, simplistic analysis of the construction of *Others* at the point where the cognizing organism begins to discriminate certain experiential objects which, eventually, will be considered 'alive'.

Having attributed the power of spontaneous movement, say, to a lizard, the child who would like to catch one will quickly come to the conclusion that her attempts would be more likely to succeed if, beyond the ability to move, the lizard were also thought of as being able to *see* and perhaps even to *hear*. On the basis of these presuppositions, one would have a better chance to avoid acting in ways that seem to cause the lizard to escape. In other words, the child's reality will soon be populated by experiential items to which the child attributes perceptual capabilities modelled after those she attributes to herself.

Some of these perceiving creatures—especially those with whom the learning and maturing subject frequently has occasion to interact—will require an even more sophisticated model than the lizard, and thus they will soon be imbued with other and more elaborate functions and mechanisms which the subject has rightly or wrongly abstracted from her own dealings with experience. That is to say, other creatures will come to be thought of as possessing cognitive structures and ways of operating that are similar to, but not identical, with the subject's own. Above all, these Others will be considered to have goals as well as ways and means of attaining them.⁵

Needless to say, the interaction with these Others will tend to be more and more of the kind that we call 'linguistic'. This kind of interaction is, in fact, based on the assumption of analogous operative, conceptual processes in the users of the particular language. Insofar as that assumption leads to successful interaction, it leads to the belief that the meanings of words are fixed and external, and this, in turn, reciprocally reinforces the notion that Others think and operate in ways that are analogous to one's own.

NOTES

- ¹ See the excellent survey given in Popkin, 1979.
- ² There is a remarkable but, as far as I know, totally accidental echo of Vico in Piaget's statement: "To know is to construct or to reconstruct the object of knowledge in such a way as to capture the mechanism of that construction" (1969, p. 356).
- ³ I owe the discovery of Georg Simmel's momentous essay to Donald Campbell, who referred to it in his outstanding review of 'evolutionary epistemology' (1974). I gratefully acknowledge Jacques Vonèche's mention, in his superb translation of my talk, of a reference to William James' paper, "Great Men, Great Thoughts, and the Environment" (1880), in which he expressed the basic idea of the evolution of knowledge.
- ⁴ The main reason why Vaihinger's remarkable *Philosophie des Als Ob*, published in 1913, did not have anything like the impact it could have had, was probably that it was presented as an 'instrumentalist philosophy' and not as an *evolutionary* theory of knowledge.
- ⁵ It should be obvious that a cognizing subject, in constructing a hypothetical model of another cognizing subject, cannot transcend its own conceptual limitations. 'Decentering' to take another's point of view, therefore, can only mean rearranging one's conceptual structures in a way in which one does not usually arrange them in one's own operating: no effort of decentering can draw on material one does not already possess in some form or fashion.
- ⁶ No one could say this more elegantly than Piaget did almost fifty years ago: "L'intelligence organise le monde en s'organisant elle-même" (1937, p. 311).

FACTS AND THE SELF FROM A CONSTRUCTIVIST POINT OF VIEW

Of what should absolute reality be independent? If you want it independent of humans, you should consider that it would then be useless for humans. (Ludwik Fleck, 1929, p. 429)

The title of a conference that begins with the phrase ‘The invention of facts...’ is bound to create a variety of reactions. For a constructivist, who has for a long time been engaged in the investigation and reconstruction of the concept of knowledge, the phrase expresses an epistemological attitude that is diametrically opposed to the philosophical heritage that has come down to us in the Western world. It also raises the awkward questions: What is a fact? What do we intend when we use the word ‘knowledge’? What follows is this constructivist’s current answer to these questions.

For some 2500 years the Western world has manifested an overwhelming tendency to think of knowledge as a cognitive organism’s representation of an outside world, its structure, and how it works. The representation might not yet be quite perfect, but, in principle, it was thought to be perfectible. In any case, its goodness was supposed to depend on the degree of correspondence between it and the outside world called ‘reality’. Today, this way of thinking is no longer viable.

THE PROBLEM OF REPRESENTATION

There are two points I want to make about this notion of representational knowledge, one logical, the other semantic. First, at the time of the pre-Socratics, when our epistemological tradition began, there already were some thinkers who held that any conception of knowledge that required correspondence to a ‘real’ world was illusory and useless. It was illusory, they argued, because there was no way of checking any such correspondence. These thinkers saw with admirable clarity that, in order to judge the goodness of a representation that is supposed to depict something else, one would have to compare it to what it is supposed to represent. In the case of ‘knowledge’ this would be impossible, because we have no access to that ‘real world’ except through further experience from which we abstract what we call ‘knowledge’; and this knowledge, by definition, would simply be another representation. In other words, we have no difficulty in comparing one representation with another to decide whether they are different, similar, or equivalent; but we cannot compare a representation with something it is supposed to depict, if that something is supposed to ‘exist’ in a real world that lies beyond our experiential interface.

The second, semantic point pertains to the word ‘representation’ and in particular how it has come to be used in English. Like many other words, it has different meanings. Speakers of the language usually handle ambiguity quite well; but in the case of the word ‘representation’ there is a peculiar difficulty: one of its ambiguities seems to have sprung, not from the word’s original use in English but from an unfortunate use that was made of it by translators of German philosophy. Perhaps already earlier, but certainly since Kant’s *Critique of pure reason* was translated, the two German words ‘Vorstellung’ and ‘Darstellung’ have been rendered in English by one and the same word ‘representation’. In epistemological contexts, this conflation is disastrous: although both the German words are used to refer to conceptual structures, they specify incompatible characteristics. The first, *Vorstellung*, indicates an autonomous internal construction, whereas the second, *Darstellung*, indicates a structure that is considered the picture or illustration of something else. An author who remains unaware of this difference is bound to get into a muddle about what the human mind can and what it cannot ‘represent’ to itself.

That I can mentally re-present certain things to myself is, I believe, indisputable. Though I have not the vaguest idea *how* I do it, I can at this moment re-present to myself the way up a mountain I climbed on a winter’s day, 40 years ago in the Swiss Alps; I can *hear* that peculiar swishing, crunching sound at each step, as one pushes the ski forward into the untouched snow and then puts one’s weight on it; I can see the track I am making, in front of me as a project, behind me as a product, as it follows the contour of slopes and gullies, and I can *feel* that constant effort to keep the track at a steady gradient; and I can *smell*, with every breath, that incomparable combination of dry, cold air and brilliant sunlight. Clearly, in this context, ‘to hear’, ‘to see’, ‘to feel’, and ‘to smell’ do not refer to quite the same activities as in a context of immediate perception. When I perceive, I would say I am registering signals that seem to come from my eyes, ears, and nose. When I re-present something to myself, it seems to come from another source, a source that feels as though it were wholly inside. Perhaps this difference springs largely from the experiential fact that when I perceive, my percepts can be modified by my physical motion; the past I re-present to myself, in contrast, is not influenced by the way I move at present.

As I said, I do not know how re-presentation works. In fact, no one, today, knows how it works. We have not even the beginnings of a plausible functional model of human memory, let alone a model of human consciousness. Yet, something we want to call memory as well as something we want to call consciousness are involved in the kind of *re-play* of past experiences that I was describing. The point I want to make is this: If I re-present to myself something that was a familiar experience 40 years ago, it is, indeed, very much like re-playing or reconstructing something that was made at another time. It is, under all circumstances, a re-play of my own experiences, not a piece of some independent, objective world.

That is the reason why I insist on the hyphen. I want to stress the ‘re’ because it brings out the repetition—repetition of something that was present in my

experiential world at some other time. (Note that, like the German word *Vorstellung*, representation may refer to a new construction that has not yet been actually experienced but is projected into the future as a possibility.)

I shall leave it at that. I am not a neurophysiologist and I can live without a model of memory and of consciousness. I take these items as what they are to me, that is, I take them as phenomena, as part of my experience. I am not arguing with the traditional view that knowledge consists to a large extent, if not entirely, of representations. I am, however, refuting the notion that representations are or could be pictures, replicas, copies of an experiencer-independent ontic world; instead, I am suggesting, that they are (and cannot be anything but) re-presentations of experiential material that was present at some other time.

The moment this is said, a question arises: If one denies that ‘knowledge’ and ‘representations’ stand in an iconic relation to the ‘real’ world and thus correspond to it, in what relation do they stand to it? This, again, is a serious question, because if we were to say that there is no such relation, we should find ourselves in the trap of solipsism, the doctrine according to which the mind, and the mind alone, creates the world—a doctrine that seems easily refuted by the simple consideration that our world is hardly ever quite what we would like it to be and, what is more, the ‘real’ world has some rather nasty ways of treading on our toes.

KNOWLEDGE AND REALITY

The kind of constructivism I have been advocating has nothing to do with solipsism because it is intended as a theory of knowledge, not as a theory of being, that is, an ontology. I have fully expounded the theory elsewhere (Glaserfeld, 1983, 1984b); here I shall confine myself to mentioning the two basic points without attempting to defend them:

- Cognitive organisms do not acquire knowledge just for the fun of it. They develop attitudes towards their experience; they begin to like certain experiences and to dislike others. Consequently they become goal-directed in that they will tend to repeat the experiences they like and to avoid the ones they dislike. The way they attempt to achieve this is by assuming that there must be regularities or, to put it more ambitiously, that there will be some lawfulness in the world of our experience. As Maturana has said, biological organisms operate inductively; they assume that what has worked once, will work again (cf. Maturana, 1970a). One kind of knowledge, then, is knowledge of what has worked in the past.
- Knowledge, from this perspective, must not be characterized as a ‘picture of the world’. It does not reflect the world at all—it reflects what one can and what one cannot do.

In other words, knowledge does not depict the real world but pertains to ways and means the cognizing organism has evolved in order to fit into the world as he or she experiences it. It follows that what we ordinarily call ‘facts’ cannot be elements of an observer-independent world but, at best, elements of an observer’s experience. But not every element of an observer’s experience would ordinarily be considered a fact. Vico, the true father of all modern constructivism, noticed that

the Latin word for ‘fact’ was directly derived from the Latin word for ‘to make’. This helped him to formulate the epistemological slogan that a human can know only what a human has made (Vico, 1710).

Although Vico offered some shrewd suggestions about the formation of certain key concepts such as ‘point’, ‘space’, and ‘time’, he did not say much about the construction of facts, except that they could be made of nothing but elements that were accessible to the maker. This question of accessibility, it seems to me, is of crucial importance if today we want to discuss what is empirical and what is not. If we go back to the beginnings of Empiricism, we find that Locke proposed two different sources for the generation of ideas: the senses on the one side, and reflection on the other. It is worth noting that Locke’s definition of reflection is precisely the same as the one Piaget used in order to differentiate ‘operative’ knowledge from ‘figurative’ knowledge, viz., “the perception of the operations of our own mind within us” (Locke, 1690, Book II, Chapter I, §4).

Where the senses are concerned, Locke did not trust them entirely. He perpetuated the distinction between primary and secondary qualities and agreed with Descartes that only the primary qualities should be considered as belonging to the real world. The secondary qualities (colors, sounds, tastes, etc.) “in truth are nothing in the objects themselves but powers to produce various sensations in us” (Book II, Chapter VIII, §10). Consequently, they could not be used as a basis for any kind of realism.

Then came Berkeley who showed that the very same arguments that had induced Locke to consider the secondary qualities mere appearances that were relative to the human experienter, could be turned with equal success against the primary qualities. The perceptual basis for realism was thus shattered. Hume continued the demolition by arguing that causal connections were a matter of an experienter’s association, and it is easy to show that what Hume said of causality can also be said of any relation that a cognizing organism establishes between two items of experience.

Finally, Kant pulled the rug from under whatever had remained of realism after the British Empiricists. By proposing that space and time should be considered characteristic forms of the human way of experiencing (*Anschnungsformen*) rather than properties of the ‘real’ world, he eliminated any possibility of envisaging or visualizing a world before it has gone through our experiential procedure, simply because we are now assumed to be incapable of seeing, touching, hearing, and, indeed, knowing anything that is not framed in space or time or both. If we accept Kant’s proposal, there is no way out: everything that we might want to call ‘structure’ depends on space and time; hence, if ‘reality’ does not comprise space and time, it makes no sense to think of that reality as having anything that we would want to call ‘structure’.¹

At this point one might say, let’s forget about the British Empiricists and let’s forget about Kant and then we might be in a position to make a case for realism. After all, there have been quite a few philosophers since Kant who tended towards realism—if not explicitly, at least implicitly. The trouble is that none of them has found a satisfactory defense against the age-old attack of the sceptics. If we think

of knowledge as a picture of reality, we would like to be reassured that it is a good picture, that is, a 'realistic' one that shows things as they really are. However, the comparison that might give us that assurance is precisely what we cannot make.

This problem has recently been formulated also in the context of 'information processing'. In that school, too, knowledge is often considered in terms of 'representations'. The cognitive organism, it is said, comes to have representations, and these representations 'encode' information that has been gleaned from reality. As Bickhard and Richie (1983) argued, a code is an arrangement of semantic links between items that signify and items that are signified by them. In order to create such an arrangement, one must have not only the signs or symbols that one intends to use but also the items one wants them to signify or symbolize. In the case of 'reality', the second condition cannot be fulfilled. Reality, as I said before, always remains on the other side of our experiential interface and on the other side of our sense organs; hence it is at best an unfortunate metaphor when people say that the signals we receive through our senses are a 'code' and contain 'information' about reality. We are in no position to know this, because we never gain access to what is supposed to have been encoded.

THE REALITY OF EXPERIENCE

The startling thing about this is that, although we cannot prove the truth of our knowledge, we seem to have a remarkably stable experiential reality in which we carry on our daily living. We formulate explanations, we make predictions, and we even manage to control certain events in the field of our experience. Much of this management involves what we call scientific knowledge, which seems to be the most solid. We rely on it, and it allows us to do many quite marvelous things.

For epistemologists, then, it may be useful to look at the method that supplies us with that kind of knowledge. From my point of view, Humberto Maturana has provided the most lucid analysis of the procedure that is usually called 'the scientific method' (Maturana & Varela, 1987). Maturana divides the procedure into four steps:

1. OBSERVATION. In order to count as 'scientific', an observation must be carried out under certain constraints, and the constraints must be made explicit (so that the observation can be repeated).
2. Observations may then be related by an HYPOTHESIS, usually an inductive hypothesis that involves causal connections.
3. By deduction a PREDICTION is derived from the hypothesis, a prediction that concerns an event that has not yet been observed.
4. The scientist then sets out to observe the predicted event; again, the OBSERVATION must take place under certain explicit constraints.

Throughout the four steps, what matters is experience. Observing is a way of experiencing and, to be scientific, it must be regulated by certain constraints. The hypotheses by means of which one then relates one's observations, connect experiences, not 'things-in-themselves'. The predictions, again, regard experiences, not events in some 'real' world that lies beyond one's actual experience.

Seen in this way, the scientific method does not refer to, nor does it need, the notion of ontological reality—it concerns exclusively the experiential world of observers.

Scientific knowledge, then, does not and could not yield a picture of the ‘real’ world; instead, it provides more or less reliable ways of dealing with experience—and dealing with experience means to be more or less successful in the pursuit of one’s goals. Hence knowledge may be viable, but even if it is, it makes no claim to Truth, if ‘Truth’ is to be understood as a correspondence to an ontologically real world. The constructivist approach thus changes the relation between knowledge and the ontic world philosophers have always placed around the cognizing subject. Unlike the notion of truth (of descriptions), which would require a match, that is, shared points, between the picture and what it is intended to represent, the notion of viability (which refers to actions and ways of thinking) requires *fit*; and it can be characterized precisely by the absence of shared points because they would be points of friction or collision. The concept of viability, however, does imply that there are or will be obstacles and constraints that interfere with and obstruct the organism’s way of attaining the chosen goals. It is not the case that ‘anything goes’, and it is precisely through its obstructions that ontological ‘reality’ manifests itself: by impeding some of our actions and by thwarting some of our efforts. The salient point in all this is that, since this reality manifests itself only in failures of our acting and/or thinking, we have no way of describing it except in terms of actions and thoughts that turned out to be unsuccessful.

THE ANALYSIS OF EMPIRICAL CONSTRUCTION

With this development, empiricism has come full circle and has returned to its original intent, namely to examine the world of experience. The examination started out with the hope that the world of experience would sooner or later reveal something of an ontic world, a world of objective reality. This hope was not fulfilled, and if we continue to investigate the world of experience, it must be in the spirit of Kant’s ‘transcendental enterprise’, which is to say, with the intent to find out how we come to have the apparently stable world in which at a certain point in our development, we find ourselves living.²

What we ordinarily call ‘reality’ is the reality of the relatively durable perceptual and conceptual structures which we manage to establish, use, and maintain in the flow of our actual experience. This experiential reality, no matter what epistemology we want to adopt, does not come to us in one piece. We build it up bit by bit in a succession of steps that, in retrospect, seem to form a succession of levels.

Repetition is an indispensable factor in that development. Without repetition there would be no reason to claim that a given experiential item has some kind of permanence. Only if we consider an experience to be the second instance of the self-same item we have experienced before, does the notion of permanence arise. This creation of ‘individual identity’ has momentous consequences (cf. Glasersfeld, 1979). If the two experiences we want to consider experiences of one

and the same item do not immediately succeed one another in the flow of experience, then we must accommodate for other experiences between the two instances. That is to say, we are obliged to think of that individual item as subsisting somewhere while we are attending to others in the flow of our experience. Thus we must construct 'existence' as a state that can take place outside our field of experience, and in order to take place it needs the space in which to be and the time to perdure while our attention is elsewhere. In other words, by creating individual identities of which we can believe that they recur in our experience, we have created a 'real' world that exists whether or not we experience its constituents.³

THE QUESTION OF OBJECTIVITY

If we do accept this way of thinking as a working hypothesis, we shall have to account for a difference in conceptual constructs which, even as constructivists, we would not like to miss: the difference between knowledge that we want to trust as though it were 'objective' and constructs that we consider to be questionable if not downright illusions. Needless to say, this constructivist objectivity should be called by another name because it does not lie in nor does it point to a world of things-in-themselves, but lies wholly within the confines of the phenomenal. I have elsewhere proposed a model that provides such a 'highest level of experiential reality' and traces its construction within a subject's field of experience (Glaserfeld, 1986). This highest level arises through the corroboration of 'Others' which the subjective observer can construct within his or her own experiential domain. This construction is, in fact, an extension of a suggestion Kant made in the 1st edition of his *Critique of pure reason*:

If one conceives of another thinking subject one necessarily imputes to that other the properties and capabilities by which one characterizes oneself as subject. (Kant, 1781, p. 223)

The creation of Others in our likeness does not happen all at once. It begins quite harmlessly with the child imputing the capability of spontaneous movement to items in the experiential field that do not stay put. It is followed by the imputation of visual and auditory senses to animals, and it is crowned by the imputation of goal-directed behavior, deliberate planning, feelings, and experiential learning to Others whom one considers 'like' oneself. Once this level of sophistication is reached, one spends a great deal of time explaining, predicting, and attempting to control these Others. That is to say, one now has populated one's experiential field with models of moving, perceiving, planning, thinking, feeling, and even philosophizing Others to whom one imputes the kinds of concepts, schemes, and rules one might oneself abstract from one's experience. These models incorporate some of the knowledge we ourselves have found useful and thus viable in our own dealings with experience. If, then, we are able to make a successful prediction about one of these Others, the particular piece of knowledge which, in making the prediction, we have imputed to the Other, acquires a second

order of viability: we now feel justified in saying that this piece of knowledge was found to be viable not only in our own sphere of actions but also in that of the Other. This, I believe, is as close as a constructivist can come to ‘objectivity’. It is obvious that the construction of a viability of which I can say with some justification that it seems to reach beyond my own field of experience into that of Others, must play an important part in the stabilization and solidification of my experiential reality. Indeed, it helps to create that highest level of which one can then believe that it is shared by Others and, therefore, ‘more real’ than anything experienced only by oneself. It is the level on which one feels justified in speaking of ‘empirical facts’. But, as Ludwik Fleck so clearly saw, it is of the utmost importance to realize that they are not facts of an impersonal, absolute reality but facts of one’s own making, facts that arose out of a particular way of experiencing and interacting with the products of one’s experience.

The corroboration of Others, one might think, is much more easily and much more generally achieved by linguistic communication. From the constructivist point of view, however, the notion of ‘sharing’ as a result of a linguistic exchange turns out to be the result of the very same kind of imputation I have discussed above. It would lead too far here to expound the constructivist approach to language and communication but I do want to substantiate the claim that not even language enables a cognizing subject to get beyond the boundaries of subjectively constructed experiential reality.⁴

In spite of the fact that it often feels as though the meaning of words and sentences were conveyed to us by the sounds of speech or the signs on a printed page, it is easy to show that meanings do not travel through space and must under all circumstances be constructed in the heads of the language users. If we then ask, what these meanings could be made of, we find that the only raw material available is the stock of experiential records the individual language user has so far accumulated. There is no doubt that these subjective meanings get modified, honed, and adapted throughout the course of social interaction. But this adaptation does not and cannot change the fact that the material these meanings are composed of can be taken only from the individual language user’s subjective experience.⁵

It may be useful to repeat that constructivism does not deny reality, nor does it deny that the living organism interacts with an environment; but it does deny that the human knower can come to know reality in the ontological sense. The reason for this denial is simply that the human knower’s interactions with the ontic world may reveal to some extent what the human knower can do—the space in which the human knower can move—, but they cannot reveal the nature of the constraints within which the human knower’s movements are confined. Constructivism, thus, does not deny the ‘existence’ of Others, it merely holds that insofar as we know these Others, they are models that we ourselves construct.

THE ELUSIVE SELF

At frequent intervals in the above text I have used the first person pronoun and much of what I have said makes it clear that this first person is assumed to be a

constructor of knowledge. Thus the question arises whether or not the subject that is supposed to reside in this first person can construct knowledge of him- or herself.

One way of coming to grips with that question was opened up by Descartes. Determined to doubt everything that could be doubted, he came to the conclusion that the one thing that remained indubitable was the fact that it was he who was doing the doubting. Thus he proved to himself that, as long as he was thinking, he, the thinker, 'existed'.

I must confess that for a long time it was not at all clear to me what the word 'to exist' was intended to mean in this context. If one believes—as Descartes seemed to—that space and time constitute an absolute, observer-independent frame of reference, it follows that 'to exist' will mean no less and no more than having a locus with specifiable coordinates in that framework. After Kant, the situation gets more complicated. If space and time are no longer considered properties of the ontic world but 'ways of experiencing', we shall have to admit that there are some things to which, though they do have specifiable coordinates in the now subjective spatio-temporal frame of reference, we may not want to attribute 'existence' (e.g., hallucinations, mirages, and, closer to home, mirror images and rainbows).

I am not suggesting that the spatio-temporal connection of 'existence' could not be unraveled; I think it can, even in the context of a theory that considers space and time to be relative. I do want to suggest, however, that 'having coordinates in space and time' is not a satisfactory explanation of what I would intend if I uttered the phrase 'I think, therefore I am'. I would want it to be interpreted as 'I am aware of thinking, therefore I am'. The Cartesian statement tacitly takes for granted that one knows what one is doing and, similarly, the word 'think' implies that the thinking subject knows what he or she is thinking. To my mind, it is precisely this awareness of what one is doing or experiencing that is the foundation of what we ordinarily call our 'self'. Given that, as I said above, we have not even the beginnings of a model of consciousness or awareness, it may seem odd to insist that awareness lies at the very root of the concept of self as an entity. Yet, I have come to the conclusion that, in the search for a sound epistemology, it is indispensable that one acknowledge this fundamental mystery.

ISOLATING THE EXPERIENTIAL SELF

There is, of course, another way to look at the concept of self, and that is to consider its experiential component. This does not concern the self as experiencer, but the self as it is experienced. Such an investigation takes for granted the fact that a self-conscious entity can arise, and it proceeds to examine how that entity could come to isolate itself from the rest of the experiential field.

If we assume that our picture of the world, the knowledge that constitutes our experiential reality, is constructed by us piece by piece on the basis of experience, then we must also assume that the picture/knowledge we have of ourselves must be constructed in a similar way. In other words, just as we construct a model of a world, externalize it, and then treat it as though its existence were independent of

our doing, so we construct a model of the entity that we call our 'self' and externalize it so that it ends up as 'a thing among other things' (Piaget, 1937, pp. 7, 82).

This construction, obviously, has many steps and takes time to accomplish. It probably begins with the infant's discovery that, having isolated moving shapes in its visual field, there is a way to distinguish some of them. When, for example, the mother's hand moves across the infant's visual field, what the infant experiences is at most the visual experience of a moving shape. However, when the infant's own hand moves across its visual field, the visual experience has the necessary corollary of a kinesthetic experience, namely the sensory signals the infant gets from the muscles that happen to be involved in generating the hand's movement. A little later, the difference is significantly increased by the realization that the hand's movement can be reliably initiated at will—whereas the movement of the mother's hand cannot.⁶

From there it is only a relatively small step to the realization that certain places in a mirror image may be taken to correspond to certain places on a surface which, thanks to the correlation of tactual impressions from different sources, one has isolated as 'one's own' (e.g., one's finger exploring the surface of one's foot). Thus a boundary of the sensorimotor self is created and can be reified. The object that results from this reification will be considered the home of that other, more mysterious entity that does the experiencing. The mirror image, thus, confirms the sensorimotor self, but it says nothing about the self as agent of experience. As we stand in front of the mirror, the self that looks and does the seeing is never in the image.

Much more complex than these very basic considerations would be the analysis of the social component in the construction and evolution of an individual's concept of self. As Paul Secord explained:

Perhaps most important to his developing idea of a person as a somewhat stable entity in his world is his realization that other persons behave in predictable ways. ... Only with time and much experience does the individual eventually identify at least some properties of a relatively stable nature associated with himself. Both self as object and self as agent are relevant here. (Secord & Peevers, 1974, p. 121)

From the constructivist perspective, there is another, more fundamental complication. If it is Others from whose reactions I derive some indication as to the properties I can ascribe to myself, and if my knowledge of these Others is the result of my own construction, then there will be a lot of re-interpreting and re-constructing to be done in order to arrive at anything like a non-contradictory notion of a person that I would call 'myself'. Fortunately, I think, the criteria of viability that we apply to the construct of our selves are a good deal less stringent than those we use in other areas of experience.

CONCLUDING REMARKS

Empirical facts, from the constructivist perspective, are constructs based on regularities in a subject's experience. They are *viable* if they maintain their usefulness and serve their purposes in the pursuit of goals.

In the course of organizing and systematizing experience, the subject creates not only objects to which independent existence is attributed but also Others to whom the subject imputes such status and capabilities as are conceivable, given the subject's experience.

As in the case of concepts, theories, beliefs, and other more abstract structures, the facts a subject has found to be viable gain a higher degree of viability when successful predictions can be made by imputing the use of these facts to Others. This additional viability is the constructivist's counterpart to 'objectivity'.

Given the constructivist belief that facts are created in the context of instrumental hypotheses and tentative models, there is no room for the assumption that there is and always can be only one solution to a problem. Instead, solutions and explanations are seen to depend on the specific concepts to which experience is being assimilated within the framework of a particular goal structure.

As to the concept of self, constructivism—as an empirical epistemology—can provide a more or less viable model for the construction of the experiential self; but the self as the operative agent of construction, the self as the center of subjective awareness, seems to be a metaphysical assumption and lies, at least for this constructivist, outside the domain of empirical construction.

NOTES

- ¹ This is one root of the difficulty we encounter in the mystics' 'metaphorical' use of words such as oneness; the word refers to separation and constitution of a bounded unit, whereas the mystic's notion intends to be infinite and all-comprehensive.
- ² The expression 'the world in which we find ourselves living' is not intended to echo Heidegger's notion of being 'thrown into the world', but springs from the Piagetian idea that some of the concepts that determine the structure of our experiential world are constructed during the sensorimotor period, prior to the age of two years, when we are anything but aware of what we are building. As adults, therefore, as Spencer Brown so elegantly said: "Our understanding of such a universe comes not from discovering its present appearance, but in remembering what we originally did to bring it about" (1969, p. 104).
- ³ The difference between the construction of equivalence as the basis of classification and the construction of permanence as the basis of individual identity springs from assigning 'subsistence' to different items. In the first case, subsistence is given to the set of characteristics that differentiate a particular experiential item from all other constructs. If this set of characteristics is maintained (given subsistence) for future use, it constitutes the template or prototype to which future experiences can be assimilated as 'members of the same class'. If, on the other hand, subsistence is given to the experiential item irrespective of its being actually experienced or not, then the item will be said to 'exist'.
- ⁴ A fuller exposition of my approach to language and communication can be found in Glasersfeld, 1983.

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- ⁵ In this respect, social adaptation is analogous to biological adaptation: it can do no more than bring out, recombine, or thwart what is already in the organism—it cannot instill new elements.
- ⁶ A similar distinction is no doubt made by every kitten that plays with her litter mates and discovers that biting its own tail is different from biting someone else's.

SIGNS, COMMUNICATION, AND LANGUAGE ¹

In everyday usage we are not often bothered by doubts as to what is meant by ‘communication’. We take it to mean that one conveys to another what he feels, thinks, knows, or does—and often also what he would like the other to do. Yet, when the life sciences investigate the phenomenon, the everyday concept quickly disintegrates. Colin Cherry (1957), in one of the most valid introductions to communication science, says: “Scientific usage frequently needs to be more restricted but should not violate common sense” (p. 6). The study of communication in living organisms has frequently disregarded both the first and the second part of Cherry’s admonition.

In these pages I shall try to point out *some* conceptual and definitional adjustments that seem indispensable if we want to clear up the vagueness, ambiguities, and contradictions that pervade the literature on animal communication and language. The stress is on *some*. It would take more than a few pages to provide the complete set of unequivocally defined terms needed to map such a wide conceptual area.

Donald MacKay (1972) covered a good deal more ground and comes, I believe, to much the same general conclusions. I shall be content to suggest a number of possible moves towards a less confounding use of ‘sign’ and ‘symbol’, and of the more general terms ‘communication’ and ‘language’. My suggestions are not particularly original. Most of them have been formulated years ago in a branch of science that, in spite of its technicality, is based upon the everyday concept of communication. Cybernetics, as Norbert Wiener (1948) defined it, is the study of “control and *communication* in the animal and the machine” (my italics). Though Wiener has been cited by authors concerned with animal communication (e.g., Altmann, 1967; Hockett, 1960; Sebeok, 1972), most of them limited their interest to two rather specific points while disregarding the general cybernetical framework within which Wiener placed all communicatory processes.

The first of these points concerns the fact that communication events do not conform to the laws of mechanics; that is, the energy inherent in the communicatory action in no way determines the energy involved in the *reaction* (Haldane, 1955; Rosenbluth, Wiener & Bigelow, 1943). For instance, the energy of the sound waves emanating from a man shouting ‘fire!’ is in no fixed or calculable relation to the energy of the event triggered by the shout, an event which may be the explosion that sends a six-inch shell on its way, the panic of a crowd in a theatre, or some bystander’s question ‘where?’. This fact is essential if we want to keep communication apart from mechanical or chemical action-reaction events. If we do not make this distinction, we get definitions of ‘communication’ which include every sort of transaction; for example, “Communication can be said to

occur whenever the activities of one animal influence the activities of another animal” (Alexander, 1960, p. 38).

The second point is the one Wiener (1948) made in his introduction to the volume *Cybernetics*: “It is completely impossible to understand social communities, without a thorough investigation of their means of communication” (p. 18).

Both these points have to do with the *observation of* communicatory events, and that, presumably, is the reason why they were readily adopted by professional observers of animal behavior. Taken out of the cybernetical context, however, they do not elucidate the function a communicatory event might have in the organisation of either a social group or the behavior of the individual. In fact, to adopt *only* these two points is to disregard the very essence of the cybernetical approach.

We do not have to look far to discover the reason why strict behaviorists were reluctant to appropriate anything more from cybernetics. The original essay that laid the foundations of the cybernetical approach had the title ‘Behavior, Purpose and Teleology’ (Rosenbluth, Wiener & Bigelow, 1943). Both purpose and teleology were outlawed in strict behaviorism. As Marler (1961) put it: “This concept (purposiveness), which may also be associated with an anthropocentric viewpoint, has bedevilled investigations of animal behavior in the past” (p. 296; and he cites Thorpe, 1956). A page earlier, Marler states that he finds many of the ideas expressed in Cherry’s book (1957) relevant to his investigation and he quotes that author as saying that communication is “the establishment of a social unit from individuals by the use of language or signs” (p. 295). The quotation is taken from the Appendix to Cherry’s book, but it constitutes only half the definition given there for the term ‘communication’. The other half reads: “The sharing of common sets of rules, for various goal-seeking activities. (There are many *shades of opinion.*)” (p. 305, Cherry’s italics). Evidently ‘goal-seeking’ was too close to the devil’s activities.

Much the same has happened with references to Shannon’s *Mathematical Theory of Communication* (1949). Though it is cited by a number of authors (e.g., Altmann, 1967; Hockett, 1960, 1963; Marler, 1961; Sebeok, 1965, 1972), they do not mention the fact that this theory was conceived, developed and formulated within a very specific context, namely the improvement of communication systems with regard to channel capacity and reliability of signal identification by the receiver, and that, for that reason, it implies the purposiveness of the communicated messages. From this point of view, then, it does not make sense to drop, as Emlen (1960) suggests, all ‘teleological’, ‘causal’, and ‘contextual’ terms from the description and classification of animal sounds. If this were done, the term and the concept ‘communication’ would have to be dropped, too.

Not all investigators of animal behavior, however, are continually preoccupied with exorcising the devil of purposiveness. Menzel, in a recent paper (1973), applies MacKay’s definition of communication (MacKay, 1972), which makes the requirement of goal-directedness quite explicit. Earl W. Count (1969), one of the very few who, before adopting the use of cybernetical terms, have gone to the

sources from which cybernetics developed, introduces the term 'information-ecology' and has no qualms about characterising a living organism as a 'purposeful machine'.

At this point it should be made clear that, in cybernetics, the words 'purpose' and 'goal' do not mean exactly what they mean in ordinary English usage.

Teleology has been discredited chiefly because it was defined to imply a cause subsequent in time to a given effect. When this aspect of teleology was dismissed, however, the associated recognition of the importance of purpose was also unfortunately discarded.... We have restricted the connotation of teleological behavior by applying its designation only to purposeful reactions which are controlled by the error of the reaction, i.e., by the difference between the state of the behaving object at any time and the final state interpreted as the purpose. Teleological behavior thus becomes synonymous with behavior controlled by negative feedback. (Rosenblueth, Wiener & Bigelow, 1943, p. 24)

The function which was called 'feedback' is, in Wiener's most general definition (1950): "The property of being able to adjust future conduct by past performance" (p. 47).

With regard to the analysis of behavior, the contemporary development of feedback theory (cf. Powers, 1973b) has made it a good deal easier to see the relationship between the purposive behavior model of the cyberneticist and the stimulus-response model of the traditional behaviorist. The S-R theorist has always sought to describe and predict an organism's behavior exclusively on the basis of directly observable stimuli. The feedback theorist, on the other hand, sees the organism's behavior as activity directed towards the minimisation of any discrepancy between a sensory input value (stimulus) and a pre-established 'reference value' to which the particular type of sensory input is compared. The discrepancy constitutes 'negative feedback'; the larger it is, the more activity the organism has to expend in order to bring the sensory input value to congruence with the reference value. In this sense, and only in this sense, can the organism's activity be called 'purposive' and the reference value a 'goal'.

In her classification of behavioral situations which can be assumed to constitute contexts for communicatory events, Busnel (1969, p. 35) lists seven types (e.g., meeting of the sexes, search for food, territorial defence, etc.). For each of them, feedback theory would posit at least one reference value. The organism's behavior, insofar as it can be reliably recognised as belonging to one of the seven situations, can then be described and predicted in purposive terms as striving towards adequation of the sensory input to the relevant reference value. The more highly evolved the organism, the richer is the variety of sensory feedback it receives and the repertoire of behaviors it employs to adequate it. Francisco Ayala (1974) has recently mapped the evolutionary beginnings of information processing:

The most rudimentary ability to gather and process information about the environment can be found in certain single-celled eukaryotes. A Paramecium

swims following a sinuous path ingesting the bacteria that it encounters as it swims. Whenever it meets unfavourable conditions, like unsuitable acidity or salinity in the water, the Paramecium checks its advance, turns and starts in a new direction. Its reaction is purely negative.... Greater ability to process information about the environment occurs in the single-celled Euglena ... it not only avoids unsuitable environments but also actively seeks suitable ones. An amoeba represents further progress in the same direction; it reacts to light by moving away from it, and also actively pursues food particles. (p. 350)

These capabilities of avoiding, seeking and pursuing can be explained as the result of different simple feedback loops, each with its specific reference value.

The transfer of signals from sense organs to a central processor that computes negative feedback, and from the processor to other organs that implement the adequating activities, is often said to involve communication; but this communication takes place within one organism. When we ordinarily speak of communication, however, we intend what Wiener (1948) called 'intercommunication', that is, the transfer of signals or messages from one organism to another. While the first is explicitly part of a feedback loop, it may not be immediately obvious that the second, too, must be viewed within the framework of a feedback mechanism. Failure to realise this is, I believe, one of the main reasons for confusion in the literature on animal communication.

Considered as an instrument of feedback adequation, intercommunication is a type of activity that appears in the behavioral repertoire of organisms above a certain level of complexity. While all interaction below that level is a matter of 'energetics' (Count, 1969), interaction above that level can be a matter of 'information'. We can distinguish the energetic type of interaction from the one involving information by means of the criterion formulated by Wiener (1948) and by Haldane (1955). But this, though a necessary condition, does not yet yield a sufficient definition of 'communication'. Just as it does not make sense to call all events involving a mechanical transfer of energy between two organisms 'interaction', it is misleading to speak of all events involving the passage of information as 'intercommunication'.

If I land with my face in the gutter, because someone rushing along knocked me down, the transfer of energy may have been very much the same as in the case where a well-aimed punch from a mugger dispatches me into a similar terminal position. Yet, the behavioral situation would be quite different. In the first case the event is 'accidental', in the sense that neither I nor my terminal position had anything to do with the rushing person's reference value nor with the negative feedback he was trying to reduce by rushing; whereas in the second case, hitting me and dispatching me into the gutter was indeed 'purposive' on the part of the mugger because he had learned in the course of previous experience that these activities were effective in bringing about a coincidence of his sensory input and his reference signal (e.g., my wallet in his hand). We might call the one 'fortuitous', the other 'instrumental'. In ordinary English we should speak of the first event as a 'collision' and of the second as an 'attack'.

The point I am trying to make, however, has to be made, not on the level of words, but on the level of concepts. We can discriminate the two behavioral situations without reference to will, consciousness, mind, or any other concept 'associated with an anthropocentric viewpoint'; they can be discriminated by specifying the operative feedback loops—and there is nothing particularly anthropocentric about feedback loops, since they are an integral part, not only of humans, but of primitive living organisms and of machines as well.

To specify an operative feedback loop and its reference value may be a difficult task under certain circumstances. Powers (1973a, b) has suggested an experimental approach to this problem that has the virtue of great flexibility. No doubt, more work will have to be done in that direction. But even if we come across behavioral situations where, in practice, we are unable to specify the organism's reference value with the desired precision, this could hardly invalidate the extrapolation of the theoretical concept, once the 'referent' of that concept has been demonstrated and measured in many other cases.

If we apply the discrimination of accidental versus purposive (or fortuitous versus instrumental) activity to the realm of communication between organisms, we find that it is largely compatible with the classification of signs worked out by Susanne Langer (1942). She distinguishes 'natural' from 'artificial' signs: "A natural sign is a part of a greater event or of a complex condition, and to an experienced observer it signifies the rest of that situation of which it is a notable feature. It is a symptom of a state of affairs". On the other hand, if "arbitrary events are produced which are purposely correlated with important events that are to be their meaning" (p. 59), we have 'artificial signs'.

As examples of natural signs, Langer gives: wet streets as a sign that it has rained (past event); the smell of smoke as a sign of fire (present event); a ring round the moon as a sign that it is going to rain (future event); thunder as a sign that there has been lightning and lightning as a sign that there will be thunder. In every case, the item we choose to consider a 'sign' is an event which, in our prior experience, has been reliably correlated with another event; and since, as Hume put it, we believe that future experience will largely resemble past experience, the occurrence of the sign-event leads us to anticipate its correlate. If the perceiving organism has associated a behavioral response with the correlate-event, the behaviorist calls the sign-event a 'discriminative stimulus'; if the anticipation, in a cognising organism, leads to an internal representation or thought, the philosopher calls it 'inductive inference'; on both levels it constitutes the simplest and most general form of learning and knowledge.

As samples of artificial signs, Langer lists: a whistle means that the train is about to start; a gunshot means that the sun is just setting. The difference between these and the natural signs is clear. Neither the whistle nor the gunshot were (or could be) correlated with the train's starting or the sun's setting before they were introduced qua signs, simply because they did not happen within those contexts; they were made to happen in order to signify (cf. Hockett's discussion of Paul Revere, 1960, p. 420).

Langer, unfortunately, blurs the issue a little by placing what she calls 'fortuitous signs' on the side of the artificial signs. Fortuitous signs are, for instance, the warning cries of animals, such as the various vocalisations of the vervet monkey that alert other members of the group to the proximity of a large or small mammalian or avian predator respectively (Struhsaker, 1967). Similarly, some of the various sounds 'produced incidentally to other behaviors' by aquatic organisms (Tavolga, 1960, p. 94) seem to function as natural signs. It would, indeed, be anthropocentric to assume that the vervets invented these cries in order to signal the presence of specific predators; but such an assumption is unnecessary. The distinction Langer presumably had in mind is this: while the signs she calls 'natural' are events which, from the human observer's point of view, are in some causal connection to other elements of the event or state of affairs which they signify, the signs she calls 'fortuitous' are not. This distinction is the same that creates some difficulty for Hockett in his discussion of the design feature he calls 'specialisation' (1960, p. 408). Whether the vervet's utterance of a cry can be causally linked to its flight effort, or whether it is to be considered merely a fortuitous corollary of flight, depends on the sophistication of our causal explanations and does not affect the semiotic status the cry has for vervets. For them it has become a sign—and not an 'artificial' one, to be sure—because it happened frequently enough in flight situations to have been associated by vervets with flight and the presence of specific predators. In other words, when a vervet hears such a cry, it responds to it on the basis of its previous experience of the associated situation and, therefore, as though it actually perceived the predator.

To conclude this investigation of the sign-concept, I should like to stress that all the items which, according to our definition, could be called 'sign', must have one feature in common: they stand in a one-to-one relationship to the occurrence of the event or situation they signify. They are 'natural' signs if they co-occur so frequently with an event or situation that they come to signify it because they become associated with that event by the receiver. Alternatively, they are 'artificial' signs if, and only if, their association with the signified event or situation has been contrived, in that the sign-event is made to happen by the sender in order to be interpreted as a sign by the receiver.

This differentiation between natural and artificial signs, in many cases, is quite irrelevant to the interpreter of the sign. From the point of view of the traveller lingering at the bar of the railroad station, there is no essential difference between hearing carriage doors being slammed, the station master's shout 'All aboard!', or the whistle of the locomotive. All three are signs that the train is about to leave. The traveller does or does not respond to them, regardless of any knowledge as to the fact that the first of them is an integral part of the departure event, while the second and third were contrived and added for purely semiotic reasons. Yet, it is precisely this distinction which enables us to differentiate between events that have the intended instrumental function of communication and events which have no such instrumental function.

The distinction is parallel to the one suggested for 'interaction'. It can be made only by reference to the sender of the sign, that is, by an analysis of the sender's

situation, his operative feedback loops, and his activities as instruments for the adequation of his input signals to a specific reference value or goal.

In this context, Tinbergen's concept of 'incipient movements' (1952; cited by Altmann, 1967) or incipient behavior in general, may be helpful in explaining the evolutionary development of artificial signs out of natural signs. It would be absurd to consider a bite the sign for a bite; but the biter's opening his mouth may become a sign that a bite is forthcoming. With more experience, an organism could learn that another's raising his head or getting up in a particular way may also herald a bite. Carpenter (1969, p. 46), for instance, describes the development of such an anticipatory sign in the interaction of an infant howler monkey and its mother. Thus the chain of connected events becomes extended and the interval between the sign-event and the signified event becomes greater. The interpreting organism obviously gains an advantage from this extension. But so does the sending organism. If biting was an activity that had become instrumental in keeping the immediate environment clear of other organisms (implying a reference value with regard to which an intruder would constitute a disturbance), the biting organism will soon learn that getting up or opening his mouth is sufficient activity to eliminate his negative feedback. From there to 'ritualised' behavior is a small step; and, once certain acts or incipient acts have taken over the instrumental efficacy that originally belonged to the whole chain of which they were merely the beginning, a similar chain may form at the response side of the transaction, replacing the interpreter's immediate response by a subsequent one, and so on until there is (from the observer's point of view) no longer any obvious experiential connection between the sign and the receiver's ensuing activity.

Once we reach such complex levels of semiotic exchange, the sign itself, no matter how we analyse it, gives us no clue as to its signification, nor can we be sure to infer its signification correctly from the receiver's response. In many situations, however, there is a fairly reliable clue and it has been intuitively used by every observer of animal communication, though, to my knowledge, it has never been made explicit. It is the simple fact that the sender will cease to emit the sign whenever his environment, which often includes the receiver of the sign, has changed in such a way that the discrepancy between his sensory signals and his reference value has been eliminated. (Note that this approach necessarily puts relatively permanent incidental changes of the environment, e.g., an animal's tracks or odoriferous deposits, into the class of fortuitous or natural signs.) Marler's compendium of aggregation and dispersal signs (1968) is an example in which the relevant reference values can be easily and accurately specified. These reference values are distances between the individual and other members of his group or between the sender's group and other groups. The heard (received) vocalisations provide feedback as to the actual distance. Whenever there is a discrepancy between that and the reference value, a 'distance-increasing' or a 'distance-reducing' vocalisation is emitted and the instrumental function, that is, the purpose, of these vocalisations is the elimination of that discrepancy.

In other situations it may be much more difficult to infer the specific reference value that gives a sign its purposive aspect. It may, indeed, require some form of

experimentation, that is, deliberate, methodical interference of ‘disturbance’ on the part of the observer (Powers, 1973a, b). But by varying the sensory input to an organism and by observing both the kind and the amount of activity with which the organism responds to the disturbance, we can eventually establish what the reference value is, to which the organism strives to adequate his input.

This isolation and specification of the reference value or goal is essential if we want to make a distinction between behaviors that have a semiotic function and all the other behaviors that have no such function and are significant only in the sense that they happen to have been experientially associated with certain events or states of affairs. In other words, it is only when we are able to specify the instrumental relation between a sign and the goal towards which the sender of the sign is striving, that we would speak of communication.

Once we adopt the feedback model of behavior, the importance of communication in the development and maintenance of social organisation can be easily explained. What welds a number of individual organisms into a social group is not merely the similarity of certain behaviors, but rather the commonality of certain reference values or goals. Because of this commonality, the activity of one individual will frequently eliminate not only his own feedback discrepancy but also the feedback discrepancy of others. This creates a mutual or reciprocal instrumentality: others are instrumental in the adequation of my input to my reference, just as I am instrumental in the adequation of their input to their reference. This mutual instrumentality, which has nothing whatever to do with the ethics of altruism, makes for the cohesion of the group.

This is not the place to discuss how concerted efforts, division of tasks, and, eventually, reciprocal dependencies develop out of an original commonality of reference values, but it should be clear that signs, signals, and communication play an indispensable role in the development of any such organisational feats. Sebeok (1972) speaks of members of a species becoming “integrated into ‘interest communities’, joined together, for instance, for mutual protection-like a school of porpoises” (p. 79). Teleki (1973) observed and described the cooperative behavior of chimpanzees in predation and meat-sharing. Malinowski (1923), who half a century ago investigated communication among primitive tribes, said: “Speech is the necessary means of communication; it is the one indispensable instrument for creating the ties of the moment without which unified social action is impossible” (p. 310). He used the terms ‘speech’ and ‘language’, and he discriminated clearly between language functioning “as a link in concerted human activity, as a piece of human action” and language as “a condensed piece of reflection, a record of fact or thought” (p. 312). This distinction has, however, been confounded in almost everything that has been written about language and linguistic communication since then.

The difference between the use of a significatory item as an instrument of action and its use as an instrument of reflection was stressed once more by Susanne Langer. She contrasts ‘signs’ with ‘symbols’ (1942): “To conceive a thing or a situation is not the same as to ‘react toward it’ overtly, or to be aware of its presence. In talking *about* things we have conceptions of them, not the things

themselves; and *it is the conceptions, not the things, that symbols directly 'mean'*" (p. 61; Langer's italics).

This is a paraphrase of the basic semantic relation Ogden and Richards (1923) illustrated by their famous triangle. The fact that the symbol is associated with a conception, and only in some cases, and then indirectly, with a 'thing', is what releases the symbol from the one-to-one correspondence with the occurrence of an experiential situation. Thus a sign becomes a symbol when it can be used without direct connection to an experiential context. The linguistic items we normally call 'words', though they can, of course, be used as signs, are primarily symbols. We can use them in operational loops where neither the input nor the reference values are perceptual and where the activities performed to eliminate discrepancies between the two are not observable behavior but private reflective operations that never become directly manifest in public results. We can use them to modify other's reference values, to supplement their repertoire of instrumental activities, to change their representation of the world they experience, that is, their 'knowledge'.

This removal from the actual experiential situation is not quite the same as the removal Hockett (1960) characterised with his design feature of 'displacement' (p. 415). The protohominoid, who spotted a predator, took fright, and "did not immediately burst out with his danger call, but first sneaked silently away towards the remainder of his band" (ibid.), may, indeed, have achieved a kind of displacement of the danger call that had survival value, but the mere delay does not change the one-to-one correspondence between the sign and the actual perceptual situation. The danger call remained wholly instrumental in a feedback loop that was disturbed by the sensory input provided by the actual predator. The danger call will become a symbol only when a later hominoid can utter it without having spotted a predator and when the other members of the band can hear it without taking flight. The displacement characterised by Hockett may well be a plausible step in that development, but it is at best a preparatory step and, as such, not a criterion for the symbolicity that is an essential feature of 'language'.

Hockett subsequently (1963) introduced an additional design feature that implies symbolicity, that is, 'prevarication', and he remarks: "It ought to be noted that without the property here labelled 'prevarication', the formulation of hypotheses is impossible" (p. 10). It seems unlikely that we could ever establish by direct observation that another organism's sign is uttered in the hypothetical mode, unless the organism already has a conventional sign for indicating this. What primatologists have called a 'play-face', that is, a sign indicating that a subsequent aggressive behavior is not to be taken seriously, is a case in point. It should also be noted that in Piaget's theory of cognitive development the symbolic function first arises in play (Piaget, 1967a, p. 90).

Though the meaning of symbols is conceptual and, therefore, by definition inside an organism and not directly observable, their function can be demonstrated. Premack, in his artificial communication system with the chimpanzee Sarah, introduced questions and negation at an early stage (Premack, 1971). Both require conceptual representations that come very close to what we call an hypothesis and

the successful handling of a combination of the two proves the presence of some symbolcity.

Successful handling, in this context, is not a question of behavior, if by 'behavior' we intend the responses which the behaviorist dogma has sanctioned as observable. That an organism handles a symbol successfully can be shown only by the use of other symbols, because the reference values as well as the 'inputs' that have to be adequated to them are not on the perceptual level. If, for instance, we show an orange to a chimpanzee and then ask him, by means of any signs he has learned to use, "Is 'apple' the name of this?", the correct response is to say "No". What makes it correct is not the perceptual difference between the apple-sign and the orange (any sign would be different from an orange), but the difference between the concept associated with the apple-sign and the concept by means of which the orange is recognised as an orange. The kind of item we mean by 'concept of an orange' has been discussed with admirable lucidity by Bronowski and Bellugi (1970). In order to respond correctly, a comparison has to be carried out on the conceptual level, which is not the case in the usual discrimination tasks given to experimental animals. In this very specific sense the combination of question and negation is a test for symbol manipulation.

When we ordinarily use the word 'language', we certainly take it for granted that we are dealing with a system whose elements are capable, not only of displacement, but also of functioning as symbols. But there is also a more easily discernible feature which we take for granted. What we call 'language' must be a combinatorial system of symbols which, to quote Hockett (1960), "provides certain patterns, by which these elementary significant units can be combined into larger sequences, and conventions governing what sorts of meanings emerge from the arrangements" (p. 418). Hockett judiciously avoids the term 'syntax' and speaks of the "sorts of meanings that emerge from the arrangements". This is important because linguists have traditionally understated, if not disregarded, the semantic dimension of syntax. The crucial feature, then is not that language has rules for stringing symbols together, but that the stringing together adds another level of meaning. 'To paint a red house' contains the very same elementary word-symbols as 'to paint a house red', but some of the relational concepts indicated by the two strings are different. It is this feature of combinatorial meaning that leads to the openness of linguistic communication systems and gives the user the possibility of producing a novel expression. Combinatorial meaning and symbolcity of the elements, thus, yield a reliable criterion for distinguishing between language and other communication systems whose elements have a necessary one-to-one correspondence to the occurrence of events or states of affairs in the experiential context in which they are used.

NOTES

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HOW DO WE MEAN?

A constructivist sketch of semantics

When people ask about the meaning of life, they have to be given a metaphysical answer. Whether the answer will seem plausible or not, depends on their mindset. It cannot be checked in the domain of experience because the question explicitly leads beyond that domain.

Questions about language and how words come to have meaning, are of a different sort and they deserve an answer that speakers of a language can check out by reviewing their own early experience of constructing meanings for the speech sounds they learned to hear and to produce.

The domains of metaphysics and semantics are essentially different in my view. Different, too, is the way most people come to think of them, if they think of them at all. Many of us reflected at least briefly (and inconclusively) about the meaning of life when we reached puberty and wondered who we were and what the world was about. And we may later have returned to these questions in times of crisis. In contrast, very few people have spontaneously come to examine the general problem of meaning in language. This may be due to the fact that by the time metaphysical problems raise their head, we are already so thoroughly familiar with the use of language as a tool in everyday interactions that 'communication' has become a commonplace and we take for granted that it works. This may be one reason why many accept the notion that language has some form of 'existence' apart from the people who use it.

At a time that is more and more often referred to as the 'information age', an examination of the assumption that language is the main carrier of information, does not seem out of place. Some fifty years ago, Claude Shannon published the first 'scientific' *Theory of Communication* (1948). It may have been superseded by more sophisticated methods of calculation in radiotelegraphy and satellite linkages, but it formulated a logical condition that is as relevant in today's communicatory interactions as it was then.

The following diagram shows some aspects of the theory. What travels in the communication channel from sender to receiver are impulses, that is to say, changes in some form of energy (kinetic, electromagnetic, acoustic, etc.).

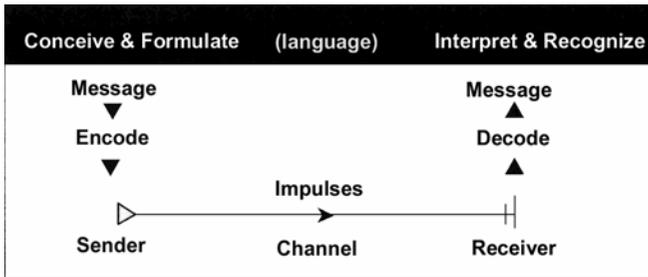


Figure 1. Communication (After Shannon, 1948)

These impulses are signals insofar as they can be identified as belonging to a code. They do not carry meaning in themselves. Both the sender and the receiver attribute to them the meanings associated with them in a list that constitutes the code. Needless to say, ‘communication’ can take place only if the code list is essentially the same at both ends of the channel. This is no problem in systems that employ technical codes—such as the one invented by Mr. Morse—because these codes are distributed to the participants before they begin to communicate.

In Shannon’s model, the ‘message’ is simply the succession of decoded signals. How the receiver converts these into meaningful conceptual structures lies altogether outside his theory. This is why I have added the level of language. It is this second level of interpretation that I intend to discuss here.

Children are not given a linguistic code in order to speak and understand language—they have to discover it on their own. They have to construct for themselves the meanings of a good many words before words can be used to expand the range of their linguistic communications. I have elsewhere given a detailed account of how one might think of this development (Glaserfeld, 1995, chapter 7). Here it will be sufficient to point out that the linguistic code necessarily consists of associations made in the subject’s experiential field. De Saussure has provided a simple diagram that shows the structure of the linguistic code:

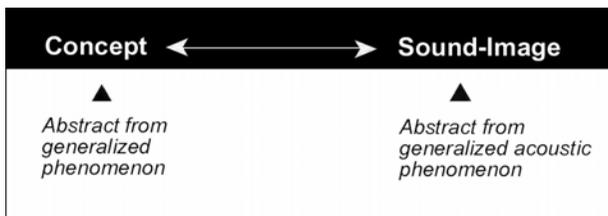


Figure 2. Linguistic sign (After de Saussure, 1916)

Whatever one assumes to be genetically determined in children, it is they themselves who must actively isolate units in their experiential field and abstract them into concepts. Having done this, they must separate in their auditory

experience those acoustic units that belong to a linguistic system from other units that do not. Only then can they tentatively associate specific items of the first type (concepts) with items of the second (sound-images of words). That children do all this unawares does not support the notion that it happens by itself without any effort on their part. The semantic connection has to be formed in their heads.

Many authors, for example Rorty (1982) and Gergen (1994), suggest that whatever we want to think of as ‘meaning’ is acquired in the course of what Wittgenstein (1953) called ‘language games’. This rightly points to the fact that children cannot guess all by themselves which sounds constitute words and what their meanings might be. It can only be done in the contexts of social interaction. Language games are the occasion for the construction of meaning, but they do not explain how children do it. Social constructionists (a term invented by Kenneth Gergen to distinguish his way of thinking from that of other constructivists) are obviously aware of the problem. Gergen (1994) explicitly states: “the constructionist is centrally concerned with such matters as negotiation, cooperation, conflict, rhetoric, ritual, roles, social scenarios, and the like, but avoids psychological explanations of microsocial process” (p. 25).

Piaget’s constructivism, and my slight elaborations of it, have the explicit purpose of proposing models to show how children may come to develop knowledge and thus the meanings of words and linguistic expressions. I see no reason why this enterprise should be considered antagonistic to those who investigate negotiation, cooperation, and other social phenomena. Such a psychogenetic model, it seems to me, could help the socially oriented researchers to ground their findings far more solidly than by assuming that the knowledge and the language of a social group could be instilled into its members through the simple occurrence of language games and other forms of social interaction.

Be this as it may, my main interest is in devising theoretical principles that might show at least one way that could lead to these important competencies. De Saussure’s model makes very clear that the semantic connection in the first place links an individual’s generalized experience of words with the individual’s generalized experience of other items. For entities that have been generalized German provides the word ‘Vorstellung’, a word that is central in Kant’s analysis of reason. In English, it has traditionally been rendered by ‘representation’, and this was thoroughly misleading. In the English-speaker the word ‘representation’ inevitably implies that somewhere there is an original which is now being represented. This interpretation makes it practically impossible to understand Kant’s theory of knowledge; and when it is applied to language it leads to the notion of ‘reference’, that is, that words refer to objects in a world thought to be independent of the speakers.

If you think about this, you sooner or later stumble over the question how you could possibly have established a semantic connection between a word and an object, if both are supposed to be independent of your experience. The answer becomes obvious in Saussure’s diagram: The semantic connection—one cannot repeat this often enough—can be made only between entities in someone’s head. Just as, for instance, the Morse code links short and long experiences of beeps to

re-presentations of letters of the alphabet, so in language, sound images are linked to concepts, that is, to re-presentations of experiential units.

The problem of meaning thus comes down to the problem of how we generate units in our experience such that we can associate them with words, and how we relate these units to form larger conceptual structures. A model to illustrate these generative operations was first suggested by the late Silvio Ceccato in the 1950s (1966, pp. 22-26) and later developed in slightly different directions by Giuseppe Vaccarino (1977, 1988) and myself (1981a). It was based on a novel conception of the mechanism of attention. Instead of thinking of attention as a kind of searchlight that illuminates parts of the experiential field, we think of it as a pulse whose beats could either coincide with sensory signals or remain empty and unfocused. Its operation can be illustrated with the help of the following design:



*Figure 3. Occasions for diverse categorizations:
The path of the beholder's attention determines what is seen*

This array can be seen as different 'things': If an uninterrupted sequence of attentional pulses coincides with sensory signals of blackness, it will be categorized as a line; if attentional pulses fall on the crests and on the white background between them, it will be categorized as three hills; if they fall alternately on black spots and the whiteness between them, it will be categorized as a collection of dots. The design itself does not determine what 'units' are to be used in the categorization you choose. It depends on your way of operating.

Ceccato's idea of attentional pulses was, one might say an inspired intuition, for at the time he knew nothing about alpha rhythm or other oscillatory phenomena in the brain. Only twenty years later a review of 'two hypotheses of central intermittency in perception' was published that surveyed research on what became known as 'perceptual framing' (Harter, 1967).

This is not the place to go into the details of the attentional theory. Besides, I want to make clear that, from a constructivist point of view, neurophysiological findings cannot confirm, let alone 'prove' theoretical assumptions; but theoreticians quite rightly feel encouraged when their conceptualizations turn out to be useful in the organization of observational (empirical) data.

The point I want to stress is that from our perspective it is attention and above all its movements that generate the conceptual structures and thus the things we talk about. These items, as I said before, cannot have an existence of their own but originate through the operations of an experiencer or observer. A striking example are the constellations we all can learn to see, name, and recognize on a clear night. Take the one called Cassiopeia. It has been known since the beginning of human

history. The Greeks saw it as the crown of a mythical queen and gave it her name. We see it more prosaically as a ‘W’ in the vicinity of the Polar Star.

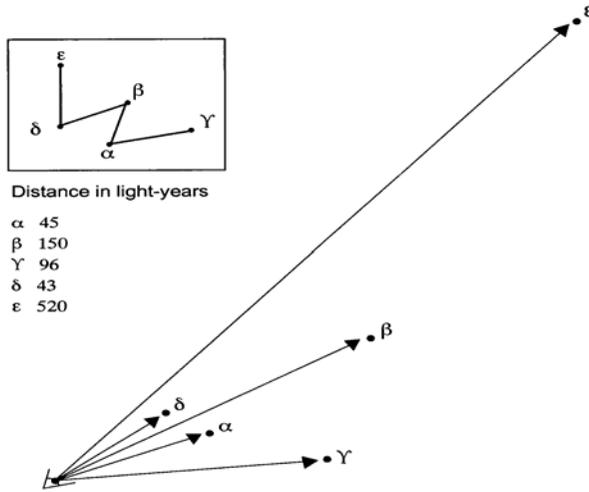


Figure 4. The constellation of Cassiopeia

If you consider the relative distances of the individual stars it becomes clear that there is only a very small area of the universe (as astronomers have taught us to conceive it) from which the five stars could be said to form a double-u. Move the observer a few light-years to the right or the left, the double-u would disappear. Move the observer 50 light-years forward, and he or she could construct only a triangle with the three stars that remained in front. One might call this the relativity of the point of view. But there is also the relativity of construction. The connections between the five stars are not in the sky. They have to be imagined by the observer—and there is nothing in the sensory material that imposes the formation of a double-u. The stars could equally well be connected differently:

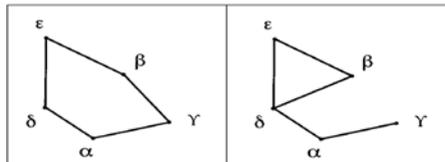


Figure 5. Alternative constructs

That Greeks called it a crown because this was a generally accessible analogy in their world. The double-u of our alphabet supplied an analogy that was more easily accessible to us. The point I want to make is that it is the experiencer who generates the image, the configuration that becomes the ‘representation’, and that

this configuration is always one of several others that are equally possible within the constraints of the sensory material. This, I claim, goes for all the experiential units or things to which we give names, and it is the reason why I maintain that ‘meanings’ are always subjective. They are subjective in the sense that they have to be constructed by the experiencer. This in no way denies the fact that the continuous social and linguistic interactions among the members of a group or society lead to a progressive mutual adaptation of the individuals’ semantic connections. These interactions inevitably bring about the fact that the members of a language group tend to construct the meanings of words in ways that prove compatible with the usage of the community. This is to say, they develop a more or less common way of ‘seeing the world’. But what they see is nevertheless their subjective construction.

That this is a viable assumption becomes clear the moment one considers more than one language. I can illustrate this by a simple example. English text books of linguistics frequently give ‘the boy hits the ball’ as example of a simple sentence that contains a subject, a verb, and an object. In the British Isles this sentence usually calls forth the re-presentation of a boy armed with a tennis racket or a golf club. In the United States he will be imagined to hold a baseball bat. This is a very minor difference. However, if the sentence has to be translated into German, it turns out to be far more complicated. The translator has to know more about the situational context, because the ‘simple’ sentence turns out to be ambiguous. It would be appropriate in several situations, each of which requires different words in German. Here are the four most likely ones:

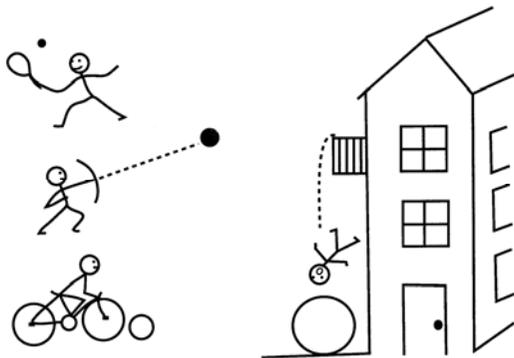


Figure 6. ‘The boy hits the ball’

If the boy hits the ball with a racket, a club, or a bat, the German verb has to be *schlagen*; if he hits it with an arrow or a bullet, it would be *treffen*; if he hits it with his bicycle, it would be *stossen* plus the preposition *auf*; and if he hits the ball when falling from the balcony, it would be *fallen ... auf* or *schlagen ... auf*. None of these verbs could be used in any of the other three situations.

The conceptual structures called up by the German verbs are more complex than the one called up by 'to hit'. They all contain the meaning of the English verb, that is, the construct of an object's sudden impact with something else; but they also contain specifications of the event that are not part of the English meaning. As a result, English-speakers who want to express themselves in German must learn not only different words but also a different way of *seeing* the details of the relevant experiences.

Between any two languages you might choose, there are innumerable differences of conceptualization. If they lie in the area of perceptual or sensorimotor construction, they sooner or later become noticeable and corrigible in practical situations of interaction. If, however, they are a matter of abstract conceptual construction, such as the meaning of the German word *Vorstellung* and that of the English word 'representation', they may cause lasting misinterpretation because their incompatibility rarely becomes apparent on the surface.

I have chosen examples of the differential construction of meaning in different languages because they manifest themselves in the daily experience of anyone who lives in more than one language. But the meanings individual users of one and the same language construct are no more homogeneous. Although individuals necessarily adapt the meanings they associate with words to what they perceive to be the usage of the community, the stuff those meanings consist of is always part of their own subjective experience. Consequently it is misleading to speak of 'shared' meanings. The terms information, communication, representation, and meaning tend to reinforce the notion that the structure of meaning is a well-known fixed entity. This, in my view, inevitably leads to trouble. What speakers of a language have constructed as the meanings of words they use, is at best *compatible* in the linguistic interactions with other speakers; but such compatibility remains forever relative to the limited number of actual interactions the individual has had in his or her past. What speakers have learned to mean always remains their own construction.

ON THE CONCEPT OF INTERPRETATION

Interpretation is a frequent term not only in literary studies. It is used by musicians and lawyers, actors and priests, translators and psychoanalysts, computer scientists and diagnosticians, and some time ago, when private airplanes began to come on the market, there appeared publications on how to interpret clouds. It is, of course, not unusual for a term to be borrowed by diverse professions and then to be used with a somewhat modified meaning, or metaphorically, or even in an unrelated way. *Interpretation* is remarkable, I believe, in that the core of its meaning has remained unaltered wherever the word was adopted. I stress *core*, because subsidiary aspects have certainly been dropped and added. To pursue these nuances would, no doubt, be an interesting and revealing investigation in its own right, but it is not what I intend to do here. The core itself is complicated enough and there is little risk that I shall exhaust it. The reason for that complexity is this: the activity of interpreting involves experience, the coordination of conceptual structures, and symbolic representation; that is to say, it involves the very activities of cognition and thus, inevitably, a theory of knowledge.

Like many nomina actionis, ‘interpretation’ designates either an activity or its results. When someone says, “I’m not sure how to interpret what she did”, it may mean that he sees several possible interpretations and does not know which to choose as the most plausible; but it may also mean that he has no interpretation because he sees no way of constructing one. In the first case, the speaker’s quandary pertains to the results; in the second, to the activity. In this chapter, I shall be concerned with interpretation as activity and only incidentally with the appropriateness or choice of its results.

Whenever we say ‘S interprets X’, we bring to mind a specific situation which, I would argue, is always composed of the following elements:

1. an active subject (S), the interpreter;
2. an object (X) which is experienced by S;
3. a specific activity (interpreting) carried out by S; and
4. the activity’s result (Y), which is not part of S’s immediate experience of X but is linked to X by some relation known to S.

Some, I expect, will disagree with this schema and consider it incomplete. There is a wide-spread tendency to take for granted that the object X must be an object of a special kind, an object that was intended to be interpreted. Thus it is implicitly or explicitly assumed that an originator or author (A) deliberately chose or produced the object X in order to express, convey, or communicate something else which we may call ‘intended meaning’. Like Y, this meaning (M) is not a constituent part of the object X; but unlike Y, which is the result of S’s interpretive activity, M is the result of an act of association on the part of A. This act of association may be

deliberate or habitual but, in any case, as far as A is concerned, it turns an ordinary experiential object, X, into a semiotic object or symbol. The addition often seems quite innocuous because we are so accustomed to dealing with conventional symbols.

The effects of that seemingly modest addition are momentous: it inevitably leads to the assumption that the result of S's activity in (3) should be considered a success only if Y constitutes a replication of M. If that condition is taken as a requirement, it vitiates all further discussion of the process of interpretation, because it at once shifts the focus away from the question 'How is interpretation achieved?' to the question whether or not the interpreter's result Y is an acceptable or *correct* replica of the author's intended meaning M. But since M is, by definition, in A's head, and Y must under all circumstances be constructed in S's head, there simply is no way of comparing M and Y in order to decide whether or not they match. One person's thought, concepts, sensations, emotions, etc., can never be actually compared with another's. At best, they can be tested for compatibility—but compatibility, no matter how well it might be established, does not warrant the assumption of *sameness*. In fact, the requirement that an interpretation of X, in order to be considered a good or correct interpretation, must match the meaning an originator has associated with X, is just another manifestation of the epistemological ingenuousness that leads realists to the unwarranted belief that what we experience should in some way correspond to an ontological reality and that, if only we try hard enough, we shall finally have a true picture of the world as it *is*.¹

Discarding the requirement of a matching replication and putting in its place the requirement of compatibility may, at first sight, seem to be no gain. One might think that, in order to establish an interpretation's compatibility with an intended meaning, one would need as much access to that meaning as if one wanted to produce a replica of it. That, however, is a false impression. Compatibility is a matter of avoiding clash, passing between obstacles, fitting into space that is **not** encumbered by the conditions that have to be complied with. The relation of fit is essentially a negative one: within the frame of reference, or grid, used to establish the fit, no point occupied by the one item is occupied by the other; a *match*, on the other hand, is a match precisely to the extent to which the two items share points within the chosen frame of reference.

The substitution of the concept of fit (and its dynamic corollary, *viability*) for the traditional concept of truth as a matching, isomorphic, or iconic representation of reality, is the central feature of the theory of knowledge I have called *Radical Constructivism* (Glaserfeld, 1975, 1980, 1981b). This conceptual shift has certain consequences for a theory of interpretation. In exploring these consequences, two questions arise at once: What constitutes *fit* and *viability* in interpretation, and how can they be established? I propose to examine these questions by first looking at simple instances of interpretation that do not involve an author and the supposition of an intended meaning.

Among the first professional interpreters were the augurs in ancient Greece and Rome. According to their various specializations they observed the stars, the flight

of birds, thunder and lightning, miscarriages and monsters, the entrails of sacrificed animals, and other phenomena, and they interpreted certain findings as omens of events to come. Although the prediction of the future was referred to as the discipline of mantics, or seers, it did not involve anything we would now call a semantic code. The particular phenomena the mantic selected were taken as symptoms (or signs) rather than as conventional symbols. The course of all events was believed to be predetermined by fate and the seer simply claimed to have privileged insights into the workings of fate. He therefore could discern connections between the events or states he took as omens and the events or states that were to follow. The connections between the two, however, were neither *semantic* (in the modern sense), nor causal; they were, as we would say today, merely correlational. The Greek mantics, for example, often chose to observe birds of prey, because, given that they soar at great heights, they had a wider horizon and could see further. Consequently they had information that was not yet accessible to an earth-bound creature. Observing them, a skilled seer could draw more or less intuitive inferences from their behavior and predict what lay ahead and still out of view for the surface dweller (cf. Burckhardt, 1952, p. 502).

In terms of the schema, the mantic, S, takes the behavior of a bird, or flock of birds, as object X and derives from it a future event, Y. The relation on the basis of which he interprets X as an omen pointing to Y may lie anywhere in the continuum from experiential induction to mystical intuition. I want to emphasize, however, that no matter how the seer has established that relation, there is absolutely no assumption that the birds fly in the way they are observed to fly because some originator intended that way of flying as an expression of the *meaning* a mantic might see in it. The flight of the birds, in that classic view, was just as predetermined by fate as everything else, and the skilled mantic was simply one who managed to *see* connections between events that remained hidden to other mortals.

Although, today, we tend to scoff at mantics and seers, we still accept a good many predictions that are based on the interpretation of signs whose relation with the predicted event is purely correlational, and some of the signs are not very different from the flight of birds (e.g., the recent Chinese successes in predicting earthquakes by observing and interpreting the behavior of wild animals who seem to know more about these events than the seismologists). Though less animistic, much of all present know-how is essentially still of that kind.

Pilots, mountaineers, sailors, and others whose life may depend on the timely prediction of the weather, will be the safer the better they interpret such atmospheric conditions as happen to be observable. In their case, too, the objects they interpret are signs only because experience has shown them to be more or less reliable indicators of things to come; that is, induction has yielded correlations, although there is as yet nothing like a comprehensive causal theory of atmospheric processes and developments. A mountain guide, for instance, may interpret a certain cloud formation as heralding the approach of a snow storm, and he might say, "These clouds mean storm". In any such case, however, it is clear that the

meaning is attributed by the weather-wise observer and not by some other originator who is using the clouds as a vehicle of expression.

Any prediction based on the interpretation of an experiential item that is taken as a sign pointing to a not yet experienced item, will be judged according to whether it is or is not *confirmed* by actual subsequent experience. The question of what constitutes *confirmation* is an extremely tricky one because it hinges on how well the predicted item has been specified. It involves all the thorny problems of definition (e.g., how many drops confirm a prediction of rain?); but in this discussion we can shortcut all that by saying that a prediction will be considered good if, within a stipulated time frame and in the judgment of the people involved, a subsequent experience fits the predictive statement. Whether such a fit can or cannot be found is essentially an empirical question. Both the description and the experiences to be tested are accessible to those who must judge the fit and, though there may be practical difficulties, the question should, in principle, be answerable. The situation is changed and becomes far more complicated when the item X is taken as a sign, not merely on the basis of the interpreter's experience, but as *expression* of a meaning given to it by an intentional originator. In the simple examples we have so far considered, the experiential items taken as signs, as well as the items they were assumed to point to and the relation between them that constituted the basis of the interpretation, were all three within the interpreter's field of experience and cognitive action. In cases where there is an originator's intended meaning, the relation between that meaning and the experiential item that is to function as the sign pointing to it are within the originator's field of experience and conceptual action and, as such, *not* accessible to any other interpreter. Nevertheless, we are firmly convinced that we can communicate with others. There seems to be a blatant contradiction between the claim of communication and the apparently irrefutable subjectivity of meaning. The contradiction, however, may be resolved if we consider what, actually, takes place when we communicate and, above all, what are the prerequisites of any communication.

On the simplest technical level, Shannon's Theory of Communication (1948) makes it clear that *meaning* does not travel from one communicator to the another. What travels is a *signal*. A signal, of whatever physical form it might be, has for the originator or source the specific meaning he or she has encoded in it. A receiver can *decode* a signal, provided two conditions are satisfied: he must (1) recognize it as a signal, and (2) have a specific meaning associated with it. On the technical level, moreover, it is usually taken for granted that the sender's and the receiver's codes are the same.

In the realm of telegraphy, Morse code, and other technical signaling systems, the 'identity' of the sender's and the receiver's codes can be assured by simple means *outside* the communication system (e.g., distributing a priori a list of permitted signals plus their fixed meanings). In non-technical, that is, not deliberately designed communication systems, the assumption of any such identity of codes and meanings becomes precarious.²

When the communication system is a natural language, we tend to ignore that precariousness. Natural language is learned in interactive situations, that is, in situations where speaker and hearer are reciprocally part of each other's experiential field and where, therefore, there is some feedback regarding the hearer's interpretation of the speaker's utterances, as well as feedback regarding the speaker's expectations about the hearer's responses.

Every child, in order to survive in its community, must learn to interpret a great many linguistic signals in terms of responses that are considered compatible by the adult speakers among whom it lives. "Shut the door!", for instance, must be responded to with a sequence of motor acts which has to be learned in a succession of experiential situations, a succession which provides occasion for the acquisition of simple but nevertheless specific skills and, above all, occasion to experience what has to be avoided. Most of us have been scolded at one time or another for slamming a door when the instruction was to shut it. In time, we have learned to shut doors so that the givers of the command are satisfied—which is to say, we learned to adapt our interpretation of their signal to their expectations. But that learning was neither intuitive nor instantaneous—it required a certain number of trials, errors, and the gradual isolation of *viable* ways of responding. We would not expect a child that grew up in igloos or tents to have acquired either the skills to comply with that command or, indeed, the meaning of the phrase. Though this example is extremely simple, the principle it illustrates is fundamental to all linguistic communication: a linguistic message, under any circumstances, can be interpreted only in terms of the receiver's experience.

At the beginning of the 18th century, Giambattista Vico formulated a constructivist epistemology by saying that humans can know only what humans can construct. That also fits the theory of interpretation. In order to understand a piece of language I hear or read, I must build up its meaning out of conceptual elements which I already possess. If I am told that a mermaid is a creature with a woman's head and torso and the tail of a fish, I need not have met such a creature in actual experience to understand the word, but I must be somewhat familiar with what is called 'woman' and what is called 'fish' to construct a meaning for the novel word. And if I am not told that the fish's tail replaces the woman's legs, I may construct a notion that is more like a fish-tailed biped than like the intended traditional mermaid. My deviant notion could then be corrected only by further interaction, that is, by getting into situations where my conception of a creature with legs as well as a fish's tail comes into explicit conflict with a picture or with what speakers of the language say about mermaids.

Once we realize that words cannot refer to things that exist independently of an experiencer but only to speakers' and hearers' representations of experiences, it becomes clear that communication is possible only within the bounds of what Maturana (1980) has called a 'consensual domain', that is, a domain in which the communicators have adapted their conceptualizations to the conceptualizations of others by a succession of interactive experiences. The notions, concepts, representations, or meanings of two communicators, however, can never be compared to establish *sameness*—they can only be tested for compatibility, and

such compatibility as has been or can be established will necessarily be relative, because the number of testing situations is, in practice, always limited.

It is important to realize that the compatibility of two items does not entail their identity. Indeed, a demonstration of compatibility cannot even be turned into a proof of likeness. We believe to have *understood* a piece of language whenever our understanding of it remains viable in the face of further linguistic or interactional experience. Only a subsequent statement or speaker's reaction to our response can indicate to us that an interpretation we have made is not compatible with the speaker's intended meaning.

The more or less permanent meanings each one of us has established for words and phrases in the course of acquiring a given language is the direct result of our individual histories of interaction with speakers of that language. Insofar as our reactions to others' recurrent use of a word have turned out to be and remain compatible with those speakers' apparent intentions, we believe to have *understood* what they intended; and insofar as we have abstracted a conceptual structure from repeated uses of a word, that conceptual structure is, for the time being, what we think of as its meaning. The more frequent the situations in which our meaning of a word seems to fit a speaker's intention, the more we will tend to believe that it is *the* conventional meaning—and almost inevitably we forget that fit, no matter how often it might recur, does not demonstrate that our understandings actually match a speaker's intended meaning. There is always a next occurrence of the word that may show us that our understanding was a misunderstanding.

To *interpret* an utterance or a written piece of language (be it a message or a text) requires something more than the construction of its conventional linguistic meaning. In fact, to interpret an utterance requires the insertion of whatever we consider its conventional meaning into a specific experiential context. In the case of a prosaic message, this is relatively easy to see. If a subject, S, let us say Susan, leaves her office, walks out on the parking lot and picks up a sheet of paper on which someone has written, 'Thursday, November 11th, 3 p.m.', she will have no difficulty in understanding the words or symbols, but she will probably be quite unable to *interpret* them. They clearly specify a particular hour on a particular day, but since she has no clue as to why that point in time is being specified, she has no way of relating that conventional meaning to the framework of her own experiential world. Had she found the sheet fixed to her car in a way that she would consider deliberate, she would search her mind for a possible sender and a plausible interpretation in terms of an experiential event or situation to which the message might refer. But the sheet of paper came to Susan from nowhere and without a pragmatic context. Hence, though she knows what it says, she cannot tell what it is intended to mean.

In the case of a text, the situation is more complicated. First of all, there are different kinds of text. If Susan, instead of a single sheet, had picked up and read a primer or a school book of any discipline, she might have acquired some new conceptual structures because the text would have led her to combine conventional meanings she already possessed in ways she had not previously combined them. In that sense, the text could modify and expand the range of her over-all conceptual

network. The mechanism of such conceptual expansion is, in principle, similar to the mechanism that enables us to acquire a concept of *mermaid* without having to construct it from direct experience. There is nothing obscure about it. It derives from the fact that the language user comes to establish 'conventional rules of language'. As in the case of word meanings, the speakers of a given language come to obey and abstract rules of word combination as a result of their continuous interaction; and again, any such adaptation and abstraction must be based on the individual construction of patterns of concepts and actions which turn out to be compatible with actions and reactions of other users of the language.

Primers, school books, and the like, usually declare their didactic purpose in the title, which might say 'Italian for Travelers', or 'Introduction to Meteorology', and they give sufficient indications as to how the concepts they explicate are to be linked to potential experiential situations. That is to say, they explicitly point out what was lacking in the message on the sheet of paper, namely, how the reader who understands the words and phrases they contain can apply his understanding to his or her own experiential world. And since that experiential world may at some time come to comprise a journey to Italy or an occasion to discuss meteorology, the conceptual structures Susan may derive from the found text are, at least in principle, liable to be tested for compatibility in interaction with others.

What, however, if Susan had found a novel? Literary writings usually do not indicate their purpose. They may, of course, have some didactic effect, but that effect is, as a rule, considered beside the point in a discussion of literary interpretation. If the novel Susan finds, for instance, describes at some point someone walking in Paris, and Susan gathers from that description how one gets from the Pont Saint Michel to the Place Vendôme, that kind of learning would surely be deemed irrelevant to the interpretation, let alone evaluation, of the novel as a piece of literature. Yet, it is far less clear whether the fact that a novel suggests to the reader a way of dealing with a fiercely jealous spouse is to be deemed altogether irrelevant from a literary point of view.

Is it the author's didactic intention that matters? One can hardly doubt that Ibsen wrote *Ghosts* to teach the public a lesson. And while there may be little, if anything, to warrant the assumption that Goethe published *The Sorrows of Young Werther* in order to warn young men against falling in love with married women, it would be difficult to maintain that, when writing *Faust*, he did not intend to impart some kind of wisdom. However, if we accept any such supposition—and it might seem quite reasonable to do so—it immediately raises a serious question: How on earth can a reader be sure that the wise conclusions he or she draws from the text do, in fact, constitute the wisdom the author intended to impart? That question, needless to say, must be raised not only with regard to wisdom but with regard to any *deeper* meaning or content that is presumed to lie beyond the conventional linguistic meaning of words and phrases.

Any proficient speaker of the language in which a literary text is composed can be expected to understand the words and phrases the text contains.³ But that kind of understanding (which, in principle, is equivalent to what Susan could bring to the found message) is not the kind literary scholars have in mind when they discuss

whether or not a certain interpretation of a text is justifiable, plausible, or correct. I submit that whatever one might choose as the measure of justification, plausibility, or correctness when one is concerned with literary interpretation lies beyond the realm of linguistic competence (which is taken for granted as prerequisite) and involves relations one establishes between the conceptual structures called forth by the text and the conceptual network that constitutes one's own experiential world. These relations, by definition, are subjective, in the sense that they cannot connect anything but the reader's own conceptual structures with the reader's own experiential world.

Again, there are three types of elements involved: the conceptual structures that constitute the linguistic understanding of the text; the over-all conceptual fabric that constitutes what we call our experiential world; and the conceptual links used to connect the two. Analogous to the way new conceptual connections are formed when we first encounter the word 'mermaid' and construct a meaning for it, reading a piece of literature may lead us to modify or extend the conceptual fabric of our world. But whereas we can test our concept of mermaid for viability in contexts where others use the word, there is usually no possibility of testing one's interpretation of a novel or a poem for its compatibility with the author's intentions.

The fact that the reader has, as a rule, no possibility of interacting with the author renders it questionable, to say the least, whether it could ever be established that a given interpretation of a text is *right* or *correct* in the sense that it embodies the author's intended meaning. In the first place, there is no way of establishing whether or not the text is, in fact, a viable expression of the intended meaning. Authors, after all, have no external reason to question the expressive adequacy of their texts, unless they become aware of the fact that readers interpret them in ways that are incompatible with what they, the authors, intended. If an author does become aware of such a discrepancy (in the contemporary scene it might, indeed, be brought home to him by a critic's review), he may still tend to blame the particular reader's insufficiency of interpretive acumen or effort rather than his own technique or ability of expression. (There are, I believe, few instances of authors rewriting their literary texts because of readers' misinterpretations.)

The question of the expressive adequacy of a text, however, becomes almost irrelevant in the face of the obstacles that preclude any verification of a reader's interpretation. If no direct interaction between reader and author takes place, there may, of course, be some indirect interaction, in the sense that the reader interprets other works, comments, or explanations of the author in question. Any such further reading may or may not lead to a modification of the reader's interpretation of the first work; but the interpretation of the subsidiary readings will, as a rule, be no less uncertain than the interpretation of the original text—and two uncertainties do not add up to more certainty.

If, indeed, the reader consults critics' or other experts' comments and explanations, this complicates the issue, because it introduces yet another interpretive step. What critics and experts say, again, can relate only to their own interpretation of the author's text and not to the author's intended deeper meaning.

The reader thus must interpret what they say about their own interpreting. At best, this may lead to some consensus about how the text *can* be interpreted, given the conceptual fabric that constitutes the reader's and critics' experiential world. But such a 'shared' experiential world exists only to the extent to which individuals have interactively established a consensus; it cannot possibly extend to include an author who has not participated in that interaction.⁴

Theoretically, then, one would expect that individuals of an interacting social group could arrive at a consensus concerning the interpretation of a given text. In fact, that seems to happen in certain places and at certain times. But since whatever consensus is achieved can be no more and no less than a relatively smooth fit of individual actions and reactions, a consensus concerning an interpretation does not, and cannot, imply that the participating individuals' interpretations have to be the same. A consensus merely requires that the manifestations of their interpretations are mutually compatible and do not give rise to perceptible clashes.

Thus there would seem to be an inevitable indeterminacy about the *correctness* of anyone's interpretation of a text. No amount of investigation of related texts and no amount of interpreting other readers' or critics' interpretations could ever establish that there is *one* true meaning of a text, let alone one that matches the author's intended meaning. By means of direct interaction, some interpretations may be eliminated as no longer viable, but they cannot confer the stamp of uniqueness or correctness on any that survive. The viability of an interpretation, after all, can be assessed only from the interpreter's point of view.

This state of affairs is analogous to the state of affairs in science. No matter how well a theory works within the framework of scientific goals—explanation, prediction, and control—it can never be shown to describe or match an ontological reality, nor can it be shown to be the only possible interpretation of the scientists' experiences. There is, however, an important difference. The scientist has, as a rule, a fairly well-defined framework of goals. He searches for explanations with a view to predicting and controlling experiential situations. In that respect, the mantic (though he may have used a different methodology) is related to the scientist, because he, too, interpreted signs and omens in terms of experiential situations. Both he and the scientist are judged according to how well some experience that is subsequent to their pronouncement can be fitted into their prediction.

By contrast, literary interpreters, though they might be said to explain the texts they interpret, cannot refer to subsequent experience as testing ground of their interpretations' viability. While the scientist's interpretation of experience or experiments and the mantic's interpretation of omens are, in the last analysis, always an instrument for the management of further experience, the interpretation of a literary text seems to be an end in itself. The constraints within which it attempts to achieve viability are set by the text alone and not by any external area of experience. Hence, the quest for *the* interpretation of a text turns out to be a futile undertaking. It would seem more appropriate to consider objects of literature, and of art in general, in the way suggested long ago by Paul Valéry (1933):

CHAPTER 6

Once published, a text is like an appliance of which anyone can make use the way he likes and according to his means; it is not sure that the builder could use it better than others. Besides, he knows well what he wanted to make, and that knowledge always interferes with his perception of what he has made. (p. 1507)

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NOTES

- ¹ This parallelism of theories of interpretation and theories of knowledge must have been noticed by literary scholars who happened to read Montaigne, Vico, Berkeley, Hume, or Kant. Recently it has been explicitly stated, for instance, by Fish (1980), Schmidt (1980), and Craige (1982).
- ² I have placed 'identity' between quotation marks because even on the technical level the identity of codes can be achieved only within the causal determinism of the machinery and becomes uncertain at the interface with the human user who interprets the machine's output.
- ³ With this I deliberately exclude works such as Joyce's *Finnegans Wake* from the discussion, because no one could reasonably claim to be a proficient speaker of the language in which they are written.
- ⁴ In this context it is imperative to remember that the interactions that are relevant to interpretation must be linguistic as well as nonlinguistic.

PIAGET AND THE RADICAL CONSTRUCTIVIST EPISTEMOLOGY

We do not experience *things*; things are a construction of ours the function of which is to emphasize the resemblance between aspects of our present immediate experience and aspects of our past experience, something which it proves enormously useful to do. (Percy W. Bridgman, 1936, p. 18)

Eight years ago, in his Introduction to Piaget's *Six Psychological Studies* (1967a), David Elkind began with an observation that is both perceptive and ambiguous: "Although Jean Piaget could legitimately lay claim to being a psychologist, logician, biologist, and philosopher, he is perhaps best understood as a genetic epistemologist" (p. v).

The phrase 'he is perhaps best understood' could be taken to mean that Piaget is, in fact, well understood as an epistemologist, and not so well as psychologist, logician, etc. But that would not have been true in 1967, nor, indeed, would it be true today. Instead, we have to interpret the phrase as an exhortation: In order to understand Piaget, one had better consider what he has to say about epistemology. In the course of his Introduction, Elkind makes it abundantly clear that this is what he intended, and his exhortation is no less pertinent now than it was seven years ago. In much of what has since been written either pro or contra Piaget, there is little evidence that his 'Genetic Epistemology' has been understood.

DIFFICULTIES OF INTERPRETATION

There are several reasons for this lack of comprehension. Piaget expresses himself with none of the clarity and precision which, we are often told, are inalienable features of the French language. His translators, therefore, often face what seem insurmountable problems. Also—and this, of course, is my subjective judgment—Piaget himself has for a long time understated the import and the reach of his more radical epistemological ideas. His works are, in fact, full of formulations that give the reader a spurious sense of security. Words such as 'perception', 'reality', 'environment', 'object', and 'cognition' are frequently used without any indication of the very special epistemological status Piaget has given them in the passages where he elucidates them as terms of his own constructivism. In the last decade, however, he has made the revolutionary aspect of his ideas a good deal more explicit and there is no longer any doubt that much of what has been said in the past about genetic epistemology will have to be revised.

Revision may, indeed, be too gentle a word for the kind of reorganization of ideas which, I believe, is indispensable for an understanding of the theory of knowledge which Piaget's constructivist formulations entail. It is not a question of merely adjusting a definition here and there, or of rearranging familiar concepts in a somewhat novel fashion. The change that is required is of a far more drastic nature. It involves the demolition of our everyday conception of reality and, thus, of everything that is explicitly or implicitly based on naive realism; it shakes the very foundations on which 19th century science and most of 20th century psychology has been built, and it is, therefore, not at all unlike the change that was wrought in physics by the joint impact of relativity and quantum mechanics. The fact that Piaget himself now and then reverts to formulations which imply an earlier 'interactionist' view (Piaget, 1972, p. 17) and which are logically incompatible with the constructivist ideas expressed elsewhere, may make it difficult to guess what Piaget actually believes, but it does not invalidate the logical coherence of radical constructivism as a model of human experience.

Early on, in the first of the *Six Psychological Studies*, we read:

The period that extends from birth to the acquisition of language is marked by an extraordinary development of the mind. ... This early mental development nonetheless determines the entire course of psychological evolutions. In fact, it is no less than a conquest by perception and movement of the entire practical universe that surrounds the small child. At eighteen months or two years this 'sensorimotor assimilation' of the immediate external world effects a miniature Copernican revolution. At the starting point of this development the neonate grasps everything to himself—or, in more precise terms, to his own body—whereas at the termination of this period, that is, when language and thought begin, he is for all practical purposes but one element or entity among others in a universe that he has gradually constructed himself, and which hereafter he will experience as external to himself. (Piaget, 1967a, pp. 8-9)

It would be easy to read this passage and put it aside as containing nothing particularly unusual. True, there is talk of a 'conquest by perception and movement', a 'Copernican revolution', and of a 'universe that he (the child) has gradually constructed'—but we are so often bombarded with pompous nonsense and esoteric metaphors that we have become quite accustomed to the need to separate what matters from the overblown phrases. In this particular case it is also much more comfortable to pick out what happens to fit our normal way of thinking and to consider the rest the kind of emphatic noise that is not at all uncommon in the more old-fashioned European writers. Taken seriously, a statement to the effect that the child constructs his universe and then experiences it as though it were external to himself, would be rather shocking. We would all like to be hard scientists, and such an 'as though' threatens to pull the rug from under our feet. It smells of an intellectual allergy; it makes us extremely uncomfortable, to say the least.

This threat is, I believe, the more serious obstacle we have to overcome if we want to understand Piaget's theory of knowledge. It is made all the more difficult, because Piaget himself sees to it that we rarely meet it face to face. His method of exposition, even in his latest writings, is such that it successfully shields the reader from a direct apperception of the radical aspect of genetic epistemology. A good example of this can be found in the same *Psychological Study*, immediately before the passage quoted above.

One can say, ... That all needs tend first of all to incorporate things and people into the subject's own activity, that is, to 'assimilate' the external world into the structures that have already been constructed, and secondly to readjust these structures as a function of subtle transformations, that is, to 'accommodate' them to external objects. From this point of view, all mental life, as indeed all organic life, tends progressively to assimilate the surrounding environment. (Piaget, 1967a, pp. 7-8)

By saying this a page before the piece I have quoted first, Piaget makes it all but inevitable that the reader will take as a mere metaphor anything that is said later about the organism's construction of reality. Nothing in this first passage raises the slightest suspicion that the 'external world', the 'external objects', and the 'surrounding environment' are, for Piaget, not exactly the same items as, say, for Skinner or for any other author who has never been troubled by epistemological considerations. Thus it is not at all astonishing that the reader will not take it literally when Piaget, one page later, says that the child himself constructs his universe and thereafter experiences it as external to himself. It is asking too much that the reader, coming to this later formulation, go back to the preceding statement, which he has already fitted into his realist views of the world, and, *instead*, adjust his ideas to it by generating a constructivist epistemology. Even an experienced and basically sympathetic reader is likely to be misled. Here, for instance, is Harry Beilin's explication of 'assimilation' and 'accommodation':

The construction of knowledge, more specifically, takes place through the operation of two general processes under the control of an internal self-regulating mechanism (equilibration). The first of these is the assimilation process. It involves the incorporation of environmental data (through physical or mental activity) into existing cognitive structures. The products of new experience are incorporated into mind only to the extent that they are consistent with existing structures. ... The other process under the control of the equilibrating mechanism is accommodation. This represents the subject's response to external stimulation by which existing cognitive structures are effectively utilized for adaptive purposes by becoming integrated with other internal structures or by differentiating as they are applied to new experience. ... In most situations in which the organism functions, both assimilative and accommodative aspects of development take part, although one process dominates, depending upon the demands of reality. (1971, pp. 88-89)

What is said here about assimilation and accommodation is directly derived from Piaget's own formulations—in the quoted passage as well as in many others. Beilin has successfully 'assimilated' it into pre-existing cognitive structures. The changes that have resulted from this assimilation are unobtrusive. Instead of 'things and people' we now have 'environmental data'; instead of 'structures that have already been constructed', we have 'existing cognitive structures' (which makes them a little less subjective and transient); and then, of course, we now have 'stimulation' and 'response' words which can be relied on to appease any conventional psychologist's doubts.

The result is a definition of the two extremely important Piagetian terms which, while making it much easier to accept Piaget, eliminates every trace of his radical constructivism. Assimilation, according to this explication, involves the incorporation of environmental data into existing cognitive structures; accommodation, on the other hand, involves external stimulation and leads to the adaptation of the organism's internal structures to an environmental reality. Who would suspect that Piaget is an unorthodox thinker? Indeed, this explication rests on the most solid traditional foundations. The cognitive structures which the organism acquires in this fashion are then considered the organism's knowledge of the world and, inevitably, there is the implication that the more the cognitive structures are adapted to the 'environmental reality' the better and 'truer' will the organism's knowledge be.

THE REALIST DILEMMA

After twenty-five hundred years of philosophical theories of knowledge, all of which attempted to deal in one way or another with that mysterious process of cognition that is supposed to adapt our knowledge to things as they *are* in an independent outside 'reality', there seems to be no reason why we should balk at a phrase such as 'incorporating environmental data into cognitive structures'. The educational processes to which we have all been subjected may have failed in many ways, but they have been eminently successful in habituating us to the acceptance of absurdities. Thus, no eyebrows are raised at such a phrase, nor does it prod us at once to ask how environmental data (by definition outside us) might get through what we should call the experiential interface so that they can be incorporated into our cognitive structures; or, similarly, how we could possibly accommodate our cognitive structures so that they become more adapted to the 'demands of reality', when the structures are, by definition, in us, while the 'reality' is supposed to be on the other side of the interface.

Psychologists, with the exception of George Kelly (1963), have preferred to ignore this basic puzzle. That is, I am sure, why Piaget does not particularly like to be considered a psychologist. He has persistently struggled with these questions and, though his exposition is far from exemplary, he does, I believe, imply an answer that makes possible a viable model of cognition. The key to this model is revolutionary, not because it leads to a new theory of how we come to have what has hitherto been called 'knowledge', but because it radically changes the very

concept of knowledge. For that reason, as Elkind recognized, Piaget must be understood, first of all, as an epistemologist.

Ever since Plato the activity of 'knowing' or 'cognizing' has been viewed as a kind of copying or replicating (Ceccato, 1949; Piaget, 1961, 1967b, 1968). The cognizing subject was thought to acquire or build up inside himself a replica or image-like representation of the outside things, that is, the 'real' object, which he was getting to know. Thus, the outside became a reality to be discovered; and, quite inescapably, it had to contain the things which the subject had already replicated and got to know, as well as those which the subject might get to know.

"When I perceive I must become percipient of something, ... the object, whether it become sweet, bitter, or of any other quality, must have relation to a percipient", says Socrates (Plato, 1949, p. 22) in his rather specious attack on Protagoras; and since then we have lived with the problem of how to specify or describe this very peculiar relation between the, as yet, unknown but existing object on the one hand, and, on the other, the subject that has to replicate this object in his head in order to know it. The senses, quite naturally, were given the task of mediating between the 'real' object and the subject's representation of it. But this immediately raised the question as to how the subject could ever be sure of having an 'adequate' or 'true' representation, if the only way he can get at the object is through the mediation of his senses. Indeed, there is no rational way around the logical difficulty that the senses cannot possibly test the veracity of their own products. The careful, Platonic subject, hence, speaks of the 'illusory appearance' of things and, to gain some kind of certainty, falls back on 'innate ideas'. The more self-confident positivist, taking a cruder path, cavalierly ignores the problem and proclaims that the world simply has to be the way we see it.

THE CONSTRUCT OF THE 'OBJECT'

Piaget approached the problem of cognition from an altogether different angle. Before attempting to deal with highly complex conceptual items such as 'reality' or 'environment', he asked rather modestly, how does a child come to have the concept of an 'object' that has some kind of permanence in his stream of experience. Piaget's answer is common knowledge today and can be found, in one form or another, in any of the more recent introductory textbooks of psychology. Whichever way it is formulated, it is generally understood that 'object permanence' is the result of the subject's coordination of experiential data from more than one source. As a rule, however, the two epistemologically most important aspects of this subjective coordination are not made very explicit.

The first of these was stated in an approximate way already by Ernst Mach before the turn of the century (Mach, 1886) and summarized in one of his last papers.

I can see an object if I look at it, I can feel it if I touch it. I can see it without feeling it, and vice versa. As a rule, however, visibility and tangibility are linked. Although the emergence of the elements of this complex takes place

only on certain conditions, these are so familiar to us that we hardly notice them. We regard an object as being always present, whether or not it is sensible at the time. We are accustomed to regarding the object as existing unconditionally, although *there is no such thing as unconditional existence*. (Mach's italics) ... to extrapolate this experience beyond the proper limits of experience, and to assume the existence of a 'thing-in-itself', has no intelligible meaning. ... We have become accustomed to regarding an object as existing permanently. (1910, p. 30)

Stressing, as he does, the subject's active part in making objects 'exist permanently', Mach must be considered a forerunner of modern operationalism as it was launched by Bridgman in 1927. Bridgman, in fact, explains 'object permanence' in a way that is identical with Piaget's analysis:

When we say that we see a thing out there in space we are exploiting correlations built, by experience and repetition, into the structure and functioning of our brains. (Bridgman, 1961, p. 46)

Both Mach and Bridgman were concerned mainly with the 'existence' of objects, and they came to the conclusion that this 'existence' is the result of our own constructive coordination of experiential data and our subsequent projection of these coordinations into an 'outside' world. Neither Mach nor Bridgman considered the developmental aspect of the subject's coordinatory activities. This is what Piaget has done, and his empirical findings concerning the genesis of object permanence in infants are a splendid example of the experimental confirmation of an idea postulated on purely theoretical grounds. Besides, Piaget has made the process of coordination a good deal more explicit. When Mach said, "I can see an object if I look at it, I can feel it if I touch it", he gave no indication that this 'seeing' and 'feeling', too, must consist in coordinatory operations on the part of an active subject. That is to say, shape, form, or, generally speaking, pattern must be considered no less the result of a subject's coordinatory activity than the concept of a permanent object.¹ As far as I know, Ceccato, the founder of the Italian Operationist School, was the first to propose this 'radical' constructivism, and he did it on purely logico-theoretical and not, as Piaget, on developmental grounds. We call this school of constructivism 'radical' because it holds that the knower's perceptual (and conceptual) activity is not merely one of selecting or transforming cognitive structures by means of some form of interaction with 'existing' structures, but rather a *constitutive* activity which, alone, is responsible for every type or kind of structure an organism comes to 'know'. And this brings us to the second epistemologically important point in the analysis of object permanence.

THE BUILDING-BLOCKS OF CONSTRUCTION

Following the philosophers' traditional use of the expression 'sense data', psychologists rarely discriminate the two possible meanings of the term. They tacitly assume an epistemological position that is nothing but a reductionist brand

of realism and, therefore, incompatible with any radical constructivist theory of knowledge. If by 'sense datum' we mean a characteristic (primary, minimal, elementary, or whatever) of an independently existing object, that is, an item that possesses that characteristic independently of the subject's sensation, we are still faced with the unanswerable question whether the datum, as sensed by the subject, does or does not correspond to a 'real' characteristic of the object. Philosophers, as a rule, perform some sleight of hand to get out of this dilemma; psychologists merely look the other way. If, however, we take the second, more prosaic meaning of 'sense datum', which can be defined as elementary 'perturbation' or 'signal' on the knower's side of the experiential interface or, simply, as 'elementary particle of experience', we can get rid of the realist implications and of the problem of the impossible comparison between an 'inside' and an 'outside' datum.

Piaget has not been very explicit about this. In the conclusion of his work on perception (Piaget, 1961), for instance, he frequently talks about the 'object' as though he had Kant's noumenon in mind, that is, an absolute entity of independent 'reality', a thing-in-itself which, though perceived only approximately by our senses, must nevertheless have a structural correspondence to the phenomenon we experience sensorially. It all sounds as though, in spite of his genetic analysis of the child's construction of object permanence, he still felt the need to see the constructs of experience as replicas, at least in a structural sense, of really 'existing' objects.² However, in the light of what Piaget has said about the child's constructive activities during the sensorimotor period ('a universe ... which hereafter he will experience as external to himself') and many of his other statements on the 'externalization' of constructs (Piaget, 1967a, 1968, 1970b), one can hardly doubt that he would agree with Hebb's formulation: "At a certain level of physiological analysis there is no reality but the firing of single neurons" (Hebb, 1958, p. 461).

From the radical constructivist point of view, 'firings' are, of course, a metaphorical expression for the minimal units of experience; and 'neurons', in which they are said to originate, and the 'nervous system' that is said to process them, are constructs which, though placed inside the experiencing organism's 'body', are no less an externalization or projection *beyond the knower's experiential interface* than is any 'object' in the further reaches of the externalized world. Some such externalization, indeed, seems to be the prerequisite of any rational construction—and presumably also of any form of self-awareness that enables an organism to view his experience rather than merely to experience it.³ The crucial difference between the realist and the constructivist, thus, is not that the one projects his cognitive structures beyond the experiential interface, while the other does not; the difference is that the realist believes his constructs to be a replica or reflection of independently existing structures, while the constructivist remains aware of the experiencer's role as originator of all structures, that is, for the constructivist there are no structures other than those which the knower constitutes by his very own activity of coordinating experiential particles.

Instead of the 'firing of neurons', Piaget occasionally speaks of 'aliments' or of the 'given'.⁴ These terms, once more, shroud in mystery rather than clarify the

basic epistemological issue. If we want to avoid the unanswerable questions to which any version of ‘replica’ theory of knowledge necessarily leads (see above), Hebb’s formulation is a much safer one. The basic unit of experience, the ‘elementary particle’ of cognition, in the constructivist interpretation, is itself a construct—if only for the reason that, in direct experience, we are never aware of particles, let alone of the ‘firing of neurons’. But just as Bridgman found ‘thing’ to be an enormously useful construct (cf. quote on the first page of this text), so do we today, after several decades of neurological conceptualization, find the firing or signal of a neuron an analytically powerful construct. The important point is that such a signal or firing can be taken as a datum in its own right that need not be considered the effect of some independent and intangible cause. The constructivist, who remains aware of the fact that this ‘elementary particle’ is his construct which he imposes upon the flow of experience, may externalize it as the signal originating in a neuron and he may then consider all cognitive structures, no matter at what level of complexity, the results of the knower’s active coordination of these signals—and he may thus provide a consistent and non-contradictory analysis of knowledge that does not pretend to reflect in any sense the ontological ‘reality’ of an independent world. Ceccato, who in his early model of mental operations (1961) posited binary flip-flop devices as ‘differentiators’, called the firing signals simply ‘differentiata’. In his later model (1966) he derived not only shapes and objects but also abstract conceptual categories from regularities in the co-occurrence and the hierarchical structuring of particles of attention.⁵

From the radical constructivist point of view, then, both the raw material (i.e., the firings or ‘sense data’) and the cognitive structures which become the organism’s reality (i.e., the invariant patterns of coordination) are from the very outset ‘inside’ the cognizing system. This might seem to entail an inexorable solipsism—but this impression, I believe, arises solely from the ontological pretensions in which traditional philosophy has steeped us. The constructivist schools, Piaget’s as well as the Italian one, are well aware of the fact that no organism is free to construct any reality he might wish to construct and that, instead, there are certain constraints with regard to all construction; but these constraints are not specifiable in terms of ontological characteristics of independently existing ‘real’ structures to which we have no access. Any specification or description of the constraints, therefore, must be formulated in terms of the availability of single, as yet uncoordinated signals (i.e., particles of experience) and of the regularities or interdependence of these signals which the knowing organism, as a result of his own cognizing activity, singles out from his initially undifferentiated continuous stream of experience.⁶

THE ENVIRONMENT AS A BLACK BOX

The difference between the constructivist’s awareness that all coordination—and thus all structure—is the result of his own activities and, on the other hand, the traditional and common sense view that the cognizing subject in some way discovers structures that belong to an independently existing reality—this

difference may seem a subtle one, but it is crucial from the epistemological point of view.

Perhaps the most enlightening demonstration of this point was supplied by the pioneering work of Lettvin, Maturana, McCulloch, and Pitts (1959). On the basis of anatomical considerations and micro-electrode recordings from single fibers of the optic nerve, they established that the frog's visual system has four types of highly specialized 'detector' networks, one for sustained light-dark contrast; a second for small dark convex shapes; a third for a moving edge; and a fourth for sudden dimming of illumination. The speed with which signals are conveyed to the brain is different for each of the 'detectors', and there are certain conditions under which a discharge from one will cancel the discharge from another. The system as a whole makes the frog an efficient fly-catcher, because it is tuned for small dark 'objects' that move in an abrupt fly-like way. In the frog's natural habitat, as we, who observe the frog, see it, every item that possesses the characteristics necessary to trigger the frog's detectors in the proper sequence is a fly or bug or other morsel of food for the frog. But if the frog is presented with a black bead, an air-gun pellet, or any other small dark moving item, it will snap it up as though it were a fly. In fact, to the normal frog's visual apparatus, anything that triggers the detectors in the right way, *is* a 'fly'.

What are the epistemological implications of this frog story? The simplest way of putting it is perhaps this: Whatever is perceived is basically composed of signals within our sphere of experience. We are, of course, free to consider these original signals the effect of some outside causes. But since there is no way of approaching or 'observing' these hypothetical causes, except through their effects, we are in the same relation to that 'outside' in which the first cyberneticists found themselves with regard to living organisms—that is to say, we are facing a 'black box'. We may observe and record the 'output' from the black box (in this case the 'sense data', the signals on our side of the interface), and we may observe and record the 'input' to the black box (in this case 'proprioceptive data' and 'feedback signals', again on our side of the interface); both are neuronal signals—but once we have imposed a differentiation of 'input' and 'output', we can establish recurrent coordinations and more or less reliable dependencies between the two. Having done this, we can construct an 'external world' and our 'selves' on the basis of input-output relations.

The history of science and, especially, of technology shows how very far we can get by exploiting such input-output relations and the inductive inferences by means of which we predict future outputs from the black box world on the basis of regularities and invariances in the recorded past. It is a black box with which we can deal remarkably well. As far as its structure, its ontological character, is concerned, it is nonetheless black, and there is no hope of a rational way of dispelling its blackness. We can no more get at the objective character of its 'ontological reality' than the frog can get at the objective character of the items to which his visual system reacts. As far as the frog could 'know'—if his brain allowed for the complex processing involved in the kind of self-monitoring we call conscious cognition—any constellation of conditions that happens to trigger the

detectors of his fly-spotting system, be it insect, pellet, or accidental co-occurrence, would be a 'fly'; his representation, his concept of 'fly' could be defined only in terms of the neuronal signals that concur in the experience and never in terms of the inaccessible hypothetical outside 'causes' of these signals.

It should be clear that, in principle, our position cannot be different from the position of the frog. To say that it is, and to argue that we can discover aspects of an 'objective reality', because we are able to experiment and to modify our 'environment', is merely to extend the realist illusion. What we experiment with—no matter how elaborate the apparatus and the conversion of different sensory modes—is, in the last analysis, never anything but the interrelation of our signals which we have come to consider input to, and output from, the black box of the 'universe'; and what we modify or control by our activities are always, as William Powers (1973a) has formulated it, our own perceptions, that is, the signals we call sense data, and the ways in which we coordinate them. We may and do, of course, project structures that result from this coordinating activity into an 'outside', but even the most spectacular successes we achieve in predicting and controlling our experience give us no logical ground whatsoever for the assumption that our constructs correspond to or reflect structures that 'exist' prior to our coordinatory activity. Even if we posit causes for the sense data (i.e., the particles into which we can break up our experience), this does in no way entail that these causes exist in the spatio-temporal or other relational structures into which we have coordinated them. The fact that we can coordinate our own sense data into recurrent structures can never prove that these structures are ontologically real—it only proves that the individual data occur frequently enough in our experience for us to establish 'invariant' co-occurrences. There may, indeed, be countless ways of operating and arriving at coherent structures that are no less recurrently impossible on our stream of experience than the ones we have come to construct. To disregard this, and to attribute ontological status to our constructs is precisely what Kant (1783), in one of his mellow moments chidingly called 'dreaming idealism'; when scientists go in for this kind of dreaming it might deserve a harsher word.

The integrated hierarchical feedback theory that Powers has developed bridges the conceptual gap which, hitherto, made it almost impossible for a traditional psychologist to accommodate to the constructivism of the Piagetian or of the Italian school. In the first place, Powers (1973b) begins by pointing out the insufficiencies and unwarranted assumptions in Brunswik's (1952) epistemological model, a model that is more familiar to American psychologists than the philosophical precursors of either Piaget or Ceccato. Secondly, Powers' demonstration that a system's behavior can be better understood as modifying the system's perceptual 'input' rather than as the result of it, is a good deal simpler and more economical than any philosophical argument. It makes the epistemological point without talking about epistemology. Thirdly, the use of well-known cybernetical terminology makes Powers' work a great deal more accessible than either Piaget's or Ceccato's. When Piaget says:

In reality, the element to which we must constantly turn in the analysis of mental life is 'behavior' itself, conceived, as we have tried to point out briefly in our introduction, as a re-establishment or strengthening of equilibrium. (Piaget, 1967a, p. 15),

it is not at all easy to glean from this the underlying idea that behavior aims, not at modifying, some unknown and unknowable thing or event in an hypothetical outside world, but aims at diminishing the difference between the present coordination of sensory signals and a previous construct. This idea is nevertheless implicit in Piaget's concept of 'equilibrium', which as he reiterates often enough, is the result of both assimilation and accommodation. But the way he formulates this frequently suggests to the realist reader that what is being balanced, by means of assimilation and accommodation, are the organism's cognitive structures on the one side of the scales, and the 'demands' of a real environment on the other. A careful reading does, I believe, show that this is not what he intends.

Already in his volume on perception Piaget (1961) says quite clearly:

Knowing consists in constructing or reconstructing the object of knowledge in such a way as to grasp the mechanism of this construction; which is the same as saying (if one prefers to use the terms which positivism has persistently but ineffectually proscribed) that to know is to produce in thought (i.e., in the thinking mode), and the production must be such that it reconstitutes the way in which the phenomena are produced. (pp. 441-442)

There is, of course, a strong echo in this, of Giambattista Vico (1710), who first maintained that man can know only what man himself has made (i.e., produced, constructed), and who, therefore, is the acknowledged spiritual father of the Italian Operationist School. In recent texts on epistemology, Piaget's constructivism is even more explicit:

By contrast, for the genetic epistemologist, knowledge results from continuous construction, since in each act of understanding, some degree of invention is involved; in development, the passage from one stage to the next is always characterized by the formation of new structures which did not exist before, either in the external world or in the subject's mind. (1970a, p. 77)

... experience deals with the connection between characteristics introduced by action in the object (and not on its previous characteristics). In this sense, knowledge is abstracted from action as such and not from the physical characteristics of the object. (1972, p. 31)

KNOWLEDGE AND EQUILIBRATION

Given the definition of 'knowledge' that emerges from these passages, we may now ask how Piaget relates the concepts of 'assimilation' and 'accommodation' to the cognizing organism's continuous activity of construction and equilibration of constructs. The following passage, though it does not contain the actual terms

‘assimilation’ and ‘accommodation’, provides one of the most comprehensive and comprehensible answers Piaget has so far given.

Rhythms, regulations, and operations, these are the three essential procedures of the self-regulation and self-conservation of structures; anyone is, of course, free to see in this the ‘real’ composition of structures, or to invert the order by considering the operative mechanisms the source of origin, in an atemporal and quasi-Platonic form, and by deriving everything from these mechanisms. In any case, however, it will be necessary, at least with regard to the building up of new structures, to distinguish two levels of regulation. On the one level, the regulation remains internal to the already formed or nearly completed structure and, thus, constitutes its self-regulation, leading to a state of equilibrium when this self-regulation is achieved. On the other level, the regulation plays a part in the building up of new structures, by incorporating one or more previously built-up structures and integrating them as sub-structures into larger ones. (1968, p. 16)

In somewhat simpler terms, equilibration, in the cognitive realm, involves the adjustment of, for instance, percepts to conceptual structures which the perceiver already has assembled; and this adjustment of the new to the old is called ‘assimilation’. But cognitive equilibration also involves the adjustment of concepts to percepts, and this second type of adjustment, which can take the form of creating a novel structure or of combining several already assembled structures to form a larger conceptual unit, is called ‘accommodation’.⁷

In Piaget’s system—and this is the important point that is often overlooked—the ‘percepts’ which I have contrasted with ‘concepts’ in order to explain the two types of cognitive adjustment are not at all what they are for conventional philosophers and psychologists, for they, too, are the result of preceding rounds of assimilation and accommodation. This is the case, not only when the adjustments concern perceptual structures (i.e., coordinations of sensory signals) but it is the case in every act of cognitive adjustment. In other words, assimilation and accommodation are operative on every level of cognitive activity; what differentiates the two is the relative novelty of the constructs to which they give rise.

At the very beginning of an infant’s cognitive development, his coordinations of neuronal firings are, in Piaget’s view, already subject to assimilation and accommodation. Assimilation, because the actually occurring sensory signals are continually selected and coordinated to fit the genetically determined or ‘wired-in’ structures of perceptual and motor activity. Insofar as this selection and coordination varies from instance to instance to allow for the recurrent application of the fixed structures assimilation is at work; insofar as the fixed structures are modified, combined, or supplemented in order to fit novel instances of selection and coordination, accommodation is at work. Repetition of any structure results in its perpetuation, in the sense that it becomes a relative fixture that can be used again, once it has been assembled. When this use or application is achieved by an adjustment to the actually occurring sense signals or their coordination, it is a case

of assimilation; whereas if it is achieved by an adjustment of the assembled structure, it is a case of accommodation.

CONCLUSION

Though Piaget's writings make it difficult to establish his basic epistemological position unequivocally—be it because his ideas have continued to evolve in the course of a long life's work, be it because he prefers to coax his readers rather than force them into understanding—he has time and again made statements that go beyond a mere suggestion of a radical constructivist basis; they can be integrated into a coherent theory of knowledge only if we interpret them from a radical constructivist point of view.

The radical constructivist's interpretation of Piaget's genetic epistemology, then, consists in this: The organism's representation of his environment, his knowledge of the world, is under all circumstances the result of his own cognitive activity. The raw material of his construction is 'sense data', but by this the constructivist intends 'particles of experience'; that is to say, items which do not entail any specific 'interaction' or causation on the part of an already structured 'reality' that lies beyond the organism's experiential interface. As a cognitive construct, this 'interface' is a corollary of the organism's externalization of his constructs, an operation manifestly inherent in every act of self-consciousness or experiential awareness. Though externalization is a necessary condition for what we call 'reality', this reality is wholly our construct and can in no sense be considered to reflect or represent what philosophers would call an 'objective' reality; for no organism can have cognitive access to structures that are not of his own making.

Cognitive equilibration must be viewed as a kind of ideal state that is never achieved. The organism works towards it by assimilating the signals he is actually coordinating at a given moment (or stage) to the structures he has formed in the past; and he also works towards it by accommodating the already formed structures, whenever the signals with which he is operating cannot be fitted into one of the available structures as they are.

What saves this epistemological model from absolute solipsism is the constructivist concept of 'adaptation'.⁸ Once again, however, we must be careful not to interpret this term in the way current among realist biologists and psychologists. What the organism adapts to, and what ultimately determines the pragmatic viability of his constructs, are certain regularities in the input-output relations the organism registers, with respect to the black box which he experiences as his 'environment' or 'world'. Since the constructivist holds that all coordination and, therefore, all structure is of the organism's own making, he remains constantly aware of the fact that, though he may project his constructs beyond his experiential interface, he must not and cannot consider them ready-made structures of an ontologically prior world. The structures he calls 'things', 'events', 'stages', and 'processes' are the result of the particular way in which he himself has coordinated his 'particles of experience'; and the fact that, at a certain level of elaboration, he can assimilate a good many 'perceptual events' without further accommodation of

his cognitive structures, tells him nothing about the ontological reality of these ‘well-adapted’ structures. At best—and if he chooses to externalize structures such as ‘neurons’, ‘firings’, and a ‘central nervous system’—he may say that the universe is such that it will supply whatever conditions trigger his ‘receptors’ frequently enough for him recurrently to coordinate their firings, and to keep the thus coordinated structures relatively permanent. He may then call this relative permanence of certain structures ‘reality’—especially once he has managed to elaborate his cognitive construction to incorporate ‘others’ and ‘communication’.

As we know well enough from our own experience, at that level of elaboration, the ‘permanence’ or ‘universality’ of our cognitive constructs tends to become precarious. But even if it did not, even if we could achieve perfect intersubjective agreement of structures, it still would not get us across the border of the black box, because all it would tell us with certainty is that we, collectively, have found one viable construction.⁹ Such a construction becomes no more ‘real’, in the ontological sense, if we share it—it would still be based solely on signals on our side of the construct we have called ‘experiential interface’, and on the particular way in which we have categorized, processed, and coordinated these signals as input to, or output from, the construct we have called ‘universe’.

NOTES

- ¹ This, of course, does not preclude that certain patterns of visual, tactual, or other sensory data are genetically determined; but if they are ‘wired in’, this does not make them more ontologically real—it merely means that the coordinatory process was carried out phylogenetically by mutation and facilitation of survival rather than ontogenetically by the individual perceiving subject.
- ² This, indeed, is the reason why Piaget is often described as an ‘interactionist’ and why his reader at times gets the impression that the Genevan constructivism does not really go any further than the relatively conservative epistemological models of ‘active’ cognition proposed, for instance, by Cassirer (1932), Reichenbach (1938), Brunswik (1952), and others who still cling to some idea of correspondence between cognitive structures and independently ‘real’ structures.
- ³ Anatol Rapoport (1949) said: “Knowledge consists of an ordered sequence of neuromotor events” (p. 185). Since knowledge implies a knower, ‘neuromotor events’ necessarily have to be taken as an externalization of elementary experiential particles, or combinations of experiential particles, and not as items existing on the other side of the knower’s experiential interface.
- ⁴ E.g., “the necessary and continual internal coordinations that make possible the integration of external cognitive aliments” (Piaget, 1967b, p. 34).
- ⁵ Ceccato’s model of the basic mental processes could, I believe, be considered a cybernetical implementation of the approach to the structure of knowledge worked out by G. Spencer Brown (1969); in both systems all structures are hierarchical arrangements of binary ‘differentiata’.
- ⁶ It is the failure to appreciate this aspect of constructivism and the ancestral belief that science must be able to discover the world *as it is* which leads, for instance, George G. Simpson (1963) to refer to Bridgman’s despairing conclusion that “the very concept of existence becomes meaningless” (p. 96). It is apparently extremely difficult for traditional thinkers to separate the concept of ‘knowledge’ from the fiction of ‘absolute existence’ and to realize that this epistemologically necessary separation does not imply the demise of ‘science’.
- ⁷ As C.D. Smock (1974) put it: “Equilibration ... is a response to internal conflict ... a matter of achieving accord of thought with itself” (p. 154).
- ⁸ For a somewhat different argumentation with regard to solipsism, see Heinz von Foerster (1973).

PIAGET AND THE RADICAL CONSTRUCTIVISM

- ⁹ This aspect of constructivism has been extensively treated by Humberto Maturana (1970a).

PART II

THEORY OF KNOWLEDGE

ASPECTS OF CONSTRUCTIVISM

Vico, Berkeley, Piaget

Constructivism is a way of thinking about knowledge and the activity of knowing. For me, this way of thinking arose out of the practice of psycholinguistics, cognitive psychology, and some ten years of studying the works of Jean Piaget. In my effort to assimilate Piaget's ideas into a coherent, non-contradictory model of what I would call our rational component, I may, as some knowledgeable Piagetians have remarked, have gone beyond what Piaget intended when he spoke of 'constructivism'. That is one of the reasons why, at a certain point, I decided to call my way of thinking 'Radical Constructivism'.

Recently the reviewer of a paper of mine commented that this approach to the problems of knowledge is 'post-epistemological'. I like this description because it accentuates the radical shift of the relation between knowledge and ontology. This shift replaces the conventional notion of ontological truth with the notion of experiential viability and leads to a far-reaching reorganization of the way we think of the world. I hasten to add that the constructivist perspective is not a new invention. I claim no credit for it, except as the coordinator of other people's ideas.

In what follows I briefly want to explain my interpretation of certain key ideas proposed by three among the thinkers from whom I drew as I was formulating the radical theory of knowing. Vico and Berkeley have been dead for almost two and a half centuries, Piaget for a decade. None of them, I suspect, would agree wholeheartedly with my interpretation of what they intended to say. This does not greatly worry me. All three tended to disagree with most, if not all, of the interpretations of their work that came to their eyes during their life time. Had they witnessed the most recent developments in the philosophy of science, they might have been radicalized themselves.

One of the reasons why they were bound to be misunderstood was the simple fact that their thinking was truly original and involved a drastic change of some very basic concepts such as 'being', 'truth', and 'reality'. When they wrote about their ideas, however, they had to use words of the given language, words which their readers were likely to understand in terms of customary meaning, not in terms of a novel conceptualization.

Conceptual change, one might think, can be announced and made clear by means of explicit definitions. This sounds quite easy. In practice, however, it often goes wrong. Vico, Berkeley, and Piaget, each in his way, provide excellent examples of the problem. I shall take the two earlier ones, who were close contemporaries, first.

VICO'S BREACH OF A PHILOSOPHICAL TRADITION

In the year 1710 two treatises on epistemology were published at opposite ends of Europe, one in Naples by Giambattista Vico, the other in Dublin by George Berkeley. Both authors broke with the age-old philosophical belief that human knowledge must in some way provide a representation of an observer-independent material reality. To many such a breach of a venerable tradition did not only seem unacceptable but was indeed inconceivable.

When Vico said that humans can know only what humans have made, whereas God can know the world because He created it, his readers did not see, or did not want to see, that this statement implied not simply a restriction, but a change of the concept of human knowledge. In the philosophical tradition of the Western world, 'to know' meant to have captured something that was true because it was 'objective'. And in this tradition, to be objective meant to represent something as it was supposed to exist in itself and for itself, something that was as it was, irrespective of any knower.

Most of the great philosophers were careful to mention that the human representation of the objective world would never be more than an approximation, but they nevertheless held out the hope that the approximation could become closer with time. This, indeed, was the basis of the general belief in the progress of knowledge and that this progress could be measured in terms of a better match with reality.

The sceptics, of course, had always argued that such a match between knowledge and reality could not be ascertained because knowledge could be compared only with other knowledge and never with reality itself. Since the sceptics' arguments could not be logically refuted, philosophers tried to get around them in the most ingenious ways. Their efforts make fascinating reading but, after more than two thousand five hundred years, it is fair to say that they did not succeed. One interesting aspect of this struggle is the fact that neither the sceptics nor the other philosophers thought of changing the concept of knowledge. In contrast, Vico's treatise (1710) suggested precisely that. He crystallized this view in the striking phrase: "God is the artificer of nature, man the god of artifacts" (1710, chapter 7, §III). This entails a radical change. If humans can comprehend only what the human mind has made, namely its artifacts or, as we might say today, its 'models', it is clear that the human mind could never comprehend God's reality.

Vico's notion that in order to know something one has to be able to account for the elements it contains and to trace the steps taken in putting them together, shifts the focus of attention from a supposedly pre-existing world to the active, generating agent of cognition. Instead of a passive recipient of data or 'information', the cognizing subject is now the maker of knowledge. This change of perspective, obviously, has serious consequences for the notion of science and the character of scientific knowledge which, as a rule, is thought to be objective and therefore capable of providing the most reliable way towards a 'true' representation of the observer-independent 'real' world. Vico saw this with

admirable clarity and did not hesitate to give us a revolutionary definition of human science as the endeavor “to make things correspond to one another in handsome relations” (1710, chapter 7, §IV).¹ The ‘things’ he has in mind are, of course, the products of the human activities of perceiving and conceiving, that is, human constructs. Hence one can see in Vico the pioneer who explicitly embarked on the road of constructivism.

BERKELEY’S DEFINITION OF EXISTENCE

Berkeley was not as explicit with regard to the subject’s constructive activity, but he was no less incisive in his contention that the things we perceive are the result of our perceptual activity and cannot be said to have ‘existence’ in themselves. Innumerable philosophical discussions have been written about his famous dictum ‘esse est percipi’, but all I have seen are flawed from my point of view. Berkeley (1710) introduces the Latin phrase that states that ‘being is to be perceived’ at the very beginning (§3) of his *Treatise concerning the principles of human knowledge* and he uses it to present as crisply as possible his view concerning “what is meant by the term exists, when applied to sensible things”:

The table I write on I say exists, that is, I see and feel it; and if I were out of my study I should say it existed—meaning thereby that if I was in my study I might perceive it, or that some other spirit actually does perceive it. There was an odour, that is, it was smelt; there was a sound, that is, it was heard; a colour or figure, and it was perceived by sight or touch. This is all that I can understand by these and the like expressions. For as to what is said of the absolute existence of unthinking things without any relation to their being perceived, that seems perfectly unintelligible. (Part 1, §3)

I do not think that this passage is difficult to understand, provided one does not feel compelled to reconcile it with a preconceived idea of what ‘existence and the like expressions’ must mean. Yet, Berkeley’s contemporaries and his later commentators were unable to overcome this compulsion. They have gone, and still go, to great lengths to unravel how Berkeley might have intended the statement *esse est percipi* when it must have been obvious to him that the table he was writing on did not have to be perceived in order to exist. His philosophical critics are unable or unwilling to see that he is simply suggesting a new definition.

Berkeley argues that, qua experience, the table cannot be generated by anything but acts of perception which the perceiver coordinates to form the thing that goes by the name of ‘table’. Where human knowledge is concerned, therefore, ‘sensible things’, that is, the sensory objects of which we are inclined to say that they ‘exist’, are the objects we generate as we perceive.

In other words, Berkeley creates a definition of what it means ‘to exist’ that seems unacceptable to traditional philosophers, because their tradition has held for over two thousand years that perception requires, as Socrates said, that something must exist before it can be perceived.

This, needless to say, is, a very partial exposition of Berkeley's epistemology. He is well aware of the fact that, once perceived, the idea of a thing can be re-presented, reflected upon, modified, and named by the human mind to serve in other circumstances—and that, once named, the name of the idea can be generalized. And in perfect consistence with this, he says that general ideas do not exist, because what we perceive is always a particular individual. He is also aware of the fact that the perceptual coordination that generates 'things' from simple sense impressions requires relational notions which, themselves, are not perceptual. Consequently, these notions, that include space, time, and number, are 'things of the mind' for him and do not belong to the realm of existence that is determined by perception.

This, I would claim, is perfectly compatible with Vico's assertion that the human mind can know only what the human mind itself makes. Indeed, in his *Alciphron*, discussing the knowledge we might have of God, Berkeley says:

Passions and senses, as such, imply defect; but in knowledge simply, or as such, there is no defect. Knowledge, therefore, in the proper formal meaning of the word, may be attributed to God proportionably, that is, preserving a proportion to the infinite nature of God. We may say, therefore, that as God is infinitely above man, so is the knowledge of God infinitely above the knowledge of man. (1732, p. 170)

One might interpret the difference between human knowledge and God's as a merely quantitative one, but given Berkeley's firm assertion that, for us, only what we can perceive has existence, I prefer to believe that the world God knows must be the result of a rather different divine perception. Hence, as a crude capsule of the two treatises of 1710 one could say that the Neapolitan proposed a change of the concept of human knowledge and the Irishman a change of what is meant by the expression 'to exist'.

PIAGET'S CONTRIBUTION

Both these proposals are contained in Piaget's famous principle: 'Intelligence organizes the world by organizing itself' (1937, p. 311). But there is a big difference between the two 18th century philosophers and Piaget, if one considers what prompted them to embark on their unorthodox ways of thinking. Vico and Berkeley were driven to approach the problem of human knowledge for two reasons. They profoundly disliked scepticism, which, though it had successfully undermined the concept of objective knowledge, was unwilling to relinquish it. Both Vico and Berkeley were convinced that Descartes had not only failed in neutralizing the sceptical disintegration of traditional values and beliefs but had indeed aggravated the situation by his pursuit of doubt in order to establish certainty. In contrast, Piaget developed his constructivism on a biological basis. In his works, Piaget frequently reiterates two points that are of fundamental importance in his epistemology:

1. knowledge is not a copy of reality but an accommodation to reality; and

2. the cognitive activity is adaptive.

If one insists on interpreting these statements within the framework of the traditional theories of knowledge (which have taken no account of either Vico's or Berkeley's proposals), one will be driven to conclude something like the following: It is no great revelation that knowledge cannot be a copy of what exists—knowledge, after all, is a mental commodity, and the world that exists is material; and to say that it is adaptive, is simply to state what we all believe, namely that our knowledge, as we work at it over time, tends to provide a better correspondence with reality.

This, I claim, disregards the fact that Piaget started as a biologist, investigated cognition as a biological function, and was not concerned with any traditional epistemology. As a boy, he transplanted mollusks from a lake to a fast flowing brook and observed that their next generation had accommodated their shape to the flowing water. Not having read Piaget's teen-age publications, I do not know whether he drew any general conclusions from the experiment at that time. However, it must have laid the foundation for his later work on the forces that operate in evolution. In any case, when he wrote *La construction du réel chez l'enfant* (1937) he had formulated the principle according to which individual organisms accommodate when their internal equilibrium is perturbed.

Adaptation, thus, does not mean that organisms become like the environment, but rather that they find a way of 'surviving' in the face of such environmental perturbations as they experience. Cognition, then, is an adaptive function, not because it might (or should) produce a mental representation of an independent objective world, but because it endeavors to produce viable conceptual structures that enable the cognizing subject to fit into the world it experiences. One of Piaget's descriptions of this process is:

The general model of equilibration shows the interaction between observables and coordinations, that is, the collaboration of empirical and reflective abstractions on all levels. (1977a, p. 14)

Piaget's 'observables' are obviously perceived and therefore belong to what Berkeley posited as 'existing'. The coordination of such *sensibilia* by means of empirical abstraction then give rise to the items that Berkeley called 'sensible things' or, more strikingly, 'the furniture of the earth' (1710, Part 1, §6). And the actions of coordinating and abstracting involved in this process are actions of the cognizing subject or, in Vico's terms, are *made* by the subject and therefore accessible to its knowing.

With regard to reflective abstraction, Piaget has made it very clear that this way of operating would not be possible without the use of symbols. This, again, is something that Berkeley was well aware of when he said: "It is not, therefore, by mere contemplation of particular things, and much less of their abstract general ideas, that the mind makes her progress, but by an apposite choice and skilful management of signs" (1732, p. 304).

In spite of such echoes and apparent compatibilities, however, it would be doing violence to Piaget's epistemology to equate it with Berkeley's or Vico's. Whereas

the Irish philosopher clearly asserts that his ‘sensible things’ do not imply the existence of things in themselves prior to the acts of perception, and in Vico’s view ‘man’s artifacts’ are in no sense replicas or copies of objects in God’s world, Piaget does not altogether remove human knowledge from the notion of an independent ontological reality—he merely changes the relationship between the two. To him, as biologist, the constructs that result from empirical and reflective abstraction are at best adapted to the experienced environment. Like the physical structures that arose in biological evolution, the concepts and the picture of the experiential world which a cognizing individual constructs must prove viable in maintaining that individual’s equilibrium. Thus the function of the cognitive capability is not to produce a ‘true’ picture of an independent objective world, but rather to produce a livable organization of the world as it is experienced.

I have often been told that this is too radical an interpretation of Piaget. All I can answer is that in my view no interpretation, not even mine, should ever be considered the only one possible. On the other hand, I am confident that there are solid grounds to justify my interpretation as a viable one. In his *Insights and illusions of philosophy*, for example, Piaget says: “What is important is the triad *reflection x deduction x experiment*, the first term representing the heuristic function and the other two cognitive verification, which is alone constitutive of ‘truth’” (1971, p. 232; my emphasis).

If we remember that experiments are, after all, nothing but controlled experience, the passage clearly states that, in order to be considered ‘truth’, the products of reflection and deduction must prove viable in the realm of our experience. And for something to prove viable does not mean that it has to match an ontic world, but only that it fits into it without causing perturbations.

In my view, Piaget should have repeated this definition of truth in every book he wrote. The fact that he did not, allows many readers to think that he uses the term in the sense of the philosophical tradition, that is, as entailing a representation of ‘reality’. This source of misunderstandings is aggravated by his frequent use of the word ‘representation’.

If one takes seriously the fundamental assertion that knowing is an adaptive function, it is clear that no product of the cognitive activity will be a representation of whatever the cognizing subject is adapting to. The results of adaptation will have to fit into the world, not to copy it. Piaget uses the word ‘representation’ for what Kant called *Vorstellung*, which is much closer to ‘idea’ or ‘conception’. Confusion arises because ‘representation’, both in French and in English, designates a picture or copy of something else that is considered the original. But the products of the cognitive activity are the constructs of the cognitive activity and not represented images of an inaccessible original. Piaget’s genetic epistemology, from my point of view, constitutes a continuation and expansion of Kant’s transcendental enterprise, that is, “the idea of a science whose internal architecture is to be designed entirely on principles derived from the critique of pure reason” (Kant, 1787, p. 27). Piaget remained true to Kant’s (and also Vico’s and Berkeley’s) axiom “that reason can see only what she herself has brought forth according to her design” (Kant, 1787, p. XI). But he goes beyond Kant, in that he

was able to show that much of what Kant had to assume as given a priori could be explained by a developmental analysis of the cognizing subject's mental operations.

THE CORE OF RADICAL CONSTRUCTIVISM

The key ideas of this post-epistemological approach to the questions what is knowledge and how do we come to have it, can be summarized as follows:

1. What we call 'knowledge' does not and could not represent a world that is supposed to be beyond our experiential interface with it. In this, constructivism agrees with the sceptics. But, like pragmatism, constructivism introduces a modified concept of knowledge. Knowledge pertains to the way in which we organize the world of our experience.
2. Radical constructivism does not deny an ulterior reality; it follows Vico in that it denies that human rational knowledge can attain a God-made world or produce anything that could rightly be called a representation of it.
3. It agrees with Berkeley that it is unintelligible to attribute existence to anything that cannot or could not at some time be perceived, because, as he said, "there is no rational evidence for the existence of an independent reality" (in Popkin, 1951, p. 230).
4. It takes from Vico the basic idea that human knowledge is a human construction, an idea which Piaget—who, I believe, did not know the Neapolitan philosopher—developed very much further by minutely mapping the constructive conceptual operations by means of which human subjects furnish their experiential worlds.
5. Constructivism drops the requirement that knowledge be 'true' in the sense that it matches an objective reality. All it requires of knowledge is that it be viable, in that it fits into the world of the knower's experience.
6. Inherent in radical constructivism is the realization that no knowledge can claim uniqueness. In other words, no matter how viable the solution to a problem might be, it can never be regarded as the only possible solution.
7. This last consideration, together with Leo Apostel's admonition that "a system should always be applied to itself" (1977, p. 61) leads to the conclusion that radical constructivism cannot claim to be anything but one approach to the age-old problem of knowing. Only its application in contexts where a theory of knowing makes a difference can show whether or not it is a viable approach.

CONCLUSION

Radical constructivism relinquishes the venerable image of the human knower as a discoverer whose task is to find the truth about a world into which he comes as an objective observer. This is sufficient to make this theory of knowing unpopular. Human self-esteem was profoundly shaken when Copernicus suggested that the human planet was not the center of the universe. Now to be asked to give up the hope that human knowledge will eventually produce a true representation of that

universe posited beyond the realm of experience, seems unacceptable, no matter how logical the arguments may be. Although everyone realizes at some juncture that life and the universe in which it has unfolded are ultimately an impenetrable mystery, few are inclined to accept the consequences of that realization for their rational efforts.

The constructivist approach to knowing is an attempt to safeguard reason from two equally pernicious fallacies. On the one hand, it counters the sceptics who have forever disparaged reason by demonstrating that it cannot lead to certain knowledge about the real world and thus, with or without intention, prepared the way for some irrationalism; on the other, it counters those (and there are many among them who call themselves scientists) who claim to have found the truth.

In my view, however, the constructivist orientation comprises far more than the critique of the traditional presupposition that objective knowledge is not only possible but also necessary. Once one accepts the view that all knowledge is a cognizing subject's construction, that subject regains such autonomy as it can find within the constraints of an unknowable world. And with autonomy comes responsibility. What we know largely determines how we act. Consequently, if we want to act responsibly, we shall have to take responsibility also for the way we see the world.

Those who propound constructivist ideas must expect criticism and sometimes even animosity. And among the critics there are usually some who react in the same way as the anonymous reviewer of Vico's treatise at the beginning of the 18th century: They demand proof that the propounded thesis is true. Thus they miss the central point of this post-epistemological way of thinking. Radical constructivism does not claim to be anything but a model, that is, a construct whose value depends exclusively on its viability. In other words, it will sink or swim according to whether it manages to establish and maintain equilibrium in the sphere of rational cognition.

POSTSCRIPT

In these pages I have simplified—perhaps oversimplified—the points of view of the three thinkers that I cited as relevant to radical constructivism. As a mitigating circumstance, I might say that if I had tried to consider each one's entire work, rather than a few brief statements, I would have had to write a book. This is not an excuse. Rather, I would maintain that the three thinkers had long working lives in the course of which, as their most knowledgeable students agree, they changed some of their ideas. It was not my purpose to produce a comprehensive account of how they developed their views. Piaget made clear that no matter how we try to accommodate to the view of others, we cannot help assimilating what we experience to our own conceptual structures. Hence, it should be taken for granted that my interpretation of the authors I have cited does not purport to be 'objective' but is a frank assimilation to my own way of thinking.

- ¹ It is characteristic that in the first English translation of Vico's thesis on epistemology, Lucia Palmer (1988) took it upon herself to add two 'explanatory' parentheses to this passage. Human knowledge, she translates, "is nothing but making things (in the mind) correspond to themselves (in the world) in beautiful proportion" (p. 97). A most efficient way of undoing Vico's insight that things constructed in the mind are not intended to be replicas of things in an external world.

THE END OF A GRAND ILLUSION

Our title speaks of ‘the end of grand designs’. For psychotherapists, I am sure, the phrase has a rather special meaning. Being myself an obstinate outsider, who has so far managed to avoid annexation by *any* discipline, the phrase immediately prompts me to ask: What, indeed, are the grandest designs in the twenty-five hundred years of our intellectual history?

Since this is a rhetorical question, I am going to answer it myself. The grandest designs—and I say this without the least hesitation—are the philosophers’ schemes to find out what the world ‘in itself’, the world apart from the human knower, might be like. And since I do not belong to any discipline and do not have to defend a particular dogma, I would add that these grand schemes have finally run out of steam.

No one, as far as I know, has yet tried to work out how many man-hours of laborious thought and conceptual computation have been devoted to this task, but anyone who has looked around in a philosophical library will, I am sure, agree that it has been an enormous endeavor. Let me say at once that I consider it to have been unsuccessful—a wild goose chase, but nevertheless a glorious endeavor. It produced a wealth of inspired writing and, what is more important, it was quite indispensable. If our history of ideas were not littered with these noble ruins, we could not see what we are seeing today. Ross Ashby (1963) said a long time ago that one should never disparage false starts and failures; because to find out that a particular approach to a problem does *not* work, is a gain in any field where the number of choices is finite: it reduces the manifold of possible approaches that remain to be tried.

THE CLASSICAL IMPASSE

Had the grand epistemological projects not been pursued with all the ingenuity and all the stamina of the great philosophers, the lesson to be learned today would still be out of reach. Hence it is well worth our while to look back and, if we can, put into focus the point at which they turned into a blind alley. From my perspective, this point lay at the very beginning.

Given the earliest records we possess concerning investigations about the questions of what knowledge is and how we come to have it, it seems inconceivable that the formulation of these two questions should not have raised a third—namely how we might decide whether our knowledge is *really* true.

These earliest records, as far as the Western world is concerned, stem from the 6th century BC, the time of the pre-Socratics. The extraordinary thinkers of that period were as universal in their interests and as versatile in their accomplishments

as the famous elite of the Italian Renaissance. Among them there were also some who clearly saw the inevitable impasse of the epistemological venture they had embarked on.

Xenophanes of Colophon, for instance, said that even if someone happened upon knowledge that represented the world exactly as it is, he could never tell that this was the case (cf. Diels, 1957, Fragment 34). This remarkably concise statement is based on the logical fact that, in order to check the ‘truth’ of any representation, one must have access to what it is supposed to represent. In the case of knowledge that purports to be knowledge of the ‘real’ world, a check would be possible only if one could step outside the field of one’s knowing. This, indeed, is what in one way or another the sceptics have been reiterating ever since.

With the beginning of the Christian era, however, the focus of interest shifted to knowledge which was to be acquired from the scriptures and revelation rather than by rational consideration of actual experience. But here, too, a logical difficulty was raised by 3rd-century theologians in Byzantium. If God was omnipotent, they said, if He was omniscient and present in every place of the universe, then He had to be fundamentally different from anything we could encounter in our experiential world; and if this was the case, there was no way to grasp His essence in human concepts. The Byzantine school became known as *apophatic* or *negative* theology and, for obvious reasons, was quickly suppressed as heresy by the Church.

Though these insurmountable logical obstacles to the acquisition of ontologically true knowledge were clearly seen early on in our history of ideas, and the sceptics, throughout the ages, never tired of drawing attention to them, the quest for ‘true’ or ‘objective’ knowledge of a world posited as independent of the knowing subject was pursued by almost all great thinkers. Somehow they hoped that reason would find a way in spite of the logical impasse. They were driven on, above all, I think, by two deep-rooted feelings. First, the reluctance to acknowledge that, while human thought had obviously managed to solve a great many problems, there were mysteries to which there was no human approach. Second, the age-old mystical conviction that Descartes much later expressed when he said, in the context of perception, that God could not have been so mean as to equip us with an insufficient reason and deceptive senses.

METAPHYSICAL CONFUSION

At this point, it should be clear that I am concerned exclusively with what we call *rational* knowledge. Mystics may well have a way of resonating to a world that lies beyond our experience. However, their resonating does not involve the workings of reason but rather what Giambattista Vico aptly put under the heading of ‘poetic wisdom’. What mystics say about their visions is couched in private symbols whose formation and interpretation is and necessarily remains in a domain of subjective invention that lies beyond the reach of rational assimilation. It may, indeed, be more important than anything reason produces, but it must be grasped intuitively because it eludes prosaic expression in logical terms.

The source of the trouble is that, in the works of the great philosophers of the Western world, the distinction between the rational and the mystical became blurred and they freely larded their epistemological investigations with implicit metaphysical assumptions that could not be rationally grounded in human experience.

The first time a sharp distinction between the two kinds of knowledge was suggested was at the birth of modern science. Cardinal Bellarmino, who had been charged to conduct the prosecution of Giordano Bruno, was an extremely cultivated gentleman who appreciated the lure of intellectual explorations. When he heard of the accusations against Galileo, he sent him a warning: Galileo should be prudent, and present the heliocentric theory as an hypothesis that could serve to explain and to predict certain experiences. This would not be considered heresy. But on no account must he present that theory as a description of God's world. True knowledge of that world was the domain of the Church, and science must not meddle with it.

Against the logical arguments of philosophers who denied the possibility of true knowledge, the Church could always pit the contention that its access to knowledge was through revelation. Now, however, when scientists produced empirical facts that flatly contradicted the sacred dogma, another line of defense was needed.

Hence Bellarmino reinstated the distinction the Greeks had made by contrasting *doxa* with *gnosis*. The first was to refer to experiential knowledge and could never get beyond the status of 'opinion' because it was derived from the necessarily limited observations in the world of actual living. The second was the knowledge of the soul, acquired directly and without contamination by everyday praxis.

But Galileo would not accept such a division. Though he was apparently deeply troubled by the conflict between his scientific work and the religious dogma, and though he recanted—as any reasonable man would have—when he was shown the Vatican's instruments of torture, he did not want to give up the belief that he was uncovering the real workings of the universe.

In retrospect, this was a strange conceit. Yet most of the scientists that followed—and most of the teachers of science today—have taken the same stance. To them, science is the way that leads to knowledge of the world as it really is. It is a strange conceit, because the stupendous advance of science since the days of Galileo is, after all, based on his brilliant gambit of calculating the behavior of observable physical objects by relating it to 'laws of physics' which physical objects were never observed to follow exactly. Nowhere, for instance, was a physicist able to demonstrate that objects of different weight fall at the same rate, and nowhere could they observe that the motion of balls rolling down an inclined plane is uniform and infinitely continuous. Such laws could not be observed in actual experiments, they had to be *invented*.

CONCEPTUAL CONSTRUCTION

In spite of the spreading myth that science provides absolute knowledge, however, there were scientists, and often the most imaginative and successful ones, who

were at least occasionally aware of the fact that it was they themselves who constructed the conceptual framework that supports the scientific picture.

From Galileo's student, Torricelli, we have this remarkable statement:

Whether the principles of the doctrine *de motu* be true or false is of very little importance to me. Because if they are not true, one should pretend that they are true, as we supposed them to be, and then consider as purely geometrical, not as empirical, all the other speculations that we derived from these principles. ... If this is done, I say that there will follow everything Galileo and I have said. Then, if the balls of lead, of iron, or of stone do not behave according to our computation, too bad for them, we shall say that we were not talking of them.¹

Some two hundred years later, in the middle of the 19th century, the extension of scientific know-how and the technological wonders that were being achieved led to a mood of unbounded confidence. There were scientists who not only thought but also said that all important mechanisms of the universe had been explained and soon no mysteries would be left for science to tackle.² Yet, a brief generation later, some students of those overly enthusiastic scientists began to see through the optimistic illusion. The continual widening of the horizon and a growing awareness of how science had been dealing with the increasing wealth of observations led them to realize that they were *managing experience* rather than explaining the universe. In 1887, Thomas Henry Huxley wrote:

Any one who has studied the history of science knows that almost every great step therein has been made by the 'anticipation of Nature', that is, by the invention of hypotheses, which, though verifiable, often had very little foundation to start with; and, not infrequently, in spite of a long career of usefulness, turned out to be wholly erroneous in the long run. (p. 56)

At much the same time, Hermann von Helmholtz, in a postscript to his pioneering paper on the conservation of energy of forty years earlier, formulated the fundamental insight that "the principle of causality is in fact nothing but the presupposition of lawfulness in all the appearances of nature".³

In physics, most of the prominent actors in the revolution brought about by the theory of relativity and quantum mechanics have at some time or other taken a similar stance and acknowledged the fact that they had first constructed a theory and only then looked for observational results to fit into it. But the textbooks from which students are to acquire 'the scientific method' are still perpetuating the earlier dogma according to which the regularities and 'laws of nature' are *discovered* by observation (cf. Brush, 1974).

A THEORY OF FICTIONS

Although the authors of modern physics were at least occasionally aware of the breach they had caused in the epistemological tradition, they had no time to tie this breach to particular strands in the history of philosophy that would have helped to

substantiate it as a generally viable position. While Helmholtz was undoubtedly familiar with Kant's assertion that "reason can see only what she herself has brought forth according to her design",⁴ neither he nor the later emancipated scientists said anything about how it might come about that *invented* scientific theories can turn out to be so eminently useful in the actual praxis of living. Yet, this is the question that has to be answered by anyone who suggests that the grand design of knowledge as a representation of the real world should be replaced by a more modest paradigm.

The first attempt to justify the use of fictions—where legitimate 'fictions' are understood as *useful*, not merely fanciful inventions—was made by Jeremy Bentham, an 18th-century prodigy who was admitted to Oxford at the age of twelve and a half (cf. Ogden, 1932). Bentham provided some truly seminal analyses of concepts such as 'matter', 'form', 'quantity', and 'space'. For me, however, the fundamental insight he provided is this: *relational* concepts cannot be absolute, because they can be *known* only when an operating subject assembles them in experiential time.

Because I consider this insight fundamental, let me try to explain it in simple words. When we relate, we are obviously dealing with more than one unitary thing. To 'relate' means that we have one focus of attention, move our focus of attention to something else, and then look at the way we moved from the one to the other. Only by operating in some such way, can we specify a relation. Hence it requires the attention of an observer, someone who *does* it or, as I would say today, someone who *constructs* it by operating in a particular way.

This insight should give thought to anyone who claims to be a 'realist'. One may disagree with some details of Bentham's conceptual analyses, but it would be difficult to ignore the general principle they embody, namely that most of our indispensable concepts are not given to us by the senses but are the result of our mental operations and our creative reflection and abstraction.

With regard to this 'constructivist' approach, Bentham is in agreement with Locke, who introduced the notion of "the perception of the operations of our own mind within us" (Locke, 1690, Book II, Chapter 1, §4). But whereas Locke was still anxious to maintain a correspondence between the subject's conceptual world and an objective reality, Bentham's construction of concepts was guided only by the notion of utility. This gave his opponents the opportunity to disparage his work as tainted with an unworthy 'utilitarianism'.

This criticism is essentially the same that was leveled more than a century later against the pragmatists, who promoted the maxim: True is what works. Both in the case of Bentham and that of the pragmatists, usefulness or workability were tacitly understood to refer to 'practical values' in a material world and to those alone. What was lacking was an unequivocal statement that practical utility in that approach was secondary to the far more important use it had in that it provided a new way towards explaining the structure of the experiential world, the only 'reality' we can rationally comprehend.

Even when this view is expressed with all the required care and clarity, it still takes a long time to counteract the inveterate belief that the structure of the

experiential world is nothing but a more or less hazy and partial reflection of a real world that lies beyond it and whose exploration, therefore, is a worthier goal for philosophy. The sceptics' cogent arguments have not been sufficient to discourage the illusory quest in twenty-five hundred years. I have earlier mentioned emotional reasons for this remarkable persistence. Now I want to suggest a more practical one.

OBSTINATE TRADITION

If, in the domain of science, a problem is approached from all conceivable angles and still resists solution, it does not take long before someone suggests that there may be something wrong in the way the problem is conceived. Then, it may happen that a concept, considered fundamental until that moment, is dismantled and replaced by a novel one that opens a new perspective in which the problem either disappears or can be solved. The concept of space, in the switch from the Newtonian to the Einsteinian view of the universe, is a recent case in point.

In epistemology, no such thing has happened since its inception in the 6th century BC. The conceptual approach has not changed in all that time. The great failing of the sceptics was that they never seriously tried to go beyond the demonstration that 'true' knowledge was impossible. They never questioned the original cognitive scenario in which knowledge *had to be* a representation of a reality independent of the knower.

Only about a hundred years ago an idea cropped up in the theory of knowledge that could eventually lead to a new scenario. In an essay published two decades before the turn of the century, William James suggested that one could apply the basic notions of the biological theory of evolution to the evolution of concepts and conceptual structures (James, 1880). The same idea was presented with great clarity and detail a few years later by Georg Simmel (1895), and when Hans Vaihinger came out with his *Philosophy of 'As If'* (1911) it became clear that he, too, had been thinking along that line since 1876.

Today, 'evolutionary epistemology' is quite a fashion. Well-known biologists are vigorously sponsoring it, but they have also managed to direct it back into conventional channels. Knowledge, this school holds, evolves through adaptation brought about by a process analogous to natural selection—and this is the original idea James, Simmel, and others suggested at the turn of the century. But then, for instance, Konrad Lorenz jumps to the conclusion that, because concepts such as 'space' and 'time' have evolved and are successful, we are justified in assuming that they correspond to characteristics of an ontologically real world (Lorenz, 1973, p. 21ff).

In my view, this is no more rationally warranted than the belief that God would not have created us without giving us the means to see the world as it 'really' is. The assumption springs from an implicit over-extension of the biological notion of *adaptation*. In the realm of living organisms, to be adapted means no less, but also no more, than to have found a way of surviving and reproducing under the present environmental circumstances. In other words, all organisms found to be alive at a

particular moment of evolutionary history are adapted—but this means no more than that they have not succumbed to the obstacles and perils the environment has so far put in their path. The ways and means they have developed to avoid these obstacles and perils cannot be taken to reflect properties of the environment. All an organism might conclude from the fact that it has survived is that it happened to find one among the countless ways and means that do not happen to come into conflict with the environment's constraints or, in other words, that it is still *viable*.

To use a simple metaphor, natural selection in evolution works like a sieve: what passes through, passes through. Successful passage provides no clues about what might have been an impediment, no clues about the character of the sieve. Thus, the fact that the concepts of space and time are useful in the management of our experiential world, entails no more than that these concepts are among the possibilities the real world leaves open to organisms with our conceptual capabilities. And I would add that, given the success of the theory of relativity, it is now clear that, even in our experience, there are reaches where our ordinary concepts of space and time are no longer viable.

I have gone into this at some length, because 'evolutionary epistemology' has in fact blocked the most promising perspective that was opened at its beginning: the revolutionary view suggested by Jean Piaget's programmatic statement that *knowing is an adaptive function* (Piaget, 1967b). If we take seriously the premise that cognition is an instrument of adaptation, we have to replace the traditional concept of knowledge as *representation*. Instead of thinking of knowledge as corresponding to an independent 'objective' reality, we have to think of knowledge as the collection of conceptual structures that have so far not clashed with the constraints of our experiential world. This is to say, knowledge does not have to *match* an ontological reality, it merely has to *fit* into the world we experience. It is well to remember that this is what 'empiricism' is about. To attribute an ontological value to 'empirical evidence' is the old misconception used in the attempt to turn science into unquestionable dogma.

THE CYBERNETIC PARALLEL

It is worth mentioning that Piaget, in his later years, found himself in agreement with many of the epistemological fragments produced by cyberneticians. In retrospect, this is not surprising. As Bateson pointed out, cybernetics differs from the traditional scientific procedure in that it operates by means of constraints rather than causal connections and that, consequently, Darwin's theory is a cybernetical one, because it explains evolution as the result of nature's 'restraints' on the random variations of organisms (Bateson, 1972a). In this context, it is tempting to adapt Paul Feyerabend's most shocking statement and to say: Anything that goes, goes.⁵

Since cybernetics is mainly concerned with self-regulation and the gaining and maintaining of internal equilibrium, cyberneticians who become interested in the process of *knowing*, will try to see it as a process of self-regulation. I have elsewhere tried to show that there are sound philosophical arguments for the

cyberneticians' contention that cognizing organism cannot receive anything that could reasonably be called 'information' from an external world, and that such 'knowledge' as they are able to acquire can be constructed by no one but themselves (Glaserfeld, 1981c).

I want to emphasize that this autonomous construction of knowledge should not be considered a form of solipsism because it is by no means a 'free' construction. All its conceptual elements and the structures into which they are built have to prove viable in the flow of experience. On the physical level, their viability is an *empirical* question in the very sense of the empiricist philosophers. On the conceptual level, on the other hand, viability is a question of logical coherence in the *rationalist* sense. On both levels, the construction is subject to constraints which separate what is viable from what is not, but the nature of the constraints is inaccessible to the constructing subject, because there is no way of telling whether a failure is due to a flaw in the constructive operating or to an obstacle of the ontological world.

CONCLUSION

Coming to the end, one might ask: Is it actually the case that the 'grand epistemological design' of the Western world has failed? There is still, after all, a majority of people who firmly believe that the knowledge we have gathered through the centuries *is* representative of an objective world beyond our experiential interface and that it is this and only this correspondence with an ontological reality that makes it true. Those who suggest that this belief is based on an illusion are often considered to be spoilsports or, by the more virulent defenders of the status quo, dangerous heretics.

In fact, the intellectual mood is reminiscent of the 16th century, when the majority of thinkers were still struggling to perpetuate the notion that, being the crowning achievement of God's creation, mankind's place in the center of the universe could not be doubted. It took much more than a hundred years before the geocentric myth was generally relinquished. But this change did not in any way weaken what I consider to be another popular myth: the belief that our reason should be sufficient to grasp at least an outline of a world *as it really is*. Unlike the earlier one, this myth is not kept alive by mere human vanity but rather by the fear of what would follow if it were given up.

As long as we cling to the notion that parts of our experience reflect an objective world that is independent of our knowing, we are not compelled to feel responsible for that world. When it would be uncomfortable, laborious, or painful to change certain things, we can simply escape by saying: There is nothing we can do, because this is how it is; or, on the personal level: "I cannot help it, this is how I am".

Thus, I want to suggest that the profound emotional reactions against the notion that it is ourselves who *construct* our experiential reality spring from the desire *not* to acknowledge that no one but ourselves can be held responsible for what we know and what we do.

If my exposition was at all intelligible, it should now be clear that I cannot possibly claim to have presented a ‘truth’ in the traditional sense of that term. I have no intention of claiming such a thing. I present my view as a possible way of thinking, and I do this because I strongly feel that, by relinquishing the ontologically ambitious ‘grand designs’ in epistemology, and replacing them with the conception of knowing as a powerful instrument for achieving a viable fit with our experiential world, we may have a better chance of saving that world before it is too late.

ACKNOWLEDGEMENT

I want to thank my wife Charlotte for continuously helping me towards a clearer expression of my ideas.

NOTES

- ¹ I owe this quotation to Silvio Ceccato who used it forty years ago in a treatise I translated for him (Ceccato, 1951).
- ² This has been documented by many authors, e.g., by Bernal (1954, vol 2., p. 665).
- ³ Helmholtz wrote this as an addition to his famous paper *Ueber die Erhaltung der Kraft: Eine physikalische Abhandlung*, the 2nd edition of which was not published until 1899; a less concise expression of the same thought, however, can be found in his essay on perception of 1878.
- ⁴ Kant, *Kritik der reinen Vernunft*, B XIII.
- ⁵ Feyerabend used the phrase “anything goes” in *Against method* (1975, p. 23).

THE SIMPLICITY COMPLEX

Ich habe mir erst später klargemacht, dass das Prinzip der Kausalität in der Tat nichts anderes ist als die Voraussetzung der Gesetzlichkeit aller Naturerscheinungen. (Hermann von Helmholtz, 1881, in Exner, 1911, p. 663)¹

We have an indomitable urge to simplify experience in order to predict and control it. I shall argue that it is this compulsion that leads us to believe that the real world must be a world of formidable complexity. Whenever we succeed in managing experience, we tend to think we are managing reality; when our management fails, we conclude that we have not yet found the rules that govern and, therefore, simplify what we take to be the baffling complexity of the universe. We forget that the complexity we believe to be facing springs from one source alone: the fact that whatever regularities, rules, or laws we construct are derived from and apply to our experience—and our experience is a world which we ourselves engender, define, and delimit by our own activity of segmenting and conceptualizing. The history of science shows, perhaps better than anything else, how mutable and relative the way of segmenting and conceptualizing has been; and I would suggest that it could not be otherwise.

I have elsewhere proposed an epistemological model that is radically different from the traditional one. Instead of the usual requirement that knowledge should match an independent, absolute reality to which we have no access, the model substitutes the relation of ‘fit’ in the evolutionary sense that our cognitive structures are required to survive in such space as they find between experiential constraints. In this shifted perspective, the traditional notion of ‘truth’ as corresponding to an ontological state of affairs is replaced by the concept of viability (Glaserfeld, 1981b, 1985). Here I shall be concerned not with the problems of ‘knowing’ as such but quite specifically with ways and means the human knower might be using in the organization of experience.

To some readers it will soon become clear that I am propagating what professional philosophers condemn as ‘genetic fallacy’. I do this quite deliberately because I hold that knowledge consists of conceptual structures and, with Piaget, to whom I owe a great many of my ideas, I believe that “there is no structure without construction” (Piaget, 1968, p. 120). I emphasize that I am engaged in exploration and that what follows should not be mistaken for the description of something that ‘really’ goes on; it is intended simply as a tentative, hypothetical model. The model itself is, of course, a manifestation of the ‘simplicity complex’—it does not even begin to deal with the more complicated aspects of cognitive construction; but the

principles it embodies could, I maintain, cover a large part, if not all, of what we call 'knowledge' of the experiential world.

I.

We can think of the unborn child in the womb as a potential organizer of experience with very little opportunity to begin the cognitive career. Yet, in the maturing nervous system there may already be some perturbations of the kind the adult observer would later call 'sensory'. (Since I, as an adult, am considering the unborn child's initiation into cognitive activities, I cannot help thinking about it in the way I have grown to think; it is a case of cognition modeling cognition and, therefore, an enterprise that is quite deliberately circular. In that respect, however, it is no more reprehensible than any attempt to know about knowing.)

The prenatal sensory world is no doubt sparse and monotonous compared to mine. As yet, there are no perturbations of the kind which, later, would be categorized as vision, taste, or smell; but those that I would call touch, and perhaps hearing, are beginning to occur. From my adult perspective, I must further assume that in some primitive way a segmentation of the amorphous flow is setting in, and that at some point the prenatal organism is beginning to experience one fuzzy sensation after another. If that is granted, there is the possibility of coordinating segments of experience. There maybe 'noises' that are often followed by some particular 'pressure' and there may be particular 'pressures' that are often followed by a 'noise'. I place these sensations between quotation marks, because it is only very much later in the organism's development that they will be categorized as such. I accept Heinz von Foerster's notion that neural signals from different sensory modalities are indistinguishable *qua* signals and could be differentiated only on the basis of an internal topography of correlations (Foerster, 1970). Nevertheless, I want to suggest that such correlations can begin to be formed at that early stage. Mummy, after all, walks about, the walking involves her abdominal muscles, and the joints which she moves in walking are not totally noiseless. Thus there is the possibility of establishing regular sequences of sensations.

This hypothesis is not as far-fetched as it might seem. Studies with infants have shown that, even a few hours after their birth, they can be conditioned to move their head one way to switch on a light, and the other to sound a buzzer (Lipsitt, 1977). Whatever else this may be taken to indicate, it does show that the new-born organism's nervous system is capable of establishing relatively stable sequential links. We do not know precisely what this 'establishing of regular sequences' means or what mechanisms are involved in achieving it; but we do know that even as unsophisticated a creature as the common earth worm can do it (Yerkes, 1912). Psychologists who, like many other scientists, are bent on reducing as many phenomena as possible to one and the same principle, invoke Thorndike's 'Law of Effect', which simply states that a living organism tends to repeat any activity that has led to a satisfying result.

The Law of Effect is eminently plausible for at least two reasons. On the one hand, it allows us to make fairly reliable predictions about the behavior of living

organisms. (It is unlikely that one will act, i.e., work, in order to obtain some unsatisfactory result. Even the earth worm will ‘learn’, albeit slowly, that it must turn elsewhere if the soil at the end of the accustomed pathway has dried up). On the other hand, the Law of Effect is plausible because it says no more and only a little less than the principle of induction: Whatever works will be repeated.²

II.

The interesting thing about the inductive principle is that it functions, as it were, in two directions. We go in the one and infer that something is going to ‘work’ (in the sense that we are ready to try it again), if we have seen it work on a number of occasions. We go in the other direction whenever we establish anything as an ‘it’, that is, an item of which we believe that it has recurred or will recur; and it seems clear that only items that are supposed to recur could be used as components in one of those sequences or patterns of which we then say that they do or do not ‘work’. In both cases there is the implicit assumption that our experience reflects an independent world that is predictable because it has a basically stable structure. David Hume (1748) saw this with admirable clarity:

For all Inferences from Experience suppose, as their Foundation, that the future will resemble the past, and that similar Powers will be conjoin’d with similar sensible Qualities. If there be any Suspicion, that the Course of Nature may change, and that the past may be no Rule for the future, all Experience becomes useless, and can give rise to no Inferences or Conclusions. (p. 47)

The inductive prediction that something will recur in future experience, thus, seems to be based on having experienced it recurrently in the past. To establish the recurrence of an item, however, requires that one be able to recognize it—and the moment we use that term, we introduce a problem and an ambiguity. The problem is that recognition seems to entail what we usually call ‘memory’ and, at present, we have no satisfactory conception of how that might function. It cannot be any kind of storage or file but must be dynamic, that is, some type of re-enactment and I shall leave it at that.³

The ambiguity, on the other hand, is this: to recognize an item can mean one of two things, and both involve repetition and ‘sameness’. Either we intend that we are experiencing something that we consider equivalent to an item we have experienced before, or we intend that we are experiencing an item that we consider to be the self-same individual that we experienced at some other occasion. In the first case we are *classifying*, in the second *identifying*. The differences that arise from these two ways of proceeding, though often confused, are epistemologically profound and I have tried to deal with them elsewhere (Glaserfeld, 1979, 1984c).

What I am concerned with here is the way in which we establish sameness in the equivalence sense. Obviously such a notion can arise only in a flow of experience that has been segmented and registered in chunks. Only if one cuts a piece of experience out of the ongoing flow can one compare that piece to some other piece

and come up with the conclusion that the two are, or are not, 'the same'. Similarly it is clear that a conclusion of 'sameness' will always be limited to the particular aspects or properties one has happened to take into account. (There are, after all, no two things in our experience, that could not be considered the same in some sense—just as there are no two things that could not be considered different.) In any such comparison that is made in order to simplify experience by lumping individual items into a category, the aspects or properties that are taken into account depend on what one intends to do with the items. The 'doing' may refer to a particular context of action, of description, or of categorization that has become an end in itself. Under all circumstances, therefore, it will be our context of action that determines how simple or how complex the particular area of experience must be made so that our actions might have an acceptable chance of success.

III.

The notion of assimilation, as Piaget elaborated it, is of the utmost importance in this regard. The way the human experiencer comes to organize chunks of experience into categories is a good example. Take a two-year-old who, for the first time, encounters the word 'pear'. She already knew the word 'apple', but when she now used it to request an item out of her reach on the table, she is told: "That's not an apple—it's a pear". As Daddy passes it to her, he repeats that it is a pear. The two-year-old's categorization of certain experiential items has again failed. She had been simplistic, and once more the world in which she might satisfy her desires turned out to be more complex than her organization allowed. To adapt that organization, more segmentation and differentiation are needed. The slight perturbation caused by the failure of her categorization may now lead her to look at the unruly item and focus attention on some sensory element that could be considered a difference relative to the experiential items for which the word 'apple' has worked satisfactorily in the past.

In practice such an accommodation may of course take many more instances of failure than just one; but if the child is ever to discriminate apples and pears, it must isolate differences between them. These differences may be in seeing, eating, or acting upon the items in some other way. The differences are differences in the way of experiencing, and once they have been made and registered, both kinds of item will be perceived in a new way.

Let me take this childish example one step further. It will not be long before the two-year-old uses appropriately not only the word 'pear' but also its plural, when three or four are lying on the table. This is a momentous step in the child's construction of reality, a step that is quite different from the preceding ones and which, though seemingly complicated at first, will eventually afford the kind of spectacular simplification that we may call 'conceptual economy'.

To my knowledge, neither developmental psychologists nor linguists or philosophers have paid much attention to the conceptual construction of pluralities. Yet, without it, we could never come to have the kind of *Weltbild* our societies require. William James (1907) put his finger on it:

Kinds, and sameness of kind—what colossally useful Denkmittel for finding our way among the many! The manyness might conceivably have been absolute. Experiences might have all been singulars, no one of them occurring twice. In such a world logic would have had no application; for kind and sameness are logic's only instruments. Once we know that whatever is of a kind is also of that kind's kind, we can travel through the universe as if with seven-league boots. (p. 119)

In order to isolate pear-experiences in one's experiential field, one must have established a reliable difference that allows one to recognize them. To recognize an experiential item as equivalent to an item one has experienced before, requires a comparison of some sort. In order appropriately to use the plural 'pears', however, one must attend to something that is of a kind altogether other than sensations and differences between sensations. To recognize a plurality of pears requires taking into account that, in a given context, one is carrying out the same comparison repeatedly and that it yields equivalence more than once. That is to say, attention must be focused not only on sensations or groupings of sensations but on what one is doing, on one's own operating. In fact, instead of creating a category that could be defined as a specific sequence of sensations, one creates a category that can be defined only as a specific sequence of operations.

Clearly, 'doing' or 'operating', in this context, does not refer to movement of hands and feet, or physical activity of any kind, but to activities that are carried out by an agent which, for the lack of a better word, we may call 'mind'. We are concerned with mental operations. I want to stress that, in saying this, one does not preclude that some physical machinery might be functioning as well. The important point, however, is that it is not the machinery that matters but the way in which its function and, above all, its results are interpreted.

Many years ago, in the 1950s and 60s, Silvio Ceccato, who one day will be recognized as the pioneer of conceptual analysis, spoke of *lavoro apportativo* (Ceccato, 1966). I like that formulation because it makes explicit the active role of the experiencer. John Locke (1690), an early proponent of mental operations, subsumed them under the term 'reflection' and said: "In time the mind comes to reflect on its own operations" (Book II, Chapter 1, §24), and he explained that, in reflection, complex ideas are formed by compounding simple ones. Yet, in the framework of Locke's general philosophy, the mind had little if any autonomy with regard to its operating. His notion of reflection was somewhat like his notion of perception—a process of passive receiving that enables the cognizing agent to understand what is already there. More recent schools of empiricism, though they claim Locke as a founding father, have tried to eliminate the mind and its operations altogether.

In contrast, Piaget, in whose genetic epistemology the concept of 'reflective abstraction' plays an important part, has throughout his work stressed the active role of the cognizing organism. Even so, it requires a considerable effort to appreciate the full extent of the generative power assigned to the knower in

Piaget's theory. One reason for this is that active construction takes place on more than one level.

There is the level of segmentation that creates chunks of experience, where we construct recurrent 'things' by focusing on similarities and disregarding differences. There is the level of relating that creates sequences and links that enable the experiencing subject to think in terms of more or less reliable 'schemes'. And there is the level of reflection, where abstraction, not from things but from the subject's own operating, creates complex conceptual structures which, then, are called theories, systems, and knowledge of the world.

IV.

From the constructivist perspective, the segmenting, the relating, and the abstracting are done by us, for our purposes and with our means. We evaluate them first and foremost according to whether or not they do what we expect them to do. Only if they work, if they achieve what we expect, are we inclined to apply other considerations, such as economy, speed, or, indeed, simplicity. In other words, the constructivist theory of knowledge is unashamedly instrumentalist. That is one reason why philosophers who still cling to the established dogma, cannot accept it. In spite of the fact that logic tells them that it cannot be done, they will not relinquish the precept that we must strive to attain knowledge that might be ontologically 'true'.

The bulk of the resistance that the constructivist epistemology is meeting, however, springs from the fact that it proposes a radical change in the conception of knowledge itself. As I said initially, constructivism suggests that the way in which the cognitive structures that we call 'knowledge' relate to 'real world' should be considered as fitting, and not as matching. That means that the relation must not be conceived in analogy to the way a picture may relate to what it is supposed to depict, but rather in analogy to the way a river relates to the landscape through which it has found its course. The river forms itself wherever the landscape allows water to flow. There is a continuous, subtle interplay between the 'logic' inherent in the water (e.g., that it must form a horizontal surface and cannot flow uphill) and the topology of the land. Both constrain the course of the river, and they do so inseparably. At no point could you say, for instance, that the river turns right because there is a hill, without implicitly presuming the logic of the water that prevents the river from flowing uphill. Thus, the river does not 'match' the landscape but 'fits' into it, in the sense that it finds its course between constraints that arise, not from the landscape or the logic of the water but always and necessarily from the interaction of both.

An analogous, irreducible interaction takes place between the 'landscape' of ontological reality and the 'course' of our cognitive constructing that generates what we call 'knowledge'. At no point could we say that a particular conceptual structure must reflect 'reality' because it helps us to circumvent some experiential constraint. It would, indeed, be a strangely ingenuous conceit to believe that, having found one path, it must be the only one possible and therefore 'real'. No

less ingenuous would it be to forget that the goals we try to attain, the ideas and theories we construe in order to attain them, and the constraints and obstacles we meet in our endeavor, are all and sundry products of our own way of conceptualizing experience.

From that point of view, then, whatever complexity we are facing is of our own making, for it can arise only from the relation between the goals we have chosen and the ways and means we construct for getting there. Attributing it to an ontological world as a property it might have in itself and apart from our conceptual activity seems as unwarranted as the pious hope that the ‘real’ universe might be ruled by comprehensible and therefore ‘simple’ laws because God would not have played with dice.

Franz Exner, the Austrian physicist to whom Schrödinger said he owed part of his own orientation, said it very well in one of his last lectures:

Nature does not ask whether man understands it or not, nor do we have to construct a nature that might be adequate to our understanding, we merely have to manage with what is given us as best we can. (1911, p. 697, footnote 1)

ACKNOWLEDGEMENT

I am indebted to Siegfried Schmidt for helpful comments on a draft of this paper.

NOTES

- ¹ English translation: “Only later did I realize that the principle of causality is in fact nothing but the presupposition of lawfulness in nature”.
- ² Humberto Maturana formulated this in his *Biology of cognition* (1970a):

A living system, due to its circular organization, is an inductive system and functions always in a predictive manner: what happened once will occur again. Its organization (genetic and otherwise) is conservative and repeats only what works. (p. 39)
- ³ For an inspired approach to this problem see Foerster (1965).

THE LOGIC OF SCIENTIFIC FALLIBILITY

As you know, ever since Galileo there have been fierce debates about whether or not certain areas are to be considered ‘scientific’. Even today, you can find people arguing that creationism is as scientific as the theory of evolution, or that astrology should count as a science.

One of the roots of this controversy is both parties’ claim that they have access to some kind of incontrovertible Truth. The ones derive their certainty from scripture and revelation, the others from a no less dogmatic view of science. To a dispassionate witness it may seem odd that those who want to preserve the purity of science revert, under pressure, to arguments that are indistinguishable from the arguments creationists bring up to defend their religious faith. The defenders of science will be inclined to say that the knowledge they attain is objective, whereas their opponents’ enterprises are grounded in myth.

This may have sounded plausible in the past, but recent developments in the history and philosophy of science have generated serious doubts concerning that objectivity. Nevertheless, the debate goes on, and frequently degenerates to an emotional clash of unexamined preconceptions.

From my perspective, there need not be a contest between religious faith and any scientific model—including the model of biological evolution that is perhaps the most comprehensive and successful one science has come up with. Any religious faith is a metaphysical attempt to pacify the soul. What science produces, in contrast, are rational constructs whose value resides in their applicability and usefulness. Believers may cling to what they are told or what they want to believe; scientific knowledge is of a different kind because it is tied to a method that involves experiential corroboration. Much of the fuel that keeps the controversy alive springs from the fact that many proponents of science cling to a conception of the scientific method that no longer seems tenable today.

To approach this point, I should like to tell a story. It’s a very simple little story. It reads like a fairy tale, but it is not. In fact, it is a rather serious story. I quote it from *Science*, the journal of the American Association for the Advancement of Science. It appeared in the issue of June 26, 1987:

In 1959, a badger broke through the security lines here at the world’s first plutonium factory (the Department of Energy facility at Hanford, in the State of Washington). The badger ignored all the warnings and dug a hole in one of the waste pits. After he left, rabbits began to stop by for an occasional lick of salt, but it was no ordinary salt they found. Before long, they scattered 200 curies of radioactive droppings over 2500 acres of the Hanford Reserve.

The rabbit mess ... created one of the largest contaminated areas, one that remains hot today with cesium-137 (half-life of 30 years) and strontium-90 (half-life 28 years).

Hanford also has trouble with ground squirrels, burrowing owls, pocket mice, insects, and plants like rabbitbrush and tumbleweed. With roots that can grow 20 feet, tumbleweeds reach down into waste dumps and take up strontium-90, break off, and blow around the dry land. If the dry weeds build up and there is a brush fire, they may produce airborne contamination. (Marshall, p. 1616)

Airborne contamination spreads over very much wider areas than even the most energetic rabbits can spread their pellets. The problem, therefore, is not a trivial one.

That badgers and rabbits dig burrows, eat certain things, and drop little turds all over the place—these are observations that our ancestors could make, and probably did make, when they lived in their caves 40 or 50 thousand years ago.

That plants grow roots and absorb chemicals from soil and subsoil has also been known for quite a while. In fact, in Milan, where I lived part of my Italian life, legend has it that Leonardo da Vinci experimented with this idea and dug some sinister substances into the soil around a peach tree, in order to see whether one could grow poisoned peaches.

How strange, you might say, that the scientists who direct the Hanford Reserve did not think of what badgers, rabbits, and tumbleweed do, when they lead their normal and quite well-known lives. I shall try to show that, given the logic of science, this is not surprising. But first, let me tell another story, a story I am sure you have heard before, but it illustrates a slight variation from the first.

For many thousands of years the river Nile flooded the Egyptian lowlands near the Mediterranean coast at least once a year. Vast amounts of fresh water seeped into the soil, fertilized it, and created a natural pressure against the water of the sea. The floods were a nuisance and, quite apart from this, using the Nile's water to irrigate parts of the desert up-stream seemed eminently desirable. So the Assuan Dam was built to solve these two problems. The Nile no longer got out of hand and new land up-stream could be irrigated and cultivated. For a little while the dam seemed a wonderful success of science and engineering. Then it became clear that the salt of the Mediterranean was slowly but steadily seeping into and devastating the lowlands along the coast which had fed Egypt for millennia.

I do not know whether, prior to this experience, hydrologists knew much about the balance of pressures on the level of the groundwater. They certainly had the theoretical equipment and the formulas to figure it out. Yet, they apparently did not do so before the Assuan Dam was built.

Well, you may say, one can't think of everything—the next time they'll do better. I would agree with this. Scientists, engineers—even members of the medical profession—are capable of learning. But that is not the problem I have in mind.

Learning is usually defined as 'modifying a behavior or a way of thinking on the basis of experience', and there is, of course, the implication that the modification is

towards effectiveness, efficiency, or, at any rate, something that makes it easier to attain the chosen goal.

I have no doubt that the next time a major dumping ground for radio-active waste is chosen and prepared, someone will think of the fauna and flora and of a way to keep them from spreading the poison. And when big dams were built after Assuan, someone, I'm sure, tried to work out how the water table in down-stream lands would be affected. Science and its professionals can, in fact, see more and further than the lay public—precisely because of the often uncommon experiences they have accumulated. The problem I have in mind is that they often do not look.

I would like to submit that it is, indeed, the logic of science and the scientific method that frequently stops scientists from looking outside a specific domain of possibilities. To show this, we have to agree on what it is that we want to call 'scientific method'. The conventional definition of 'science' is woolly, to say the least. However, I believe that a more adequate definition has now been produced. This definition of 'scientific method' stems from Humberto Maturana. It is not only relatively simple, but it also serves my purpose—which is to show how scientists lead themselves up a garden path.

Maturana divides the scientific procedure into four steps¹:

1. OBSERVATION. In order to count as 'scientific', an observation must be carried out under certain constraints, and the constraints must be made explicit (so that the observation can be repeated).
2. By relating the observations, a MODEL is inductively derived—usually a model that involves causal connections. (Often an idea of the model precedes the observations of step (1) and to some extent determines their constraints.)
3. By deduction, a PREDICTION is derived from the model, a prediction that concerns an event that has not yet been observed.
4. The scientist then sets out to observe the predicted event, and this OBSERVATION must again comply with the constraints that governed observation in (1).

I am confident that all who have been trained or engaged in 'doing science', will recognize in this description the famous 'hypothetico-deductive Method'. In fact, I have not heard of any scientists, conventional or not, who could not agree with this definition of 'science'. Some might want it to include more or to formulate it somewhat differently, but all can accept it as a minimal description of what scientists, by and large, are actually doing.

What is new in Maturana's break-down is that it illustrates the epistemological implications in a way you will not find in any of the textbooks on 'scientific method'. The four steps make clear that what matters is experience. Observing is a way of experiencing and, to be scientific, it must be regulated by certain constraints. The inductively constructed model relates experiences, not 'things-in-themselves'. The predictions, too, regard experiences, not events that take place in some 'real' world beyond the observer's experiential interface.

Seen in this way, the scientific method does not refer to, nor does it need, the assumption of an 'objective' ontological reality—it concerns exclusively the experiential world of observers.

The constraints placed on ‘observation’ are not intended to imply—as is so often stressed in textbooks—that the scientist should be ‘objective’ in the realist sense. They are stipulated to assure that what the scientist does is described in such a way that another scientist can come and replicate it. If another scientist is able to do this, it obviously corroborates the first scientist’s observations. But any such corroboration in no way demonstrates that what has been observed is an observer-independent reality. Corroboration merely establishes an element of consensus, that is, the kind of element that helps to create what Maturana has called a consensual domain among interacting organisms.

The conventional approach to science has always maintained that the more people observe a thing, the more ‘real’ that thing must be. Yet, the sceptics, ever since Pyrrho in the 3rd century B.C., have produced quite irrefutable arguments against this view. And, in our time, Paul Feyerabend has spoken against it from a different perspective. In his essay *How to be a Good Empiricist*, he argues as follows. (I have taken the liberty of leaving out a couple of paragraphs that refer to quantum mechanics and of changing one word—I have substituted the word ‘model’ where he has ‘theory’):

... assume that the pursuit of a (theoretical) model has led to success and that the model has explained in a satisfactory manner circumstances that had been unintelligible for quite some time.

This gives empirical support to an idea which to start with seemed to possess only this advantage: It was interesting and intriguing. The concentration upon the model will now be reinforced, the attitude towards alternatives will become less tolerant.

... At the same time it is evident, ... that this appearance of success cannot be regarded as a sign of truth and correspondence with nature. Quite the contrary, the suspicion arises that the absence of major difficulties is a result of the decrease of empirical content brought about by the elimination of alternatives, and of facts that can be discovered with the help of these alternatives only. In other words, the suspicion arises that this alleged success is due to the fact that in the process of application to new domains the model has been turned into a metaphysical system. Such a system will of course be very ‘successful’ not, however, because it agrees so well with the facts, but because no facts have been specified that would constitute a test and because some such facts have even been removed. Its ‘success’ is entirely man-made. It was decided to stick to some ideas and the result was, quite naturally, the survival of these ideas. (1968, p. 30, 31)

More explicitly than Feyerabend’s approach, Maturana’s break-down further demonstrates—and this is relevant to my proposition—the importance of the observer’s active relating (in the construction of the model) and that this relating is an inductive process.

When philosophers discuss induction, they are usually concerned with the question whether or not there is a way of demonstrating that inductive inference can lead to objective ever-lasting ‘truth’. As a constructivist, I am, to put it mildly,

not interested in that sort of ‘truth’. In contrast, I am very interested in how we construct the kinds of rule we use to organize our experience. In this, I go back to the original empiricists, to Locke, Berkeley, and Hume. All are no doubt familiar with these authors’ names, but I wonder how many would claim to have actually read them. I say this, because what one reads about them in the textbooks of contemporary American psychology makes nonsense of the most important contribution the original Empiricists have made to Western philosophy.

Reading Locke (1690), one soon discovers that, while he does indeed argue against the notion of innate ideas, he also states that the source of all our ‘complex ideas’ is not the material we get from the senses, but the mind’s reflection upon its own operations. The behaviorists, who often claim Locke as the father of their naive realist brand of empiricism, never mention this second source of ideas. They consider it heresy to mention the mind. Yet—and this is a point of enormous importance—if there were no mind to relate experiences and to become aware of its own particular ways of relating, there would be no complex ideas, no models and no scientific theories.

Hume, then, in spite of some nasty things he said about his predecessor, took Locke’s injunction seriously. “It becomes”, he says, “no inconsiderable Part of Science to know the different Operations of the Mind, to separate them from each other, to class them under their proper Divisions...” (1748, p. 17). And he explicitly says that this can be done when these operations are made the ‘object of reflection’. Hume admittedly simplifies that task quite drastically. He reduces the relations that the operations of the mind produce to three: contiguity, similarity, and cause/effect.

In the roughly two hundred years since Hume quite a few thinkers have advanced the analysis and classification of mental operations. Two, however, stand out above all others: Immanuel Kant and Jean Piaget. The one I want to draw on here, in the discussion of the operations that constitute the ‘scientific method’, is Piaget. Although he did not invent the notions of assimilation and accommodation, it was he who refined them and made them generally applicable.²

The basic principle of these operations is this: The cognitive subject organizes experience in terms of conceptual structures. The subject’s attempts at organization are always goal-directed and if one wants to treat these goals, whatever they may be, collectively, one may subsume them by saying that the subject is intent upon maintaining some form of equilibrium. At any given moment, the subject ‘sees’ and categorizes its experience in terms of the conceptual structures that it has available. Hence, the seemingly paradoxical assertion that an observer sees only what he or she already knows. This, in fact, is called ‘assimilation’.

Conventional psychologists have tried to make hay of that apparent paradox—and they have succeeded whenever they were able to obscure the fact that, in Piaget’s theory, assimilation always goes together with accommodation. They frequently announced that if the notion of assimilation were correct, infants could never acquire new behaviors or new ideas.

Let me try to correct this misinterpretation as simply as I can. When an infant picks up a spoon and shakes it as though it were a rattle, she assimilates her experience of the spoon—which is still a very limited experience from the adult

point of view—to her prior rattle experiences. If, then, she shows disappointment, because in spite of her vigorous shaking there is no rattling noise, and she looks at the spoon in her hand, she opens the door to accommodation. This is so, because this new examination may lead her to notice a visual or tactual difference between the spoon she is holding in her hand and the rattle she thought it was. This perception of a difference may lead to novel experimentation, and before long the infant will develop a new scheme for producing a very satisfactory noise—namely by banging the spoon on the table.

Let me insert a parenthesis about satisfactory noises. If you have had kittens in your house you may have seen the following. At a certain stage of their development, the kittens chase one another all over the place, and sooner or later one will land on your coffee table. If there happens to be a match box on the table, or a packet of cigarettes, the kitten may start playing with it in the way kittens play with such things: it will tap the box lightly so that it slides a little way. Inevitably, the moment comes when the box falls off the table. For the kitten this is a most enthralling event. It will stand at the edge and look down on the box in unmistakable fascination. If you then pick up the box and place it beside the kitten, the procedure is likely to be repeated; the kitten will push the box to the edge and, with wide-eyed enchantment, it will watch it fall. I believe that this is just another demonstration of a basic principle inherent in all cognitive organisms: You deliberately carry out some action and, suddenly, the action produces a seemingly spontaneous and therefore enormously satisfactory input in another sensory modality. You push—and the box falls by itself and makes its own noise as it hits the ground; you flail about with a spoon, and it bangs as it hits the table; you pull a trigger, and some distant item is shattered. All this proves to you that you have control over your experience.

As a non-professional observer, it has struck me that many marriages are held together by little more than such routines of apparent control; they seem reduced to appallingly simple procedures aimed at producing some visible effect in the partner.

To return to the notion of accommodation, it may take place whenever an act of assimilation does not lead to the expected result or, indeed, produces an unexpected result. In the Piagetian theory it is only when things do not go in the expected way, when there is disappointment, surprise, or, quite generally speaking, when there is a perturbation, that reflection is triggered, and the cognizing organism may be led to accommodate and to try something new. This is a very important point. It is important for what I intend to say about science. Also—although I am not a psychotherapist—I would submit that this aspect of Piagetian theory has its use in the practice of psychotherapy.

One more thing has to be said about assimilation: There are two kinds of it. When an infant picks up a spoon as though it were a rattle, the ‘as though’ is only in the observer’s mind. It is only the observer who sees the spoon as a spoon. The infant takes the spoon to *be* a rattle and, consequently, is disappointed when the spoon does not produce the expected noise.

In contrast, when the switch of the reading lamp on my desk refuses to function, and I use the letter opener as a screw-driver to open it—because I am too lazy to go down to the workshop—I am deliberately ‘assimilating’. I am well aware of the fact that the letter opener is not a screw driver, and that using it as such may lead to all sorts of disappointments or surprises. If it’s made of silver, I may twist it or break it. But, at the moment, I may be prepared to take that risk. Our adult lives are full of such deliberate assimilations, and we very often ‘make do’ with substitutes that turn out to be inadequate. This, too, may lead to accommodation—but not accommodation concerning the basic categories of the items we deliberately use as substitutes. We may simply learn that letter openers are less reliable screw-drivers than we thought.

So far, I have been giving you stories, that is, accounts of things that have happened and accounts of conceptual patterns that have been constructed. I have given you examples of scientists going wrong and a theory of learning that shows one way how errors may be corrected—namely by accommodation. At this point, I want to make some conjectures on the basis of those stories.

As proponent of a constructivist theory of knowledge, I want to stress that when I speak of ‘science going wrong’ or of ‘correcting errors’, I do not intend that science can or should discover some absolute TRUTH. ‘Wrong’ and ‘right’, in this context, always refer to a specific goal; for instance, a problem one wants to solve. The solution one comes up with, either works or it does not. In my way of speaking, the answer to a problem is either a viable solution or it is not.

In the case of the rabbit droppings and the tumbleweed, the problem of getting rid of radio-active waste was not solved in a viable way. In the case of the Assuan Dam, the problem of bringing water to the desert was solved magnificently—but the solution created a new, unforeseen problem. And this unforeseen problem—the salting of the Egyptian lowlands—seems irrevocable and unsolvable, at least at the moment.

In both cases, something was learned for the future, and there is hope that the same mistakes will not be made again. This sounds quite reasonable—and I believe it is. It is reasonable to suppose that we learn from our mistakes, and we can safely assume that scientists do, too.

The question I want to raise now is: how is this possible, or more precisely, what assumptions are involved in assuming that we learn from experience?

First of all, there is the assumption that things are related, especially the assumption that certain events not only precede but effectively cause others. As you know, David Hume, in the middle of the 18th century, suggested that if event B happens often enough after event A, we tend to think of B as the EFFECT, and of A as the CAUSE.

As Hume clearly saw, this involves the belief that the real world is essentially an orderly world in which events do not take place randomly. A hundred years after Hume, this was expressed very beautifully by the German scientist von Helmholtz, when he wrote in 1881: “It was only quite late in my life that I realized that the law of causality is nothing but our presupposition that the universe we live in is a lawful universe” (in Exner, 1911, p. 663).

That is to say, we expect the world we live in to be a world in which there are certain regularities, a world that runs according to certain rules. Since scientists call some of these regularities ‘Laws of Nature’, it may not be a silly question to ask how we come to know, or how we construct regularities?

Let me return to Maturana’s methodology: The second step in his break-down, was relating observations, and relating them in such a way that one comes up with a model. A scientific model, of course, is no more and no less than the crystallization of specific regularities. In order to speak of specific regularities, one must be fairly precise about the events that are claimed to be regular. That is to say, one must define certain experiences so that one can recognize them when one experiences them again. There can hardly be regularity before one has noticed repetition.

Two points are important here:

- The model connects specific observations, that is, experiences made during the step that is Maturana’s step 1;
- Experiences are specified by separating from, or isolating them in, the entirety of one’s experiential field—and this separation is made explicit and repeatable by setting up constraints on the acts of observation.

That is why I said earlier that the model in step 2 tends to determine the constraints set up in step 1.

Let us try to get some idea as to what those constraints are and how they are set up. An example that is often used is the initiation of physiologists, say, neuro-physiologists, into the working routines of the discipline or, as one might say a little irreverently, into the tricks of the trade. First of all, neuro-physiologists must learn to look through a microscope; then they must learn to use various dyes in order to make certain things visible; finally, they have to learn to see things in terms of the colors that were used to dye the preparation.

‘Preparation’ is a key word. Physicists speak of the ‘preparation of an experiment’. What they mean is that steps must be taken to make visible whatever it is they intend to observe. Sometimes these preparations include building a cyclotron or an accelerator that may cost many millions of dollars.

In the social sciences, ‘preparation’ takes a slightly different path. Experimenters know that, in order to be taken seriously as scientists, they must control for what are considered ‘interfering variables’ in their experiments. That means that they must eliminate unwanted elements from the observations. Just as the physiologists, by their techniques of coloration, divert attention from all sorts of details that could be observed, so the social scientists disregard features of the experimental situation simply because, from the point of view of their explanatory model, they are considered irrelevant.

For example, the rats, hamsters, and squirrel monkeys that serve in experiments in my Psychology Department, live on the 6th floor in our building. Thanks to the powerful airconditioning system, the temperature in their environment never changes, and whatever light they see is produced by neon tubes. There is no dawn or dusk, no sun or moon, no wind or rain, and instead of the sounds and smells of the wilderness, they hear and smell whatever is produced by electronic devices,

chemicals, and graduate students—a wilderness of a different kind. In most experiments this is considered totally irrelevant because the models used to explain the animals' behavior do not encompass these experiential elements. Don't misunderstand me—I have no specific hunches that, for instance, the repeated experience of sun rises, rain storms, or moonlight has a specific influence on the problem-solving abilities of hamsters; I am merely suggesting that even in these relatively simple forms of scientific research there are dozens of 'variables' which are considered irrelevant a priori for no other reason than that, so far, there has been no need to incorporate them into the kinds of model animal psychologists use to explain the behaviors they are studying.

Until the more or less accidental discovery of the rabbit disaster at the Hanford Reservation, the scientists who developed models for the safe disposal of radio-active waste had apparently looked at that problem in a way that focused exclusively on a few variables pertaining to experiments that were carried out in wholly artificial circumstances. The thought that they were dealing with a rather ordinary piece of natural landscape in North America did not occur to them until Nature forced it upon them. (As it happens, they apparently did not even anticipate what rain would do to the plutonium dump; it now transpires that radio-active particles have seeped into the ground water and are slowly but irrevocably being transported into the Columbia River.)

CONCLUSION

I have called this talk 'The Logic of Scientific Fallibility' and I have attempted to give some examples of how science proceeds. I can summarize this in a few sentences.

In order to observe anything, in order to 'collect data', one must have some notion—no matter how primitive and preliminary—of the particular experiences one intends to relate to one another. It is, obviously, these experiences that one will be looking for. In order to find them, one necessarily assimilates and disregards all sorts of differences in individual observations. The longer this goes on successfully and the more often the model one has constructed proves useful, the stronger becomes the belief that one has discovered a real connection, if not a 'Law of Nature'. And once that belief has been established, there is a powerful resistance against any suggestion of change and—as Thomas Kuhn has so nicely shown with examples from the history of science—there will be powerful efforts to sweep any observed irregularity under the rug.

Thus, anyone not practiced in scientific tunnel vision, say, a farmer or trapper at Hanford, who would have asked: "What about badgers or rabbits burrowing and tumbleweed roots soaking up the poison?" would have had little chance of being taken seriously.

Science proceeds by modeling limited experiential situations—by focusing on a few variables, and deliberately disregarding many. It cannot do otherwise, and it should not do otherwise. But scientists must never drift into the misplaced religious belief that they are discovering what the world is really like. They should always

remain aware of the fact that what they are dealing with is not the real world but an observer's experience—and it is not even all an observer could experience, but deliberately constrained experiences or experiments that happen to fit the scientific model the observer is working with. And sometimes these very constraints exclude elements that afterwards rear their ugly head.

NOTES

- ¹ Maturana has developed this break-down throughout the last decade and whenever I heard him present it, it had become a little more complete and comprehensive. The way I am reporting it here is not verbatim but rather the way I have come to understand it.
- ² The first to use the terms 'assimilation' and 'accommodation' systematically was Mark Baldwin, at the turn of the century. The concepts, however, are older. The German psychologist Steinthal used it in his *Einleitung in die Psychologie und Sprachwissenschaft* (Berlin, 1871) and Hans Vaihinger, in his *Die Philosophie des Als Ob* (written during the 1870s, published in Berlin, 1913), formulated it in its most general form: "The psyche is an organic shaping force which modifies, to suit its purposes, whatever it takes in and is also capable of adapting itself to the new" (p. 2, my translation).

THE INCOMMENSURABILITY OF SCIENCE AND POETIC WISDOM

Eighty years ago, in his book ‘Mysticism and Logic’ Bertrand Russell explained the basis from which he was developing his discussion of the problems inherent in this juxtaposition. Although the areas are not quite the same, the problems raised by the juxtaposition of scientific and poetic knowledge are very similar and can, I believe, be approached from the same starting point. Indeed, Russell made clear that the attempts to deal logically with elements of mysticism have always taken place in the domain of metaphysics and that the term ‘metaphysics’ comprises not only the mystical heritage of religions but also everything that is mysterious in the sphere of art.

I have found Russell’s definition useful because it enabled me to formulate precisely the point on which I disagree with most philosophers. I therefore want to begin by quoting a relevant passage from Russell’s text (1917):

Metaphysics, or the attempt to conceive the world as a whole by means of thought, has been developed, from the first, by the union and conflict of two very different human impulses, the one urging men towards mysticism, the other urging them towards science. Some men have achieved greatness through one of these impulses alone, others through the other alone: in Hume, for example, the scientific impulse reigns quite unchecked, while in Blake a strong hostility to science coexists with profound mystic insight. But the greatest men who have been philosophers have felt the need both of science and of mysticism: the attempt to harmonise the two was what made their life, and what always must, for all its arduous uncertainty, make philosophy, to some minds, a greater thing than either science or religion. (p. 20)

My disagreement springs from the last sentence. I agree with the observation that the great philosophers have tried to find a way to integrate the mystical and the scientific. But from my point of view it was precisely the preoccupation with mysticism that blocked their progress in epistemology. The attempt to analyze the mystic’s wisdom with the tools of reason invariably leads to a twofold failure: on the one hand it destroys the mystic’s vision of unity because it segments experience into separate, specifiable parts; on the other, it compromises the rules of rational thought because it admits terms whose definition remains questionable because it is based on private experience.

Wittgenstein (1933) has expressed this impasse in his famous maxim: “Whereof one cannot speak, thereof one must be silent” (p. 189). Put that way, it may give

the impression that it is easy to distinguish the things of which one can speak from those of which one cannot. But this distinction is not at all obvious. It dawns on one whenever one finds oneself in a situation where nothing one could say expresses what one would like to say.

This dilemma probably crops up most often when we would like to explain to someone precisely what it is that moves us in a painting, a poem, or a piece of music. We try to speak of the colors, the merit of the composition, the articulation, the associations, the power of the symbolism, and all the things we have picked up from erudite historians and critics—but we realize that we are not even coming close to the actual cause of our emotional reaction and attachment to the work of art. In the end we can only say: I cannot explain it, but it's out of this world.

Fifty years ago, we might have used the expression 'sublime'. It would have meant the same, even if we were not aware of the Latin root of that word, which is *sublimare* and means 'to raise above'. In this case, of course, above the limits of what is specifiable in literal language. In order to convey what we feel, we have to use metaphors.

Though the study of metaphor has been fashionable for some time, I have not found anywhere in the contemporary literature a reference to the particular distinction of different types of metaphor that, to me, seems indispensable if we want to examine the relation between science and mysticism.

The Italian philosopher Giambattista Vico was the first to propose a criterion that allows us to separate the rational scientific use of language from the poetic discourse of mystics and metaphysicians (cf. Vico, 1744). To recognize the value I attribute to this criterion, you need a brief, general explanation of what I believe to be the pattern of metaphor.

Metaphors are constructed by referring to one concept in order to describe another. Such a substitution requires some similarity or analogy between the two. This is to say, a metaphor works to the degree that we are able to transfer one or several customary characteristics of the first thing to the second, to which it is not habitually attributed. The fact that there are always two items involved, is the basis of Vico's distinction.¹

On the one hand, he says, there are linguistic expressions that use words associated with some common experience in order to evoke another experience that would also be accessible to whoever hears or reads the metaphor. We all know this type of metaphor quite well because it is frequently used in everyday language. To clarify its difference from the second type, I shall give an example. If I tell you that the other day I met my friend Robert, and it was with him in his Ferrari that I flew to Boston, you understand my metaphoric use of 'flew' because both flying and traveling by car are within the range of your actual (or at least potential) experience and you have no difficulty in gathering that speed is the characteristic to be transferred.

For the second type, I shall turn to a poet, for instance the author of a psalm, who wrote: "If I take the wings of the morning and dwell in the uttermost parts of the sea ...". You have no possibility of interpreting these words as a description of experiences you have had. Between the wings you have known and your concept

of morning there is a mysterious gap; and the uttermost parts of the sea are altogether outside your experiential range. This hiatus is characteristic of Vico's second type of metaphor. It projects something known into a domain beyond experience or, vice versa, it attributes a mysterious property to something familiar.

Poets, of course, use both types, and Blake (ca. 1770), for example, was a master at linking them.

Smile on our loves, and, while thou drawest the
 Blue curtains of sky, scatter thy silver dew
 On every flower that shuts its sweet eyes,
 In timely sleep.

Blue curtains, sky, and silver dew, flowers and eyes are familiar things in our experience and we have no difficulty in combining them in novel ways. But in Blake's poetic imagination the acts of smiling, drawing, and scattering are attributed to the Evening Star and thus evoke an ineffable mystery.

With the magic metaphor the poet alludes to what is not communicable in literal language because it is not part of the speakers' common experiential world. He wants to share one of those phenomena that William James (1901-1902) has called private and personal.

The difference that distinguishes the poetic metaphors from those that one might call prosaic, gains considerable importance if we want to recognize different kinds of knowledge. If we define the scientific kind as abstractions drawn from experiences or experiments that are repeatable and accessible to other scientists, two things become clear. Poetic metaphors are not compatible with scientific discourse, and, second, the discourse of poets and mystics cannot be translated into the language of the sciences.

In any case, poets and mystics have an aim that is altogether different from that of scientists. They use their poetic metaphors to evoke images which, as Bertrand Russell would say, should make manifest the unity of an absolute world. This, in Paul Valéry's words, is:

... a matter whose diversity and complexity confront the intellect and its will to represent and to dominate by means of symbols the insuperable obstacle of the real: the indivisible and the indefinable. (1936, p. 822)

The concept of a unity that comprises everything is not feasible according to the rules of rational thought. Reason can cut a piece out of the flow of experience. If then it reflects upon just this—a 'something' made discrete by cuts—it creates the concept of unit. As Husserl (1887) noted, this is also the first step in the generation of 'things', if it is followed by reflections on what lies between the cuts. The point that is relevant here, is that the concept of unit is dependent on the endless flow of experience. (Our experience is without ends, because as rational observers we awoke only long after it began, and we shall no longer be there when it ceases.)

This requirement of a background against which a discrete entity can be posited, is the source of a problem in the Big Bang theory. The theory would like to cover the genesis and the development of the universe. Yet, precisely because it purports

to be a history, it leads us at once to ask what was there before the Big Bang. This turns out to be a metaphysical question, and science cannot deal with those. It uses rational thought to construct conceptual models that are to help us organize and systematize the phenomena of our experience.

Vico's distinction of two types of metaphor has provided a way to separate the scientific enterprise from that of poetic wisdom. Much earlier, however, a Byzantine school of theology established the impossibility of capturing the mystical in rational concepts. If God is omnipotent, omniscient, and present everywhere, these theologians argued, then God is entirely different from all things we encounter in our experience. And since our concepts are but abstractions from our experience, we cannot hope to form an adequate concept of God. It is essential to realize that this argument in no way diminished the Byzantines' faith or the value they attributed to revelation. It merely made them aware of the incommensurability of mystical wisdom and rational knowledge.

One of the clearest images of the power of mystical metaphor is formulated in a scene of cross examination in Bernard Shaw's *St. Joan*, where Robert Beaudricourt, the Inquisitor, urges her to reveal the instructions she claims to have received:

Joan: ... you must not talk to me about my voices.

Robert: How do you mean? Voices?

Joan: I hear voices telling me what to do. They come from God.

Robert: They come from your imagination.

Joan: Of course. That is how the messages of God come to us. (1923, scene 1)

There can be no doubt that, in their initial conception of rational models, scientists, too, draw on poetic imagination. It falls under what Peirce called 'abduction' and is essential in the way they configure and then relate experiences. Their hypotheses are generated in the form of 'as if' conjectures. In order to become viable theories, they must be able to serve others as a useful interpretation of observations. Hypotheses and theories, therefore, must be couched in terms that refer to 'data' that are public in the sense that they can be deliberately brought about, recognized, and communicated. No matter how well this succeeds, however, the scientific theories and models concern the rationally segmented world of human observations and experience, not the unitary world of the mystic's revelation.

Nearly all active physicists and a good many philosophers have come to see that there is a mysterious side to the world which, of its nature, will remain out of the reach of science. But there is still the trend of the 19th century to replace religion with science. In the press and in beautifully produced programs on television, in classrooms and lecture theaters, science is celebrated as the golden path to TRUTH. This generates a fundamentalism that is no less pernicious than the religious kind.

If humanity is to find a viable equilibrium for survival on this planet, both scientists and mystics will have to acknowledge that although the rational coordination of actual experience and the wisdom gleaned from poetic metaphors

THE INCOMMENSURABILITY OF SCIENCE AND POETIC WISDOM

are incommensurate, they need not be incompatible. The most urgent task seems to be to develop a way of thinking and living that gives proper due to both.

NOTES

- ¹ A more extensive discussion of Vico's theory of metaphor can be found in my *Radical constructivism* (1995).

FAREWELL TO OBJECTIVITY

To the generation that lives through them, changes in the environment, social revolutions, and the break-down of concepts that formed the basis of a world view, always seem to be deeper and vastly more portentous than all the earlier upheavals history has recorded. Nevertheless I would say that during the eight decades that Heinz von Foerster has so far witnessed, more traditional beliefs were superseded and new perspectives opened than ever before. And among the pioneers who initiated new ways of thinking he deserves a prominent place. This becomes clear above all if one considers the intimate domain where each thinker chooses how he or she should view that treasury of facts, notions, and fixed points that we call knowledge.

The very act of knowing has become questionable in our time. There have, of course, been individual questioners in other centuries who tried to move in the direction that now seems inevitable, but their attempts were brushed aside by the momentum of philosophical tradition and their writings remain *curiosa* at the margins of the history of ideas. Heinz von Foerster did not start as a philosopher but as a physicist with an uncommon interest in the processes of thinking. His epistemological ideas did not spring from quietly reading ancient authors but from his scientist's intuition and his experiences with people in a rather turbulent world. It would take a meticulous biographer to map von Foerster's intellectual itinerary. Instead, I shall try to show that his constructivist theory of knowledge can be substantiated also by snippets from the history of philosophy.

The conceptual revolution that has shaken the 20th century is more profound than the one initiated by Copernicus, who dislocated the human being from the cherished position at the hub of the universe. But even if mankind was relegated to an insignificant minor planet, it could still maintain the belief that it represented the crowning achievement of God's creation and that the human mind towered over everything else because it was able to perceive and understand God's work, at least in its great lines. The 20th century has shown this belief to be illusory. Whatever the stuff is that we call knowledge, it can no longer be considered a picture or representation of an experiencer-independent world. Heinz von Foerster has said this in a conversation with consummate elegance and precision: "Objectivity is the delusion that observations could be made without an observer".

This is not just a quip. Like many other brilliant formulations with which Heinz von Foerster has directed the development of cybernetics, this one is not only an effective slogan but also the expression of a practical consequence of the drastic remodelling that the last hundred years have brought about in a variety of sciences. The diverse 'break-throughs' are of course well understood by the specialists in the

disciplines where they have happened. But so far only a handful of thinkers have begun to appreciate the collective weight of the implicit conceptual changes.

A LONG TRACK OF DISSENT

The general view of the activity of knowing, the knowledge it produces, and the theory of knowledge that is to describe how it works is by and large still conditioned by the age-old belief that what we come to know is there before we know it. It is seen as a process of discovery. As long as we imagine it in this fashion, we inevitably slip into some form of realism that hinges on the belief that we could 'recognize' things as they are, as though experiencing and cognizing had no significant influence on what we come to know.

Once one has understood that perceptions and observations do not drift like snowflakes into a passive receiver but are the result of actions carried out by an active subject, one cannot but wonder what precisely these actions are and how they work. That the active subject and the nature of its rational equipment are relevant, is not a novel idea ('ratio', after all, means relation). Protagoras, in the 5th century B.C., already explained that man is the measure of all things and determines *that* they are and *how* they are (Diels, 1957, p. 122). Socrates, in Plato's dialog *Theaetetus*, takes the opposite position and declares that what is perceived must be there as perceivable beforehand (*Theaetetus*, Line 160). The mainstream of Western philosophy has always interpreted this in the realistic vein and insisted that the results of perception and observation have to be images of things that exist in themselves, independently of the human subject. But throughout history Protagoras, too, has found followers.

If I mention some of these dissenters who did not agree with the conventional wisdom of philosophy, it is not to show that the present revolution merely warms up old ideas and does not offer anything new. In my view, this would be a wrong interpretation. Rather, I want to document that a more relativistic point of view which, at the time of the pre-Socratics, some thinkers adopted on the grounds of intuition and informal logic, has been bolstered up by additional arguments in the course of history and, during the last hundred years, by unexpected 'empirical' findings in the sciences.

Although the history of ideas can hardly be considered to have run along an orderly linear path, it is possible to isolate some details which, in retrospect, can be seen as a development.

Before Protagoras, Xenophanes had already remarked that even if a human mind succeeded in visualizing the world as it really is, that mind would have no way of knowing that it had done so (Diels, 1957, p. 20). This paradox bedevils all those who would like to assume that our knowledge can reflect a world that is independent of the knower. George Berkeley (1710) characterized the situation when he said that we can compare ideas only with other ideas but never with what they are supposed to represent (Part I, §8). There is no logical way around this impasse and all the philosophers of the West, having acknowledged it more or less

explicitly, side-stepped into metaphysics. That is to say, they tried to gain knowledge of a real world by means that are not accessible to reason.

DIFFERENT VIEWS OF KNOWLEDGE

The sceptics barricaded themselves behind the logically irrefutable assertion that it is impossible to acquire certain knowledge about reality. By this stubborn denial, however, they merely contributed to the traditional conviction that the concept of knowledge could not be put in question. They also made themselves unpopular because at all times it was quite clear that there are kinds of knowledge which we simply have to trust. Whether you are a sceptic or a realist, you are constantly drawing conclusions from your experience which you cannot afford to doubt in the conduct of daily living. If you consider that this knowledge can hardly be imported ready-made from an external world, it becomes clear that it must in some way be constructed. This conclusion inevitably raises the question how reason can succeed to produce usable knowledge. This is the topic on which I have collected some outstanding remarks.

The Irish mystic John Scottus Eriugena, who rose to fame in the 9th century A.D., wrote a few lines that could serve as epigram for today's radical constructivism:

For just as the wise artist produces his art from himself in himself and foresees in it the things he has to make ... so the intellect brought forth from itself and in itself its reason, in which it foreknows and causally pre-creates all things it desires to make. (in Moran, 1985, p. 102)

Eriugena's work was placed on the Vatican's Index almost as soon as it was published. Hence it is unlikely that either Vico or Kant had read it. But this is irrelevant, because both these philosophers would have arrived at Eriugena's conclusion along the path of their own thinking.

Vico summarized the notion of cognitive self-organization in the principle that God knew the world because it was He who had created it; humans, similarly, could know what they themselves have made, but not the world that was God's creation (1710, chapter 7, §3). Vico also noted that when we speak of 'facts', we literally refer to something that has been made, because the word *factum* is the past participle of the Latin verb *facere*.

In Vico's view, we ourselves have constructed the things with which we furnish our experiential world. Beginning with the concept of point, we build up shapes, and with the concept of unit we generate numbers (*ibid.*, chapter 1, §1). All this and what follows from it is the work of human imagination, a term he uses in the original sense of creating images. Human science, he suggests, is simply "the endeavour to bring things into agreeable relations" (*ibid.*, chapter 7, §3). He frequently refers to mathematics as an operative science, and I therefore feel justified in interpreting the 'agreeableness' of relations as a matter of smoothness, simplicity, and elegance.

Vico does not explicitly trace all the steps of the reasoning that led him to his theory of knowledge. It was Kant, half a century later, who made the analysis of reason a major objective. He methodically and logically dissociated the activity of knowing from the notion of discovering a pre-existing reality. In the preface to the 2nd edition of his *Critique of pure reason* he states his own version of Eriugena's intuition:

Reason can comprehend only what she herself brings forth according to her design. Following the principles of her judgements she must proceed towards lasting laws and compel Nature to reply to her questions, rather than allow herself to be led, as it were, on a leash. (Kant, 1787, p. XIII)

Later, in the course of his analysis of concepts, Kant expands this in a way that anticipates one of the corner-stones of radical constructivism. He picks up a suggestion first made by David Hume and concludes that all relations, be they consciously grasped or not, are the result of our acts of understanding. He explains:

... we cannot represent anything connected to an object, unless we ourselves have made the connection, and among all representations the representation of a *connection* is the only one that is not given by the object, but can be achieved only by the subject itself because it is the result of its own activity. (*ibid.*, p. 130)

Two things must be made clear in this quotation. First, when Kant speaks of connection (*conjunctio*), he includes every kind of relating and joining our thinking can accomplish. It covers not only the composition of objects out of single sensory properties, but also the mutual coordination of the composite objects, the perceiving or visualizing of spatial arrangements or temporal successions, and the relating of one experience to another. In short, every kind of linkage we use in order to build up concepts and conceptual structures. Consequently, everything that, on the grounds of some analysis, is considered a composite, everything to which we attribute 'structure', is the product of our own capability to present things to ourselves. This is the implicit conceptual basis for von Foerster's 'construction of realities'.

The second point concerns an aspect of Kant's formulation that is likely to mislead unwary readers. A connection, he wrote, is the only representation that cannot be given by the object. This has been taken to imply that there are properties and thus a form of particular existence that could be ascribed to the object and through it to the 'thing in itself'. But this cannot be what Kant had in mind. He explicitly says that we cannot think of anything connected with an object, unless we ourselves have made the connection. Indeed, the object, insofar as it consists of more than one sensory perception, arises in the act of our representation and must not be thought of as preformed in any way. The 'thing in itself', therefore, is a construct (Kant calls it a heuristic fiction) which we can project into 'reality' (i.e., the *ontic* world) only after we have conceptually composed it.

Today, this Kantian insight is particularly relevant when we come to Piaget and read: "l'objet se laisse faire" (*the object allows itself to be treated*) (Piaget, 1977a,

p. 64). In this regard, Piaget's genetic epistemology is in full agreement with Kant: the child must first conceptually construct an object before it can begin consciously to act upon it.

In *La construction du réel chez l'enfant* (1937), Piaget provides a fairly detailed developmental model of how children may construct objects and consider them as 'permanent' furniture in their experiential world. Neither Kant nor Piaget, however, asked what formal logical procedure could yield such a fundamental result. In one of his seminal papers, Heinz von Foerster (1976) proposed a mathematical model that opens a path in this direction. It shows how the permanence of objects may arise as 'eigenvalue' from the recursion of experiences each of which is but a fleeting partial reconstruction of a preceding one.

Continuing the report on dissenters who stepped out of the philosophical tradition and relinquished the dogmatic belief that knowledge must in some way represent an independent ontology, I want to mention Hermann von Helmholtz, one of the most successful scientists of the 19th century. In a postscript to his famous paper on the notion of conservation that became a corner-stone of the 'First principle of thermodynamics', he added some twenty years later that he had come to realize that the notion of causality "is in fact nothing but the presupposition of lawfulness in all appearances of nature" (Helmholtz, 1881, p. 181). A full exposition of this point can be found in his posthumously published papers:

Compared to other hypotheses concerning laws of nature, the causal law is in the following respects an exception: (1) It is the prerequisite for the validity of all others. (2) It gives us the only possibility to grasp something that has not yet been observed. (3) It is the necessary basis for goal-directed behavior. (4) The natural mechanics of connecting our representations leads us towards it. Hence we have the strongest motives to wish that the law of causation be correct; it is the foundation of all thinking and behaving. (in Koenigsberger, 1902, p. 247)

These four points are a somewhat expanded reformulation of the approach to causality that David Hume had taken in the 18th century. Kant reports that it was Hume's ideas that shook him out of a dogmatic slumber. While most philosophers and scientists of the 18th century slumbered on, Helmholtz took both Hume's and Kant's analyses to heart. In a famous lecture on perception, he expounded not only Kant's view that the specific character of our sensory organs determines our sensory impressions, but also the more fundamental idea that space and time are inherent in the conceptual functioning of our reason rather than in the real world (Helmholtz, 1878).

Taken seriously, this brings about a radical shift in our conception of all knowledge, including the knowledge we call scientific and consider more reliable than any other. If space and time are imposed by us as the coordinates that serve to order and systematize experience, then we have no way of representing to ourselves anything that lies outside the domain of our experience. What appears as pattern to us, as sequence of events, as structure of any kind, simply disintegrates and becomes unthinkable if we take away the spatial and temporal coordinates.

What we call knowledge, therefore, cannot possibly be a picture or representation of a reality that has not undergone the transformation of being experienced.

I want to emphasize that this change of view in no way diminishes the importance of knowledge. Its meaning and its value are replaced by a meaning and a value that are indeed more vital. Instead of the supposed correspondence with an unfathomable reality, it is the service knowledge renders that can now be seen as its testable justification. As Humberto Maturana has said, to know means to act effectively (Maturana, 1980, p. 53). To this I add as a complement: Knowledge means to understand. In both the domain of action and the domain of thought, we actively endeavour to fit elements into sequences that we can use as models to gain and to maintain an equilibrium. On the one side, the structures we compose consist of sensorimotor elements, on the other they consist of concepts—and since our concepts usually have roots in sensorimotor experience, the two domains are nearly always intermingled.

In the field of cybernetics, the term ‘model’ has a special meaning. In everyday language it usually refers either to a kind of prototype, a master design according to which something has to be built, or to a replica made in another material or a different size. In cybernetics it is a construct intended to function more or less like an item whose dynamic structure one cannot examine or copy. We have this second sense in mind when we say that our knowledge is made up of conceptual models that enable us to orient ourselves in the experiential world, to foresee situations, and sometimes even to determine experiences.

From this premise we come to the ‘radical’ conclusion that the role of knowledge is not to reflect an objective reality but to empower us to act effectively in the world of our experience, which is to say, to act so that we achieve a goal we have chosen. Hence the constructivist axiom that knowledge must *fit* reality but does not represent.

At first sight it might seem that this modification of the concept of knowledge, though it might require changing an idea here and there, does not drastically alter the accustomed picture of the world. One could think, for instance, that a theory found to fit the world need not be an exact picture, but since it fits it must in some sense reflect that world. This, however, would be a false conclusion. The judgment that a theory fits is always based on the observation that it has been appropriate in its actual applications.¹ It is a judgment drawn from experiences and not from encounters with things in themselves. Besides, it is always relative to specific goals and limited to instances in the past. And the way we see and know experience is determined by the properties of our reason which, as Kant said, “can comprehend only what she herself brings forth according to her design”.

RECENT DEVELOPMENTS

For science and the philosophy of science the constructivist redefinition of the concept of knowing entails a radical shift that would never have been considered had science itself not opened perspectives and produced facts that were no longer compatible with the traditional theory of knowledge.

As early as the 1930s, prominent physicists left no doubt in their writings that the theory of relativity and quantum mechanics had led to findings that seemed incompatible with the notion of science as a search for objective truth. But this did not immediately change the accepted picture of the world. As Gotthard Günther remarked in 1958: “Modern philosophy has so far made little effort to acknowledge the downright world-shaking consequences of the contemporary scientific situation” (p. 9).

This delay in coming to terms with new conceptions in other disciplines is characteristic not only of philosophy. The study of perception, which is a very lively branch of psychology, is divided into subsections, each of which focuses on a specific sensory modality. The psychologists who concentrate on visual experience, rarely take note of the work of their colleagues in the auditory domain. In that branch, there is a respectable literature on a phenomenon called ‘the cocktail party effect’. To experience this effect, one does not need a laboratory. As the name suggests, it happens at mundane events such as cocktail parties. While you are listening to someone’s rather tedious account of a trivial experience, you suddenly become aware that a more interesting conversation is going on behind you. You shift your attention to the voices at your back and follow the bore’s tale just enough to be able to utter a polite noise whenever he or she pauses.

The fascinating feature in this event is the fact that you are able to switch your attention from one stimulus to another, without any ‘objective’ change in the stimuli. This contradicts the naive stimulus theory according to which attention is guided by conditions in the environment. As I discussed at some length in another context, a similar effect has been observed in the visual domain. We are able to move the focus of attention in the visual field without moving our eyes (Glaserfeld, 1981a). Yet, although this spectacular finding was independently reported thirty years ago by prominent experimentalists, it has not yet had the slightest impact on the passive model of perception that is generally presented by psychologists and philosophers.

An even more powerful argument for the contention that it is the subject that actively constructs what it perceives was put forth by Heinz von Foerster when he formulated the ‘Principle of undifferentiated coding’ that applies to all perception:

The response of a nerve cell does *not* encode the physical nature of the agents that caused its response. Encoded is only ‘how much’ at this point on my body, but not ‘what’. (1973, p. 38)

Because it has been known at least since Johannes Müller mentioned it in 1838, textbooks on perception, especially those that take into account the experimental history of the field, occasionally state the fact that neural signals do not indicate the quality of the stimulus. To my knowledge Heinz von Foerster was the first to draw attention to its epistemological implications. It is, of course, more comfortable to perpetuate the myth that our senses provide us with ‘information’ that tells us what the world is like, rather than to ask how it comes about that we perceive it the way we do.

Today, research into cognition has become a wide-spread enterprise and it is somewhat depressing to see that most of it proceeds from a model of perception that has long ago been revealed as untenable. The constructivist orientation, launched by Piaget in the 1930s and greatly enhanced and put on a much wider scientific basis by the work of Heinz von Foerster, still meets a great deal of resistance. In recent years, however, a change of attitude has begun to emerge. That the pursuit of objectivity in the ontological sense is a delusion, has been confirmed by studies in the relatively new discipline of sociology of science and the conceptual paradoxes that have come to the surface in quantum physics have shaken several philosophers of science out of their dogmatic slumber. They no longer see the goal of science as the unveiling of the universe but rather as the invention of models that allow us to cope with the problems presented by our experience (Rorty, 1982).

The concept of knowledge is thus actually changing. Traditional philosophy was forever searching for timeless truths that would be independent of the thinking subject. With the spreading of the Kantian insight that what we rationally grasp is always what human reason builds up according to its own rules, interest has been growing in *how* this building-up proceeds. And this is the area in which we celebrate Heinz von Foerster as a pioneer. There can be no final answers in this area but only models that, for the time being, satisfy our demands. Hence he reminds us that the responsibility for the world we conceptually construct and enact rests with us. Since the task of human thinkers can no longer be to understand a world that God has made, they may at last begin to work towards a viable equilibrium in the world that they experience.

NOTES

- ¹ This is, indeed, the implication of Popper's proposal to substitute falsification for the conventional notion of verification that is logically unattainable (cf. Popper, 1962). For an exposition of the differences between constructivism and Popper's critical realism, see my *Radical constructivism: A way of knowing and learning* (1995).

THE RADICAL CONSTRUCTIVIST VIEW OF SCIENCE

During the last decade of the century that has just come to a close, the term ‘constructivism’ became popular, and that has certain disadvantages. Constructivism cropped up in daily papers as well as in official publications of educational offices. What it stood for was presented as a panacea by some and as a pernicious heresy by others. Some of the propagators and many of the critics had not taken the trouble to consider what constructivism was intended to do. A clarification may therefore be appropriate. To begin with, it has to be pointed out that it does not purport to describe characteristics of the world but proposes a way of thinking that may be useful in dealing with a good many problems that face us today. This paper focuses specifically on the change of epistemological attitude the constructivist orientation might suggest to the practitioners of science and those who write about the results of that practice.

The ‘radical’ theory of knowing I have been working at for the last forty years is a developmental theory, based to a large extent on the work of Jean Piaget. He called his theory *Genetic Epistemology*, and his main purpose was the design of a model to show *how* children could possibly build up the knowledge they manifest as adults. He used the term ‘genetic’ in its 19th-century sense of ‘developmental’ and not in today’s biological sense of ‘depending on genes’. To most traditional philosophers, any such theory must be wrong, because it is based on what they call the ‘genetic fallacy’. True knowledge, to them, is a commodity supposed to exist as such, independent of experience, waiting to be discovered by a human knower. It is timeless and requires no development, except that the human share of it increases as exploration goes on.

THE GROWTH OF KNOWLEDGE

If philosophy of science is to give a plausible account of how scientists acquire what they consider to be knowledge, the conception of steady growth and expansion is clearly inadequate. The history of scientific ideas shows all too blatantly that there has been no over-all linear progression. The shifts from the geocentric to the heliocentric view of the planetary system, from Newton’s spatially stable to Einstein’s expanding universe, from the notion of rigid atomic determinism to that of a statistical basis and the principle of uncertainty, from an Earth with permanently arranged land masses to Wegener’s continental drift—and other upheavals could be mentioned—are incontrovertible signs that fundamental concepts were relinquished and replaced by ideas that were incompatible with earlier pictures of the world.

Karl Popper incorporated this observation in his *Conjectures and Refutations* and added as subtitle ‘The growth of scientific knowledge’ (Popper, 1968). He thought that this process was bound to lead science to a more and more adequate understanding of the real world. But he was unable to indicate how one could ever ascertain that the new conjectures were actually getting closer to such unquestionable ‘Truth’. This was one of the problems that had prompted Thomas Kuhn to try another approach.

One can certainly argue against details in Kuhn’s description of ‘scientific revolutions’ (1962), but no one can deny that every now and then the invention of wholly unforeseeable concepts has relegated previously held convictions to the growing scrap heap of explanatory theories. The image of the scientist gradually unveiling the mysteries of a world that is and forever remains what it is, does not seem appropriate.

THE NOTION OF SCIENTIFIC MODELS

One key to the puzzle was offered in the form of a metaphor proposed by Einstein:

Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world. In our endeavor to understand reality we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even hears its ticking, but he has no way of opening the case. If he is ingenious he may form some picture of a mechanism which could be responsible for all the things he observes, but he may never be quite sure his picture is the only one which could explain his observations. He will never be able to compare his picture with the real mechanism and he cannot even imagine the possibility or the meaning of such a comparison. (Einstein & Infeld, 1938, p. 31)

Later Einstein (1955) formulated the guiding principle:

The object of all science, whether natural science or psychology, is to co-ordinate our experiences and to bring them into a logical order. (p. 1)

This principle forms the core of the constructivist epistemology and determines the constructivist view, not only of the results of scientific endeavor but also of all the ordinary knowledge we glean from everyday experience. Scientists struggle to bring their experiential world into rational order, and so do most other human beings, except that their notions of order and the methods to create it are less coherent and less explicit. Roughly speaking, the scientist’s task can be seen to consist of two alternating phases: the formation (invention) of conceptual structures and the attempt to demonstrate that experience can be fitted into these structures. Like the ingenious observer in Einstein’s metaphor, scientists invent theoretical models of mechanisms and test their viability in repeated and ‘controlled’ experiences that are called ‘experiments’. Non-scientists gather rules of thumb and attempt to apply them in their living practice. For both the actual

purpose is not to obtain a 'true' picture of an observer-independent 'reality', but to provide tools for the management of experience.

Humberto Maturana has characterized the scientific method as a succession of four steps scientists enact when they intend to explain a specific phenomenon:

1. They define the conditions under which the phenomenon can be observed, in the hope that others will be able to confirm the observation.
2. They propose a hypothetical mechanism or model that could serve as explanation of how the phenomenon might arise.
3. From this mechanism they deduce a prediction concerning an event that has not yet been observed.
4. Then they proceed to define and generate the conditions under which the mechanism is expected to lead to the observation of the predicted event.¹

Thanks to the frequent abuse of the word 'phenomenon' active scientists will hardly disagree with this analysis. Though it is not explicitly said, in this breakdown the word is used in its proper sense; that is, it does not refer to things in an independent world, but to what observers isolate in their experience. Scientists are prompted to use their method of inquiry when they experience something which for one reason or another, they feel, requires an explanation. In the history of science, this happened quite often when observations were made that could not be explained by current theories. As Kuhn remarked, however, established theories usually manifest resistance against observed anomalies. Quite often an additional mechanism could be patched on to an existing model in order to cover a disturbing phenomenon. But such patchwork tends to become cumbersome and sooner or later more radical rethinking is unavoidable. This, of course raises the question *how* explanatory models (and the less ambitious rules of thumb) are created in the first place.

PATTERNS OF CREATIVE THINKING

I would propose two ways that seem different at first sight but on closer examination turn out to be somewhat related. The first is not unlike what infants do when, overwhelmed by amorphous experience, they begin to isolate pieces that appear to turn up repeatedly. Repetition and regularity are the elementary tools for the structuring of an experiential world.² Some recurrent things can be coupled to form relatively reliable correlations or, better still, causal connections. The infant, without conceptualizing these connections through reflection, tries to re-enact them because they produce an 'interesting result' (Piaget, 1937, chapter 3). The adult scientist is able to reflect and abstract, and therefore can deliberately search for correlations among events and test them to find reliable patterns of the form P: if A (cause) - then B (effect). Where he can see a causal link, he formulates a rule and uses it as explanation, prediction, and, if possible, to control the sequence of experiences.

The other way of postulating a model is based on what Charles Peirce called 'abduction'. He considered this a third form of inference and defined it in the pattern of a syllogism:

The surprising fact C is observed;
 But if A were true, C would be a matter of course;
 Hence, there is reason to suspect that A is true. (Peirce, 1931-1935, 5.189)

In Peirce's formulation, 'A' stands for a hypothetical rule invented at the spur of the moment. To become viable as explanation and for making predictions, this new rule must be tested in the course of further experience—a kind of induction in reverse. If it turns out to be false, another rule has to be invented, until one is found that fits the experiential facts and can be considered a viable explanation.

This, of course, leaves the question of how such hypothetical rules are invented. Peter Medawar, by all accounts a serious scientist, thought that it required "a sanguine expectation of success and that ability to *imagine* what the truth might be which Shelley believed to be cognate with the poet's imagination" (Medawar, 1984, pp. 17-18). In other words, it is a procedure that has so far defied rational explanation. Most people who have thought about it seem to agree that analogy plays a role in it. But *seeing* an analogy is itself a rather mysterious process that is not unlike a minor abduction.

The reason why I said earlier that the two ways of constructing rules are not altogether dissimilar is precisely this: any coordination of experiences (or 'data') requires focusing on at least some of their particular characteristics. The choice of characteristics is usually very large, but the selection of those that are helpful in the quest for regularities and rules is not always random. Sometimes one has a hunch, and it is this form of intuition that distinguishes the productive scientist from the humdrum collectors of pointless data.

THE ILLUSION OF OBJECTIVITY

It is not surprising that concepts and their concatenation in causal chains that constitute theories, have to be modified and sometimes substituted when the range of experiences is enlarged and begins to incorporate areas that were hitherto not considered. The sudden development of shipping and sea voyages in the Renaissance, the invention of telescope, microscope, and x-rays, and many other technical achievements, generated experiences that exceeded the range of available theories and required fundamental conceptual changes.

In spite of these upheavals the tacit assumption persists that a theory that continues to fit experience and to yield satisfactory results must in some way reflect the structure of an independent reality. From the constructivist point of view, this illusion springs largely from the confusion of, on the one hand, the experiential reality composed of whatever concepts and knowledge are found to be viable in the practice of living and, on the other, a world supposed to exist, describable in itself, irrespective of any experiencer.

The way science is written about, and popularized, does much to reinforce this illusion, because it reiterates that the scientific method and its results are 'objective'. This is an irresponsible play on the ambiguities of the words 'object', 'objective', and 'objectivity'. The first is usually intended as an item isolated as

part of experience; for example, the chair you sit on, the keyboard in front of you, the hand that does the typing, the deep breath you have just taken. In short, any item of the furniture of someone's experiential world can be called an object. In contrast, the philosophically minded also use the word for items to which they ascribe 'existence', which is to say, they posit them as entities supposed to be independent of anyone's experience. In this vein, some mathematicians speak of numbers as 'mathematical objects' as though they existed without anyone generating them by reflection on an activity such as counting.

The other two words, 'objective' and 'objectivity', show a no less tricky ambiguity. On the one hand, they are intended to indicate the belief that the objects you have isolated in your experience are identical with those others have formed. From the constructivist point of view, this, too, is an illusion. It arises from the fact that we can recognize them and to a large extent agree on their description. None of this, however, requires an exact match of what we have individually abstracted from experience. Such commonality and communication shows no more than a relative compatibility of concepts in the situations in which we have had occasion to compare our individual uses of the particular words. Consequently, it would be preferable (and more accurate) if in all these cases we spoke of 'intersubjective' and 'intersubjectivity'. This would preclude any fanciful flights into the realm of ontology. But in philosophical discourse, 'objective' and 'objectivity' are deliberately intended to imply direct knowledge of things as they are 'in-themselves', that is, knowledge of items as they might be prior to being experienced. As Heinz von Foerster put it in conversation, "objectivity is the delusion that observations could be made without an observer".

THE INTERPRETATION OF SYMBOLS

But what about measurements, you might say, or formal derivations—are they not objective? That this is an illusion was remarked a long time ago by Berkeley: "to be of service to reckoning and mathematical demonstration is one thing, to set forth the nature of things is another" (Berkeley, 1721, §18). There is no measurement or computation without units. And to assume that units, be they discrete unitary objects or units of measurement, exist ready-made, prior to the segmenting and coordinating operations of an experiencer, is contingent on the metaphysical presupposition of an independent but knowable reality (cf. Glasersfeld, 1981a). As for the objectivity (or certainty) of computations with symbols in either mathematics or logic, it pertains to the mental operations carried out and, as Berkeley remarked, not to the ontological nature of things.

The symbols used in mathematical computations designate operations that someone has to carry out. As Reuben Hersh put it: "Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later" (1986, p. 19). Even on the simplest level, for instance of $2 + 2 = 4$, the symbol '2' is meaningless for someone who has not abstracted the concept of 'one' from experiential items such as fingers, chocolates, or poker chips, and has then learned that sequences of these items can lead to the abstraction

of compound units that are symbolized by ‘2’, ‘3’, ‘4’, etc. All other mathematical symbols similarly can be *understood* only by a thinker who knows and is able to execute the designated mental operations (cf. Steffe et al., 1983).

As for measurement, it, too, is contingent on the creation of units—units in the form of things to be counted or units of measurement to count ‘continuous’ items that are experienced without articulation of their own. In both cases it clearly is an active experiencer who creates the units. What is not so obvious, is that the discrete entities that are counted, as well as the continuous ones to which units of measurement are applied, are also an experiencer’s creation.

THE SEGMENTATION OF EXPERIENCE

Nowhere have I found this better described than in the aphorisms on language and thought which Wilhelm von Humboldt wrote in 1795:

1. The essence of thinking consists in reflecting, that is, in distinguishing what thinks from what is being thought.
2. In order to reflect, the mind must stand still for a moment in its progressive activity, must grasp as a unit what was just presented, and thus posit it as object against itself.
3. The mind then compares the units, of which several can be created in that way, and separates and connects them according to its needs. (p. 581)³

The expression ‘the progressive activity’, I suggest, is to be interpreted as the mind’s segmentation and coordination of the flow of the raw experiential material that Kant called ‘the manifold’ (*das Mannigfaltige*).

The sensory perceptions (conscious empirical presentations) can only be called internal **appearances**. Not until understanding is added (and makes **order** in the manifold) does empirical knowledge, i.e., experience, arise from it. (Kant, 1800, p. 144)⁴

This is an amplification of Kant’s earlier formulation “that reason can comprehend only what she herself has brought forth according to her design” (Kant, 1787, p. XVI).

Thus, what we ordinarily call ‘experience’ has already been ordered and structured into discrete ‘things’ by perceptual and conceptual operations which endless repetition has rendered unconscious, and by assimilation to more complex conceptual configurations that have been formed in past experience.

Piaget adopted Kant’s general orientation, but disagreed with the notion of the ‘a priori’. He replaced it with a developmental model of the child’s construction of space, time, permanent objects, and causal relations among them (Piaget, 1937). The generation of these fundamental concepts begins with the construction of objects that appear recurrently in the child’s experience.

FUNCTIONAL FIT INSTEAD OF REPRESENTATION

Apart from the focus on *how* the mind could generate the conceptual structure of knowledge, Piaget provided a reason why one should assume that it endeavors to do this. Note that Humboldt, in the 3rd aphorism I quoted, says that it “separates and connects [the units it has created] according to its needs”, but does not specify what these needs might be and where they come from. Piaget’s model provides an answer to these questions which, in my view, is his most important contribution to epistemology. He proposed that the purpose of cognition, since it could not be the discovery and representation of an independent world, should be considered a tool in the organism’s adaptation to the world as it is experienced. Suggestions in this direction had been proposed earlier and somewhat generically by William James (1880), Georg Simmel (1895), Alexander Bogdanov (1909), and Hans Vaihinger (1913). I do not know whether Piaget was aware of these earlier conjectures. In any case, he posited adaptation as the main goal of cognitive activity and extended the function of the concept from the domain of biological survival to that of the organism’s internal mental equilibrium.

To recognize the full power of this position, one has to realize that adaptation is not an activity but the result of the elimination of all that is *not* adapted. Consequently, on the biological level, anything that manages to survive is ‘adapted’ to the environment in which it happens to find itself living. Once this is understood, it follows that what matters is not to *match* an ontic world, but to *fit* into the experiential one, in the sense of being able to avoid whatever obstacles or traps it might present. Taken out of the biological context and applied to cognition, this means that ‘to know’ is not to possess true representations of reality, but rather to possess ways and means of acting and thinking that allow one to attain the goals one happens to have chosen. To know, thus, is to have viable procedures or, as Maturana said “to operate adequately in an individual or cooperative situation” (1988, p. 53).

This modification of the role of knowledge, from ‘true’ representation to *functional fit*, requires an enormous effort because it goes against a traditional belief that is at least three thousand years old. Some of the pre-Socratics saw that this shift was possible, and the sceptics of all ages have reiterated that a true view of the real world could not be attained. But they were unable to specify a relation between knowledge and experience that could replace the conventional one of representation. The common sense view today is a peculiar hybrid. When people say they know such and such, it is presumed—and frequently taken for granted—that they are describing a state or an event of the real world. Yet most people are also aware of the fact that what we experience need not be objective (in either sense of the word). The great scientists of the 20th century have all stated in one way or another that they see themselves in the situation that Einstein described by his metaphor of the man and the watch. Yet, when they sit down to write generally accessible accounts of their achievements, they quickly repress that epistemological insight.

THE RELUCTANCE TO FOREGO ONTOLOGY

Stephen Hawking, to give one example, writes in his Introduction to *A brief history of time* (1988): “A physical theory is always provisional, in the sense that it is only a hypothesis: you can never prove it” (p. 10). But throughout the following ten chapters there are many statements that reflect the belief that, in principle, physics can devise theories that describe the universe *as it is*.

Einstein implied the same belief in his famous dictum “God does not play dice”. This is a metaphysical assumption because it takes for granted that human reason can recognize and understand an observer-independent structure of the universe. Considering that, like other rational knowledge, scientific theories are derived from human experience and formulated in terms of human concepts, it seems no more than a pious hope to expect that these theories reflect anything that lies beyond the experiential interface.

The constructivist conclusion is unpopular. The most frequent objection takes the form of the accusation that constructivism *denies* reality. But this it does not. It only denies that we can rationally know a reality beyond our experience. Constructivism has no quarrel with the mystics who express their intuitions about a transcendent world in poetic metaphors which, of their nature, are not translatable into scientific language. From my point of view, the trouble is that most critics seem to be unwilling to accept the explicit, programmatic statement that constructivism is a theory of knowing, not of being. That a model of the construction of knowledge could be designed without making ontological claims about what is known, is apparently difficult to accept.

It is clear that fundamentalists, who claim to possess the one and only ‘truth’, cannot abide such a notion. And among the scientifically minded the reluctance may spring from the fact that to see the construction of theories as based on autonomous abductions and conceptual assimilation brings with it the realization that the responsibility for the gained knowledge lies with the constructor. It cannot be shifted to a pre-existing world. This deprives scientists of the comforting belief that what they do, can simply be justified as steps necessary for the growth of knowledge. The awareness that it is they who are responsible for their theoretical models and thus at least to some extent for actions based on them, might change the widely held belief that the direction of scientific research must not be fettered by ethical considerations.

NOTES

- ¹ Maturana presented this analysis at the Symposium in honor of Eric Lenneberg (Maturana, 1978, p. 28) and continued to sharpen its formulation in later talks and publications from which I composed the concise form presented here.
- ² From the constructivist perspective, the flow of experience does not present recurrent things as such, but repetition is created by the operations of ‘assimilation’. A full account of assimilation is given in my 1995 book on *Radical Constructivism*.
- ³ The English translation is a slightly modified version of Nathan Rotenstreich (1974).

- ⁴ Kant used the German word *Vorstellung* in the first parenthesis of this quotation. It is usually translated as 'representation,' which also covers the German word *Darstellung* (an image or replication of some original) and gives the misleading impression that the senses convey something that is already structured.

CYBERNETICS AND THE THEORY OF KNOWLEDGE

FIRST ORDER CYBERNETICS

Historical Roots

The term ‘cybernetics’ was introduced in the twentieth century by Norbert Wiener as the title of his 1948 book. In the subtitle he presented his definition: “Control and communication in the animal and the machine”. The word was derived from the Greek ‘Kybernetes’ which referred to the steersman of a ship and is the etymological root of our word ‘governor’. Historians have found prior use of the term in the writings of the French scientist Ampère; and suggestions of control functions similar to those intended by cybernetics could be seen in a paper the famous British scientist Clerk Maxwell wrote in the nineteenth century.

On the practical side, control devices had been invented long before any cybernetic theory or mathematics was formulated. James Watt’s governor which, by shutting a valve at a certain rate of revolutions, prevents steam engines from running faster than they should, is the best-known example. In its ingenious design, the rotational speed of the engine itself provides the ‘feedback’ that reduces the intake of steam. Very much simpler systems, based on a float that ‘governs’ the level of liquid in a container, have existed ever since the water clocks and the self-filling oil lamps of the third century BC.

The Notion of Feedback

The basic meaning of ‘feedback’ is simply this: something that is produced by a machine or organism is led back to modify the process of production. If it increases the output of that process, it is called ‘positive feedback’. It is implemented, for example, in the amplifiers of electronic sound technology. If feedback is used to regulate or limit the process that generates it, it is called *negative*. This second kind of feedback constitutes the core of the control mechanisms that first-order cybernetics is primarily concerned with. In the examples mentioned above, negative feedback originates from an inherent physical force. In Watt’s governor, for instance, it is a set of rotating weights driven outward by the centrifugal force that ‘sense’ the speed of the engine; in oil lamps or water closets there are floats that sink with the consumption of a liquid and ‘sense’ the near emptiness of a container. Their ‘sensing’ is of course purely metaphorical. They have no sense organs, but are constructed in such a way that, on reaching a certain position, they

respectively close or open a valve by means of a physical connection of levers or chains. (An excellent review of mechanical feedback devices can be found in Otto Mayr's book *The origin of feedback control*.)

In all these gadgets, the feedback is mechanical and does not involve signals or symbolic communication. Nevertheless the more sophisticated among them have features that manifest important theoretical characteristics of cybernetics. For this reason the thermostat was used as the prime explanatory example by the early cyberneticians. In the case of an air conditioning system, the role of the thermostat is to keep the temperature in an enclosed space at the desired level. A human agent sets a specific temperature as reference value, and the thermostat 'senses' the actual temperature by means of a thermometer and has the ability to compare it to the set value. If what it registers is lower than the reference, it activates the heater, if it is higher, it activates the cooling system. Inherent in this function are two principles.

The Function of Difference

The first of these principles is that whatever action the thermostat initiates, it is not caused by the sensed temperature as such, but by its difference relative to the reference value. Consequently any of these actions may cease for two reasons: either because the relevant space has reached a temperature equal to the reference value, or because the reference value has been changed and now equals the temperature the thermostat senses.

It is intuitively convincing that this pattern of acting and reacting provides a useful theoretical model to explain behaviors of living organisms. The notion of feedback resolves a major problem of stimulus-response theory, namely that whatever is categorized as a stimulus does not always elicit a response. As a rule, an internal condition also has to be considered, and this condition can be seen as a discrepancy relative to a *reference value*. If there is no relevant discrepancy, the perception of the stimulus does not trigger action. Farmers have always known this. They say that you can take horses to the well, but you cannot make them drink.

Self-Regulation and Equilibrium

Besides, the feedback model makes conceptually explicit what Walter Cannon, an important forerunner of cybernetics, called 'self-regulation'. His book, *The wisdom of the body*, is still one of the pillars of biological cybernetics. Indeed, the various types of homeostasis Cannon studied, mainly in mammals, all demonstrate the ability to compensate for an environmental perturbation by an internal modification rather than by an action on the environment.

A second principle is not quite so obvious. In order to be a satisfactory regulator, a thermostat must not be too sensitive. It must allow for a reasonable space around the set temperature, so that it does not switch on the heater the moment it senses a temperature just below the set value, and then switch on the

cooling system as soon as the temperature has risen above it. In other words, there has to be a range of equilibrium in order to avoid unbearable oscillation.

The realization of this requirement leads to an important shift of focus. Interest is no longer concentrated on isolating one external cause of an organism's perturbation, but rather on the conditions that limit its equilibrium, that is, the constraints within which equilibrium can be maintained. Gregory Bateson applied this idea to the theory of evolution and thus opened a highly productive perspective on the processes of adaptation.¹ As this constituted one of the steps towards second-order cybernetics, we shall return to it in the later context.

The Domestication of Teleology

Historically, the most important effect of the study of such control mechanisms was the legitimization of the concept of purpose in the domain of science. Notions such as intention and purpose had been declared out of bounds for explanations that wanted to be considered scientific. These notions, it was held, involved something that was logically impossible because they suggested that a goal that lay in the future could influence the course of events in the present. Positing such a paradoxical influence was branded as 'teleology', a pattern of thought invariably associated with the metaphysics of Aristotle. A closer examination shows that this proscription was mistaken on two counts. Re-reading Aristotle, it becomes clear that he separated two kinds of teleology. On the one hand, his metaphysics did, indeed, contain the idea that all development would eventually lead to perfection because it was guided by the blueprint of an ideal world. On the other hand, however, he left no doubt that he saw goal-directed behavior as something eminently practical that involved no mystical assumptions whatever.²

Aristotle left no doubt about this when, in Book II of his *Physics*, he discussed the fourth of his explanatory principles that translators later termed 'causes'. He called the fourth principle 'final'—not because it was the last, but because it involved the desired end of the activity in question and not, as do the other three, only the initiation or the stuff acted upon. Aristotle defined the final cause by giving the example of someone walking 'for the sake' of his health, and he added the explanation that, in this case, health is the cause of the person's walking about.

He did not think it necessary to state in so many words how people had acquired the belief that walking would be good for them. It was common knowledge that exercise loosens the joints, reduces fat, stimulates the heart and other functions, and could therefore be considered beneficial to one's health. This had long been established by inductive inference from the domain of common experience. It was no different from the knowledge that food will alleviate hunger and that water will quench thirst. It was one of the countless rules of thumb that have proved to be quite reliable, that we use to get rid of discomforts or to attain pleasures. All of them are based on the implementation of an efficient cause that has regularly produced the specific desired effect in the past and is therefore expected to produce it in the future. But it is we who project this effect into the future, not something that exists in the future and affects the present.

Purpose and Goal-Directed Behavior

Once the analysis of feedback mechanisms presented a model showing how goal-directed behavior could actually work and attain specified goals, the inadequacy of the behaviorist's stimulus-response theory became quite obvious. Although B.F. Skinner in 1977 still persisted in stating that "The variables of which human behavior is a function lie in the environment" (p. 1), it was apparent that the relation between a thinking organism and its environment was only very rarely explicable in terms of direct causal links. The inner state of the organism, its particular cognitive structures, its individual mental focus and interests, including its goals, had to be taken into account, and the notion of reference values and feedback provided powerful tools in the articulation of this new view.

In retrospect, it becomes apparent that not all the ideas that played a part in the development of the cybernetic paradigm were as new as they seemed. In 1921, Ralph Barton Perry, a philosopher of admirable erudition, published a sequence of articles in an attempt to reconcile the behaviorist approach with the notion of purpose. They are documents of a heroic struggle, and it is fascinating to see how close Perry came at times to the cybernetic concepts of goal-reference and negative feedback. In one of his papers he said, for example, that an act is performed because its implicit sequel coincides with the incomplete part of some course of action that is at the time dominating the organism. What he did not mention (and presumably did not see) was that the assessment that something is 'incomplete' requires a mental representation of the item in its state of completeness.

Forty years earlier, in his fundamental textbook on psychology, William James had already distinguished two kinds of teleology: that of an agent who deliberately acts to attain a goal; the other, the goal-directedness an observer attributes in order to explain the agent's behavior. This foreshadows the distinction Gordon Pask introduced into cybernetics. Applying Pask's distinction to the thermostat, we can say that its internal purpose is the elimination of differences between the set reference value and the temperature it senses, whereas for an external observer its purpose is the maintenance of a desired temperature.³

To sum up this brief survey: first order cybernetics was primarily concerned with the analysis and engineering implementation of goal-directed behavior. It formulated a viable theory of purposive mechanisms and provided its mathematical formalization. On the practical side, it succeeded in designing and actually constructing a great variety of mechanisms that manifested purposive behavior. The realization of automatic pilots, target-finding missiles, chess-playing computers, and robots capable of guiding their actions by their own perceptions, is ample proof of the power of the cybernetic approach. From a theoretical standpoint, however, the most significant achievement was that the practical success of cybernetic constructs brought with it the rehabilitation of the concept of purpose. This opened the path towards the study of purposive agents, the domain of second order cybernetics.

Communication

While the analysis of feedback was being developed to account for control mechanisms, a no less important theoretical model was worked out as a technical approach to the phenomenon of communication. Communication was the second key term in the title of the book that launched cybernetics, and its problems had been tackled some years earlier by Claude Shannon with some acknowledged contributions from Norbert Wiener. The *Mathematical Theory of Communication* had an enormous influence in the development of communication technology. Far more relevant to the present survey, however, is the conceptual clarification the theory provides for communicatory processes in general.

A message can be sent from point A to point B only if there is a medium that allows such transmission. This medium has to be a 'channel' in which pulses of some form of energy can travel. In old-fashioned telegraphy, it was a wire and pulses of electrical energy; in radio and television, it is electromagnetic waves and the modulation of their frequency or amplitude; in speech, it is sound waves and their modulation; and in writing or printing, it is marks on some physical surface that can be taken from one place to another. But these pulses or marks do not carry a message, unless it has been encoded in them. For this to happen, three things are necessary. First, the sender must have a code, that is to say, a list that indicates what kind or combination of pulses or marks corresponds to the elements of the message that is to be sent. Second, the receiver of the message must also have such a list in order to decode the pulses or marks he receives. Third, if communication is to succeed, the code-lists of the sender and of the receiver must obviously be the same.

This last condition was never seen as a problem in technical communication systems, because it was taken for granted that the established code would be distributed to all participants in the system. However, the technical analysis highlights a point that was rarely considered in the study of linguistic communication. Although there are lexica for natural languages, their contents are accessible only to readers who already have a basic vocabulary. Children are not handed a code that displays the connections between words and their meanings—they have to develop it for themselves, largely by trial and error. It is true that the meanings of a number of words can be conveyed to them by parents or care givers, but the bulk of their vocabulary is formed on the basis of subjective experience in the course of interactions with other speakers.

As a result of this inherent looseness in the acquisition of the linguistic code, linguistic messages and texts in general leave a great deal of space for individually divergent interpretations. The realization of this fact had a considerable influence on some of the authors of second-order cybernetics.

SECOND ORDER CYBERNETICS

The difference that separates the two kinds of cybernetics was most succinctly stated by Heinz von Foerster, whose work initiated the new direction. The first order is the cybernetics of observed systems—the second, the cybernetics of

observing systems.⁴

Questions about what it means to observe and what kind of knowledge we glean from observation, were raised by the pre-Socratics at the very beginning of recorded Western philosophy. In the course of this history, innumerable theories of knowledge were proposed, ranging between two extremes. On the one side there is naive realism which is based on the assumption that what we come to know must be a more or less ‘true’ representation of an independently existing reality. On the other side, there is the form of subjective idealism that is called solipsism and holds that there is no reality beyond the human mind. At the realist end of this axis looms the problem of how our knowledge could ever be demonstrated to be true relative to a reality posited to be independent of its observers; at the other, there is the no less daunting puzzle why so many things concocted in the domain of our ideas turn out to be patently false in the world which we actually experience.

The Epistemological Problem

Scattered throughout the history of philosophy there are thinkers who came to see that it was impossible to find a rationally tenable position anywhere on the established axis. Whatever was proposed contained one or more elements of either one of the two extremes and could therefore be demolished by well-established arguments. There seemed to be no way to counter the sceptics’ solidly founded contention that true knowledge of either the world or the mind was impossible. Consequently the nature of what we consider to be justified beliefs remained a troublesome problem.

In the sciences, problems that resist solution for a long time are usually solved in the end by the drastic modification of one or more concepts that until then were unquestioningly taken for granted. The conceptual changes were sometimes dramatic and their proponents faced fierce resistance before the established leaders of their respective field gave in (or died) and a new way of thinking gradually became general. The shift from the geocentric to the heliocentric theory of the planetary system is probably the most obvious among the historical examples. In general it was either the accumulation of empirical evidence, or the wider applicability or simplicity, that gave the new conceptualization the winning edge. Suggested before the Second World War by the Polish author Ludwick Fleck, this theory of scientific procedure and change was elaborated and presented by Thomas Kuhn (1962), in his book *The Structure of Scientific Revolutions* that became the scientific best-seller of the post-war period.

Philosophy, and epistemology in particular, do not show this pattern. The unsolved problems in these disciplines are largely the same as they were two and a half millennia ago, and so are the concepts involved in the problems’ formulation. One of these is the very concept of knowledge. It has been, and generally still is, taken for granted that what we want to call knowledge must in some way correspond to a reality that lies beyond our experiential interface. Like the notion that the earth must be at the center of the universe, it is an idea that is difficult to give up. Yet no one seemed to be able either to demonstrate such correspondence with an independent reality, or alternatively, to give a convincing account of how,

without it, we could come to have all the knowledge that we confidently trust when we make decisions about our actions.

A New Theory of Cognition

Some years before cybernetics was born as a discipline, Jean Piaget formulated a principle of self-organization as: ‘The mind organizes the world by organizing itself’ in his 1937 book on the child’s construction of reality.⁵ In his theory, this autonomous process of organization forms the core of the capability of producing knowledge and is the highest form of adaptation. He took the concept of adaptation out of the context of evolution, where it does not involve an activity, but concerns the biological capacity to survive within the constraints of the physical environment; and he transposed it into the cognitive domain, where it concerns the active striving for, and maintenance of, equilibrium among concepts, schemes of action, and in the generation of knowledge as a whole.

Talcott Parsons was among the first to remark on the relation between Piaget’s theory, Cannon’s concept of homeostasis, and the revolutionary notion of self-regulation. But it was Gregory Bateson’s analysis of the process of adaptation that allowed us to see clearly the conceptual connection between Piaget’s theory of cognitive equilibrium and cybernetics.

In his seminal article on ‘Cybernetic Explanation’, Bateson (1972b) wrote that:

Causal explanation is usually positive. We say that billiard ball B moved in such and such a direction because billiard ball A hit it at such and such an angle. In contrast to this, cybernetic explanation is always negative. We consider what alternative possibilities could conceivably have occurred and then ask why many of the alternatives were not followed, so that the particular event was one of those few which could in fact occur. The classical example of this type of explanation is the theory of evolution under natural selection. According to this theory, those organisms which were not both physiologically and environmentally viable could not possibly have lived to reproduce. Therefore, evolution always followed the pathways of viability. As Lewis Carroll has pointed out, the theory explains quite satisfactorily why there are no bread-and-butter-flies today. In cybernetic language, the course of events is said to be subject to *restraints*, and it is assumed that, apart from such restraints, the pathways of change would be governed only by equality of probability. In fact, the ‘restraints’ upon which cybernetic explanation depends can in all cases be regarded as factors which determine inequality of probability. (p. 399)

In the theory of evolution, the biological living space of each organism is hemmed in by the limits entailed by its physiological make-up and by the obstacles presented by its environment. Both these are given conditions over which neither the individual nor the species has control. In contrast, in Piaget’s theory of cognition, a relative, labile equilibrium is possible only in the space generated by the active avoidance of, or continual compensation for, perturbations. The conceptual difference between the two essentially parallel theories resides in the

source of the restraints. On the biological level the factors that limit survival are in no way determined by the organism itself. On the cognitive level, however, perturbations that impede equilibrium spring from the mutual incompatibility of goals the organism has chosen and/or of the means used to attain them.

The Construction of Knowledge

In this view, cognitive agents themselves are clearly determining factors in the generation of knowledge. For if the goal is conceptual equilibrium, only the conceivers themselves can determine when it is reached and when not.

There is yet another, quite different consideration that has brought the role of the cognitive agent to the fore. In his book, *'Cybernetics — or control and communication in the animal and the machine'*, Norbert Wiener (1948, pp. 156-157) wrote: "All the great successes in precise science have been made in fields where there is a certain high degree of isolation of the phenomenon from the observer". In Astronomy, he goes on to explain, the scale is 'enormous', in atomic physics 'unspeakably minute' compared to the scale on which we live. In both cases, he says, "we achieve a sufficiently loose coupling with the phenomena we are studying to give a massive total account". At the end of the passage, however, he warns that "the coupling may not be loose enough for us to be able to ignore it altogether".

Second order cybernetics could be characterized partially by saying that it originated from the doubt expressed in Wiener's warning. The relationship between observers and what they observe became its primary object of study. This study, clearly, did not have to begin from scratch. Beginning with the famous statement "Man is the measure of all things", made by Protagoras in the fifth century BC, there is a chain of records indicating that some thinkers had come to realize that the observer plays an active part in the process of observation, and that anything he observes bears his mark. But they have had relatively little influence on the philosophical tradition.

Even the clear statement that Immanuel Kant made in the introduction to his *Critique of pure reason*, namely that "reason can understand only what she herself has brought forth according to her design" (1787, p. XIII), did not greatly shake the general belief that scientists succeed in unveiling the mysteries of the universe.

Rational Models and the Role of the Observer

Second-order cybernetics is the only Western discipline that has fully accepted this view and subscribes without reservation to the general description of the scientist which Albert Einstein and Leopold Infeld provided by means of the famous metaphor of the man and the watch in their *Introduction to relativity*:

Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world. In our endeavor to understand reality we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even

hears its ticking, but he has no way of opening the case. If he is ingenious he may form some picture of a mechanism which could be responsible for all the things he observes, but he may never be quite sure his picture is the only one which could explain his observations. He will never be able to compare his picture with the real mechanism and he cannot even imagine the possibility or the meaning of such a comparison. (1938, p. 31)

This metaphor brings home the fact that the real world is unknowable or, as cyberneticists came to say, a 'black box'. The twentieth century revolutions in physics, especially that provoked by quantum theory, prompted all their foremost exponents to declare, in one way or another, that the knowledge they had gathered concerns the organization of experience rather than the objective structure of an independent reality. But the attitude in most physics departments and of the writers of textbooks still tends to be that of realists.

Operational Definitions

Another development that, in retrospect, could have accentuated the role of the observer, was that of 'operational definitions' by the physicist Percy Bridgman. He succinctly characterized the ideal attitude of the researcher in his 1936 treatise on *The structure of physical theory*:

It is the task of theoretical physics to compress all experimental knowledge into an understandable point of view; the theorist can never foresee what the experimenter will find when his range is extended to include fields at present inaccessible, so that he must always regard his last and most successful theory as a structure of limited validity, always subject to the necessity for radical alteration when extended to include such new experimental facts as may be later discovered. (p. 2)

Bridgman formed the operationist idea in the context of Einstein's theory of relativity by an examination of the concept of simultaneity. He explained that the germ of the theory had been the examination of what we do when we compare the times indicated by clocks in different places. Einstein's revolutionary recognition was that the relation of two events which hitherto had been unthinkingly called simultaneity involves a complicated sequence of physical operations which cannot be uniquely specified unless we specify who it is that is reading the clocks.

Every observer, be he or she reading a clock, looking through a telescope, or simply watching an event, has a specific position, not only in the spatial sense. Like the optical instruments scientists use, all observers have their own observational characteristics and their specific way of seeing. They also have a 'point of view' that determines the concepts with which they grasp what they observe and how they formulate it when they want to communicate it to others. The 'coupling' Wiener spoke of, between the agent and the object of observation, therefore, cannot be disregarded.

Several Parallel Developments

Once interest was focused on the cognitive processes involved in observation, cyberneticians found themselves facing the problems that had bedeviled epistemologists during the entire course of history. The protagonists of the new discipline, however, had the advantage of a highly technical background. The successful engineering of purposive devices that manifested a practical solution of the puzzle of teleology helped to generate the confidence to break with other traditional philosophical assumptions. The most fundamental of these dogmatic fixtures was the belief that human knowledge ought to mirror a timeless, independent reality.

If the Piagetian principle that the mind organizes itself is taken as a working hypothesis, it becomes very clear that the primary purpose of knowledge is not the representation of an external world but rather the establishment of ways of thinking and ways of acting that serve the purposes the knower has formed in the world of his or her experience. This realization led to different but essentially parallel developments within the framework of second order cybernetics.

For some of the pioneers, George Spencer Brown's book *Laws of form* (1969) provided additional conceptual foundation. The 'calculus of distinctions' presented in this book can be seen as the most elementary basis of all logical thinking. According to Spencer Brown, the act of making a distinction is the first step in any sequence of rational thoughts. This offers an ideally simple starting-point for conceptual construction and, indeed, led the author himself to the striking statement: "Our understanding of... a universe comes not from discovering its present appearance, but in remembering what we originally did to bring it about" (p. 104).

Linked by the common goal of a constructivist epistemology, individual cyberneticians went their own way in their struggle with the problems of cognition. In the narrow frame of this survey only the three relatively complete theoretical models can be acknowledged.

Radical constructivism. Heinz von Foerster started from the fundamental insight that there can be no observation without an observer. What we call 'real', therefore, is always rooted in an observer. In his seminal 1973 article 'On constructing a reality', Heinz von Foerster explained his use of 'a' in 'a reality'. The indefinite article, he said, implies the ridiculous notion of other realities besides 'the' only one that we cherished as our Environment.

There is a deep hiatus that separates the 'The' school of thought from the 'A' school of thought in which respectively the distinct concepts of 'confirmation' and 'correlation' are taken as explanatory paradigms for perception. The 'The-School': My sensation of touch is *confirmation* for my visual sensation that there is a table. The 'A-School': My sensation of touch in *correlation* with my visual sensation generate an experience which I may describe by 'here is a table'. I reject the The-position on epistemological grounds, for in this way the whole problem of cognition is safely put away in one's own cognitive blind spot and its absence can no longer be seen. (p. 39)

The statement that it is the cognitive agent's active correlation of sensory impressions that creates the notion of objects would be somewhat dubious if it were taken by itself. But von Foerster supports it by citing the 'Principle of undifferentiated coding', formulated by Johannes Mueller before the middle of the nineteenth century and confirmed by neurophysiologists ever since. The principle summarizes the finding that the neural signals sent from an organism's sensory 'receptors' to the brain are qualitatively all the same and differ only in intensity. In von Foerster's formulation, "the response of a nerve cell does *not* encode the physical nature of the agents that causes its responses. Encoded is only *how much* at this point of the body, but not *what*" (1973, p. 38). This well-established empirical finding presents a serious stumbling block for all realist theories of knowledge.

The epistemological position of radical constructivism is primarily based on the logical consideration that observers necessarily conceptualize what they observe in terms of concepts that are of their own making (as Kant said, according to reason's own design); but the fact that the 'data' of vision, hearing, touch, smell, and taste are (from the neurophysiologist's point of view) all indistinguishable is a welcome empirical corroboration of the perceiver's autonomous constructive activity.

The constructivist theory of knowing, one of the cornerstones of second order cybernetics, can be briefly summarized in the principles:

- Knowledge is the result of a cognitive agent's active construction.
- Its purpose is not the representation of an external reality, but the generation and maintenance of the organism's equilibrium.
- The value of knowledge cannot be tested by comparison with such an independent reality but must be established by its viability in the world of experience.

The theory of autopoiesis. Humberto Maturana developed his theory of cognition as a biologist involved in the study of perception. Investigating vision in frogs and color vision in pigeons and primates, he came to the conclusion that responses in these organisms were not triggered by specific external stimuli but by the co-occurrence of neural events that showed no one-to-one relation with conditions or events in their environment. In experiments done by Lettvin, Maturana, McCulloch, and Pitts in 1959, a frog, for instance, would respond with its 'bug-catching' behavior whenever three or four neural signals created a specific pattern in its brain, irrespective of the fact that, from an observer's point of view, what caused the individual signals in the frog's visual organ may have nothing to do with a bug that could be eaten by the frog. The 'what' that caused the response was far from fully determined, and this finding required a radical revision of the generally accepted theory of more or less direct perception.

A partial conceptual skeleton of the 'autopoietic' model of cognition,⁶ which Maturana worked out during the subsequent decade, can be summarized by the following statements:

- a) Whatever is said, is said by an observer to another observer who may be the speaker himself.

- b) Cognition as a process is constitutively linked to the organization and structure of the cognizing agent.
- c) Autopoietic systems are closed homeostatic systems without input or output.
- d) The changes of state through which an autopoietic system goes while compensating for perturbations can be seen by an observer, for whom the system is in the context of an environment, as the system's actions upon the environment.

From this perspective, it becomes clear that the observer should remain aware of the fact that the observed organism, and the environment in which it is being seen, are parts of the observer's experiential field and therefore not an objective reality.

When Maturana published statement (a) for the first time in 1970b, it immediately seemed to be a perfectly obvious statement to his readers; but a look at the histories of philosophy and science shows that the quest for descriptions of the world that could be considered 'objective', in the sense that they are not dependent on the characteristics of the observer, was never given up.

Statement (b) can easily be translated into Piagetian terms by saying that what a cognitive organism comes to know is necessarily shaped by the concepts it has constructed.

The term 'closure' in statement (c) is intended to indicate that the equilibrium of the autopoietic system may be perturbed from the outside, but there is no input or output of 'information'; its actions are in the service of its homeostasis.

Statement (d) speaks for itself. It is an application of statement (a) in that it makes explicit that whatever is conceptualized and said about an observed system is an observer's description of something within that observer's experiential field, not a description of a world as such.

Maturana's autopoietic model is a highly complex and comprehensive theoretical edifice. The four points listed here may serve to render an idea of its general direction but they cannot convey the variety of original ideas that the edifice contains. The many applications that have been developed from it in areas as diverse as family therapy, immunology, and management science are testimony to its inherent richness.

The Italian operational school. One of the first centers of cybernetics in Europe focused, from the very beginning, on the problems of conceptualization and its role in the semantics of linguistic communication.

Traditional semantics has always been limited to using words in order to define the meaning of words. For the rest, it relied on the theory of reference, based on the belief that words refer to things in an external, speaker-independent world. Ferdinand de Saussure, the Swiss founder of modern linguistics, had already shown at the beginning of the twentieth century that the semantic linkage was not between words and things, but between the concepts of words and the concepts of things. Both the signs and what they signified were wholly within the experiential world. The illusion of external reference sprang from the fact that meaning could to a large extent be considered intersubjective. Concepts were explained as abstractions the speakers of a language learned to make in the course of their common experience. Piaget called this process 'empirical abstraction' where it could be

shown to originate from sensory experiences; and he added the level of 'reflective abstractions' which derive from mental operations. The idea that mental operations are a source of knowledge goes back to John Locke. But neither Locke nor Piaget nor Guy Cellérier, who wrote about the connection between Piaget's theory and cybernetics, further analyzed the mechanisms of abstraction that might yield results that could then be named by words. This analysis was undertaken by Silvio Ceccato but has remained virtually unknown because it was published only in Italian.

Silvio Ceccato's main objective was 'the mechanization of the mind', by which he intended the design of a model that could carry out mental operations.⁷ Early on, he had stumbled on Bridgman's idea of operational definitions and it determined the course of his work. If the meaning of words was conceptual, a valid semantic analysis required the specification of the medium out of which concepts could be made before they were associated with words. This position became the basis of several projects of language analysis by computer in the 1960s. Ceccato posited an active process of attention as material for the conceptual constructs. Unlike the general notion that attention functions as a kind of spotlight that illuminates objects, he saw it as an oscillatory process producing regular pulses. These pulses could either focus on other signals in the neural network or remain unfocused to mark intervals and distinctions. This attentional activity provided a mechanism for the composition of conceptual structures.

His team at the Milan Center of Cybernetics worked extensively on the minute analysis of mental operations that constitute the meaning of words. Like any effort to produce a comprehensive lexicon, it was a gigantic project. When funds dried up, the team dispersed in the mid 1960s. Giuseppe Vaccarino, who carried on single-handed for forty years, has now brought the work to a conclusion with several volumes on the conceptual foundations of the Italian language. Ceccato's theory of 'operational awareness' is kept alive, applied, and further developed in the electronic age by Felice Accame and the Società di Cultura Metodologico-Operativa which he directs.

APPLICATIONS OF CYBERNETIC PRINCIPLES

The idea that the experiencing subject shapes its experience according to its own ways of perceiving, conceiving, and feeling was implicit in the writings of many authors long before cybernetics proposed cognitive self-organization. But it remained a marginal idea and never became an insight that determined general philosophical views. Recent philosophers, such as Nelson Goodman and Richard Rorty, whose epistemological views are partially compatible with the theory of knowledge developed in second-order cybernetics, use only arguments generated within the tradition of their field and do not mention the parallels to this other contemporary area of research.

In a few disciplines the situation is different. The cybernetic theory of knowing has begun to play a noticeable role in anthropology, sociology, psychotherapy, and, most importantly, in education. What follows is no more than a sampling of conceptual parallels.

Anthropology and Sociology

Gregory Bateson began his career as an anthropologist with a thorough preparation in biology. His cybernetic analysis of the theory of evolution and his clarification of the concept of adaptation, at first a by-product of his studies of natives in New Guinea, led to the notion of self-organization and his cybernetic view of knowledge. Owing to his work and that of others such as Harold Garfinkel and Clifford Geertz, the perspective of anthropologists was slowly shifted. The earlier attitude, founded on the European notion of scientific objectivity gave way to the realization that viable knowledge of other cultures could be attained only by a participatory understanding of their conceptual and social structures.

This development was in keeping with the cybernetic maxims that there are no observations without an observer and that the observer's explanation of the observed is at best a model that proves viable in the experience of others. Geertz (1973) formulated the new attitude in his book *The interpretation of cultures*:

We (anthropologists) begin with our own interpretations of what our informants are up to, or think they are up to, and then systematize those... In short, anthropological writings are themselves interpretations, and second and third order ones to boot. They are, thus, fictions; fictions, in the sense that they are 'something made', 'something fashioned',—not that they are false, nonfactual, or merely 'as if' thought experiments. Cultural analysis is (or should be) guessing at meanings, assessing the guesses, and drawing explanatory conclusions from the better guesses, not discovering the Continent of Meaning and mapping out its bodiless landscape. (p. 20)

The influence of cybernetics on sociology has been far more direct. Niklas Luhmann, whose work has become quite familiar beyond the German-speaking sphere, adopted and adapted Maturana's autopoiesis and added an intricate model of communication in his construction of a complex and comprehensive theory of society and societal manifestations. His personal interactions with both Maturana and Heinz von Foerster brought out some disagreements about his use of their ideas. Maturana objected that societies could not be considered autopoietic systems because one could not ascribe to them the biological structure and organization which, from his point of view, is indispensable for autopoiesis. Von Foerster, who had contributed much to the clarification of the concept of information, could not accept the notion of communication as a reified element in Luhmann's theoretical edifice. Nevertheless Luhmann's work on social systems constitutes a major, albeit idiosyncratic, application of second-order cybernetics.

Psychotherapy

Considerations not unlike Wiener's admonition that the relation between observers and what they observe cannot be altogether disregarded have wrought a significant change in the theory and practice of psychotherapy. Traditionally, it was held that there is a clear, demonstrable difference between the sane and the insane, and that mental insanity could therefore be detected and objectively characterized with

relative ease. Empirical studies, by Rosenhan (1984) and others, however, have shown that an objective observation of behaviors and their categorization as 'abnormal' is very often problematic. A large-scale investigation of what happened to 'normal' people who were committed to psychiatric hospitals shook the discipline to its foundations. Among other things, the study made two points appallingly clear: first, the observation of behaviors always involves a particular interpretation of what are considered empirical facts; second, both the facts and their interpretation are to a large extent determined by the observer's expectations. Thus, normal reactions of a pseudopatient were interpreted as symptoms of schizophrenia by the hospital staff, for no other reason than that the person had been categorized as a schizophrenic when he or she was being admitted.

As a corollary of the realization that observations could not be considered to be independent of the observer's concepts, theories, and contextual assumptions, the conceptual fictions of patients were no longer seen as totally erratic. Instead, it was assumed that they had their own, albeit 'abnormal' logic and systematicity and that at least in some cases therapy had a better chance if it explored the patient's ways of thinking. This approach, of course, contrasts sharply with the common practice of categorizing patients as mentally ill and then treating them pharmaceutically.

Gregory Bateson and Paul Watzlawick introduced the cybernetic way of thinking into psychotherapy and the development of different therapeutic methodologies on the basis of second-order principles is still going on.⁸ To give an instance, constructivism and Maturana's autopoietic model in particular had a considerable influence in the area of family therapy. Its general approach has been guided by the notion that each member of a family constructs his or her own 'reality' of the family, and that the problems of, and conflicts among, the individuals often spring from the incompatibility of their constructions.

Education

The cognitive psychology of Jean Piaget had a first bout of influence on the practice of teaching some sixty years ago. His specification of stages of development was picked up by designers of curricula, and the notion of the role of biological maturation in the ontogeny of mental development became a kind of dogma for educators and educational researchers. The epistemological core of Piaget's theory, however, was largely disregarded. Not until around 1970 did a number of researchers focus on the idea of self-regulation. By then Piaget himself had become aware of the affinity of his theory and basic concepts of second-order cybernetics. Above all, they shared the principle that whatever we call knowledge has to be actively constructed by the knowing subject.

From then on, this principle of self-organization gained some attention among educators. By now, it has a firm foothold in the areas of mathematics and science education. An extensive literature concerning the individual and social construction of knowledge has been produced and there is considerable evidence that its practical applications are successful, but it is still far from being universally accepted.

Among the points stressed by advocates of constructivism are the following:

- If knowledge consists of conceptual structures learners have to form in their own heads, verbal communication (by teachers' speech or textbooks) does not guarantee a positive result. What is required is thought, that is, reflection on both practical experiences and whatever teachers and books try to communicate.
- Two excellent ways for teachers to foster students' reflection are the imposition of collaboration with others and the persistent demand that students verbalize their thinking in their attempts to solve a problem ('Team problem-solving').
- The implementation of the constructivist approach requires two things of teachers: they have to credit students with the ability to think and they have to provide the students with opportunities to discover that they are able to solve problems without the teacher providing a ready-made solution.
- Perhaps most importantly, the insight that linguistic communication cannot replace students' active abstraction of knowledge from their own experiences.

These four points are sufficient to indicate the need for a radical change of educational attitude: namely the concession of a great deal of autonomy to the student in order to develop their own capacity for thinking and learning.

A serious argument against such a change is that it would require tests that are very different from the ones given to students now. This is indeed a problem. Testing for understanding is far more difficult than testing for the correct repetition of verbal statements heard from the teacher or read in a textbook. On the other hand, there is sufficient evidence by now, that the motivation to learn grows by itself once students realize that learning is not a passive but an active process and that the ability to solve problems by one's own thinking yields satisfactions that are at least as enjoyable as winning a game.

CONCLUDING REMARKS

First order cybernetics originated in 1948 with Norbert Wiener's publication of his book. It was baptized as an independent discipline when the prestigious Josiah Macy Foundation decided to devote meetings to the new area of research during the years that followed. Before it was given its name it had already started, and now continued at a growing pace, to revolutionize technology by introducing self-regulating mechanisms that could fly planes, guide the actions of robots, and enable computers to prove theorems and play chess. Today nearly all the machines that serve us in the conduct of our daily lives contain cybernetic devices—from the braking systems of the cars we drive, and the traffic lights that control our driving, to the networks of electric power and the photographic cameras we use.

Two conceptual revolutions went hand in hand with the technological innovations. On the one hand, the successful analysis of feedback mechanisms made the notions of purpose and goal-directed behavior respectable elements in scientific explanation; on the other, the theory of communication substantiated the old suspicion that language by itself was not a vehicle for the transportation of knowledge—it could stimulate conceptual construction, but it could not carry concepts from one head to another. From these premises developed second order cybernetics which, by means of the concept of self-regulation, was able to propose a novel approach to the age-old problems of the theory of knowledge. From this

new perspective, human knowledge is defined as the repertoire of ways of thinking and rules of action that are found to be successful in the domain of experience. So viability is put in the place of ontological truth. This momentous change is justified by the fact that we gather our rational knowledge from experience and the only way we have of testing it is again through experience. This in no way diminishes the role of that other kind of knowledge which the religious and the mystics of all ages claim to possess on the basis of revelation or intuition. That knowledge, however, owing to its origin, is beyond the purview of rational analysis.

The epistemological proposal of second order cybernetics is still viewed with suspicion by traditional philosophers, and it will take time to overcome their resistance. One reason why the notions of cognitive self-regulation and experiential viability, instead of ontological truth, are difficult to accept may be that it is easier to put up with the contention that one's solution to a problem may be wrong, than with the idea that no solution will ever be the only 'true' one.

Nevertheless, the focusing on self-regulation in an area of possibilities within constraints has led to considerations that seem eminently appropriate at the present moment in human history. In one of the papers that launched his notion of a second order cybernetics, Heinz von Foerster (1974) formulated a guide-line for society by referring to the rehabilitated concept of purpose. His admonition, made a quarter of a century ago in *The cybernetics of cybernetics*, seems no less pertinent today:

Social cybernetics must be a second-order cybernetics, in order that the observer who enters the system shall be allowed to stipulate his own purpose: he is autonomous. If we fail to do so, somebody else will determine a purpose for us. Moreover, if we fail to do so, we shall provide the excuse for those who want to transfer the responsibility for their own actions to somebody else: "I am not responsible for my actions; I just obey orders". Finally, if we fail to recognize autonomy of each, we may turn into a society that attempts to honor commitments and forgets about its responsibility. (p. 8)

NOTES

- ¹ See Bateson (1972). This collection of essays laid the foundation for new ways of looking at psychotherapy, ecology, learning, and epistemology. See also Ceruti (1994), a very readable exposition of the principles of evolution and the constructivist approach to cognition.
- ² For an explanation of the relations between Aristotle's 'final cause' and the cybernetic analysis of goal-directed behavior, see Glaserfeld (1990a).
- ³ See Pask's collection of essays (1969) reviewing the notion of goal-directed behavior.
- ⁴ Winter (1999) presents a comprehensive survey of second-order cybernetics.
- ⁵ This book provides a detailed analysis of the ontogeny of the concepts of object, space, time, causality, and external world and, therefore, the original foundation of constructivism.
- ⁶ See Maturana (1970b) for an early exposition of the theory of autopoiesis.
- ⁷ See Ceccato (1964) which is the only complete, early statement of his approach to conceptual construction.
- ⁸ In "*The language of change*", Watzlawick (1978) presents an introduction to the cybernetic perspective in psychotherapy.

PART III

CONCEPTUAL ANALYSES

NOTES ON THE CONCEPT OF CHANGE

In the United States, where I have been living for the last quarter of a century, Psychology is usually defined as the science of behavior—and behavior, as the followers of Watson and Skinner preached with enormous and devastating success, is what we can observe an organism do. Proceeding from this basis, it is easy to avoid dealing with any intelligent organism's more complex and more interesting capabilities.

Among the activities that can never be directly observed is thinking or reflecting. One can at times infer thoughts or reflections from a facial expression or a position—as Rodin hoped when he moulded his *Penseur*—and sometimes one can infer them from subsequent acts or speech. But one can neither see the actual process of thinking going on, nor how it works.

The title of this year's Seminar is 'Conceptions of Change'—and conceptions, whatever they may be, are products of a mind and therefore they, too, are not accessible to direct observation. Yet, as you know better than anyone, concepts are the business of psychologists.

Although I am not a psychologist, my main interest was and still is the construction of concepts, and then also the discrepancies entailed by the use of different languages. To explain this, I have to mention a few bits of biography. That I taught for almost twenty years in a department of psychology, was due to the imagination and the open-mindedness of my late friend and colleague Charley Smock. He had spent a year or two here at Geneva and it is to him that I owe my introduction to the work of Piaget. This introduction turned out to be of enormous importance, because it provided a perspective that allowed me to pull most of my preceding conceptual analyses together under one overarching model of cognition.

Piaget broke with the tradition of Western epistemology when he cast aside the assumption that knowledge had to be a more or less truthful representation of an independent external world. Instead he proposed that knowing is an adaptive enterprise to produce patterns of acting and operating that avoid perturbations and fit into the world of our experience the way animals fit into their environment. One of the conceptual structures that plays a major role in providing a fit with the experiential world, is the concept of change.

When I said I was interested in the construction of concepts, I did not mean to imply that one could examine how a language-user builds up his or her concepts. All one can do is try to unravel, from a logical point of view, what elements a given concept must incorporate.

If we do this with the concept of change, we can say straightaway that we could not conceive of change if we had no memory. In order to speak of change, we have

to consider at least two moments of experience. This was implicit already in Zeno’s paradox of the arrow, but, as far as I know, Silvio Ceccato was the first to use it methodically in the analysis of concepts.

I worked with Ceccato for twenty years informally most of that time and officially from 1959 to 62. Ceccato was among the original editorial committee of Piaget’s *Études d’épistémologie génétique* and he was still listed as such in the 36th volume that came out in 1977; but I never learned anything about Geneva from him. To discover Piaget, I had to move to the United States and meet Charley Smock at the University of Georgia.

In the history of ideas, it is something of a tragedy that Piaget and Ceccato could not work together. In my view, they complement each other to perfection. Ceccato’s model of the functioning of attention is just the sort of thing that is needed to underpin Piaget’s crucial notion of reflecting abstraction. But this is not what I want to discuss here. However, the analysis of the concept of change may give you an inkling of what I mean.

Ceccato devised a very simple way of representing conceptual structures that involve more than one experiential moment. This comprises most of the concepts that language designates by verbs, but also some that are associated with nouns—for instance ‘change’. And change is what I want to focus on.

Change, as I said, requires two moments:

$$t_1 \qquad \text{and} \qquad t_2$$

To speak of ‘change’, we also need the perception or conception of a difference. For example a difference of color, shape, size, or the like. But the difference alone is not enough—it has to be attributed to some *thing*.

If I showed you a small green plum and then another larger purple one, you would not be inclined to speak of change. But if the green plum were on a tree, and a few weeks later you looked at it again and found it purple, you might say that it has ripened or, indeed, that its color has changed.

In other words, the concept of ‘change’ requires a difference perceived in an object that is considered the same object at two moments in the flow of experience.

This ‘sameness’ is not simply the sameness that you might ascribe to the glasses on this table or to the chairs in this room. That would be the sameness of equivalence. Instead, what is involved in the construction of change is *individual identity*.

I can now complete the schematic representation of the concept:

t_1		t_2
X	\equiv	X
green	\neq	purple

Let me correct what I just said: Strictly speaking, this diagram is not a representation, because we cannot represent unobservable items. But as cyberneticians we can construct models to fit into black boxes. This, then, is a model of what one might call the structure of the concept. Silvio Ceccato developed this idea of modeling concepts that involve change as a sequence of experiential frames, sometime around 1950, and it played an important role in the conceptual analyses that I worked on in the two following decades.

When I finally came to read Piaget’s *La construction du réel chez l’enfant* in the early 1970s, I was struck by a remarkable compatibility. Ceccato’s model of change could be fitted into Piaget’s analysis like a key that unlocked several perspectives.

As you know, Piaget’s book contains chapters on the construction of the concepts of object, space, causality, and time. Because it is a book, these chapters have to be read sequentially; but as developments of the child’s mind, Piaget assumed that they happen simultaneously. The reader, therefore, has to make a major effort to grasp the coherence and the interdependence of the developmental processes that produce the four fundamental concepts out of the same basic material.

The key to *object permanence* is the constitution of an individual identity. It is the posited identity of an experiential object that, during one or more attentional frames, is not present in the child’s perceptual field. I can illustrate this by means of Ceccato’s graphic method. There is the following sequence of steps:

t_1	$t_2 \dots t_n$	t_{n+1}
item in perceptual field	attention focused elsewhere	item in perceptual field
X_1 (posited individual identity)	X_2

At time t_1 the child isolates an item in its perceptual field; at time t_2 the item is no longer perceived and the child’s attention is focused elsewhere; at time t_{n+1} , the child again isolates an item X_2 in its perceptual field and considers this second item the self-same individual item as X_1 at time t_1 .

Note that if the two items are not considered to be the self-same individual, but merely equivalent, the sequential structure becomes the model of classification. The difference between the two conceptual constructions lies in the kind of sameness used to link the two experiences: *individual identity* in one, *equivalence* in the other.

In both cases, the conceptual structures require a further element for their completion: The child must be able to visualize the object when it is *not* in the perceptual field. This ability yields what is properly called a re-presentation, which

is in fact a re-play of a past experience. This is the point that was overlooked by many critics of Piaget and by all the experimenters who tried to demonstrate *object permanence* in rats, cats, and rhesus monkeys. They did not and, I believe, could not, show that the animal had a re-presentation of the object in question.

To be fair, I want to stress that Piaget did not always make things easy for his readers. Since he repeated many times in his works that knowledge does not ‘copy’ or ‘replicate’ an external reality, it would have been a help to his readers if he had always spelled re-presentation with a hyphen. But let us return to ‘change’. One of the perspectives opened by the structural pattern of change is the following. On the one hand, the co-involvement of both the notions of sameness and difference creates the appearance of contradiction and can, therefore, generate a perturbation. On the other hand, if the first notion of causality arises, as Piaget said, from the reiteration of actions that lead to an interesting result, it seems plausible that a novel property in an object that is nevertheless considered the self-same individual would stir some interest. This interest, at some later stage, would lead backwards to the situation at t_1 —the remembered situation prior to the interesting result—in order to discern something that could be held causally responsible for the object’s novel property in the later frame. Such an exploration would be the beginning of research, and it would quickly lead to the strategy that underlies the kind of scientific experiment that aims at establishing the cause of a given phenomenon.

A second perspective is the following: The fact that there are attentional frames in which the object that is considered one-and-the-same is not present in the actual experiential field, raise the question where it might be and what it might be doing while one’s attention is focused on other things. As I have suggested elsewhere, this requires the conception of a kind of resting place where objects can maintain their identity without being perceived. I have called this resting place a proto-space because, as yet, it has no metric, no dimensions, and no spatial relations. Similarly, the fact that the object is considered the self-same individual in attentional frames that are not consecutive, requires that its identity be stretched across an interval. This means that a continuity has to be constructed that lies outside the experiential field, a continuity that links the object to an earlier experience, when it is re-perceived. This outside continuity that runs parallel to a sequence of actual experiences, I have called proto-time. It is an undifferentiated continuity and is then furnished with temporal relations when the sequence of the subject’s actual experiences that were lived during the interval, is mapped onto it and gives it a vicarious segmentation. It also gives it directionality or, as it is sometimes called, ‘time’s arrow’. Finally, when this projected sequence of experiences is a regular one, such as night and day, the seasons, or the movement of stars, the segmentation of the continuity becomes regular and thus proto-time turns into ‘real’ time that can be measured by counting these regular segments and, eventually, by a clock.

All of this, I believe, is implicit in Piaget’s account of the construction of reality. I have used a language that is different from his, but it leads to the very same extraordinary result. He spoke of a Copernican revolution that prompts the child to construct a world of *external* things, that is, things beyond the realm of immediate experience. It is the world of space and time in which the concept of individual

identity assures the continuity of all the things we know but are not attending to at the moment. In short, this construction is the world of *being*, the world that philosophers call ontological reality.

ABSTRACTION, RE-PRESENTATION, AND REFLECTION

An interpretation of experience and of Piaget's approach

The understanding, like the eye, whilst it makes us see and perceive all other things, takes no notice of itself; and it requires art and pains to set it at a distance and make it its own object. (John Locke, 1690, p. 1)

As adults we are constantly deceiving ourselves in regard to the nature and genesis of our mental experiences. (John Dewey, in McLellan & Dewey, 1895, p. 27)

One of the remarkable features of the behaviorist era in American psychology is that so many leaders and followers of that creed could claim to be empiricists, cite John Locke as their forefather, and get away with it. Had they read the first chapter of Book II¹ of his major work, *An essay concerning human understanding*, they would have found, among many others, the following enlightening statements. Paragraph 2 has the heading:

All Ideas come from Sensation or Reflection.

Paragraph 4 has the heading 'The Operations of our Minds', and it is there that Locke explains what he means by 'reflection':

This source of ideas every man has wholly in himself; and though it be not sense, as having nothing to do with external objects, yet it is very like it, and might properly enough be called *internal sense*. But as I call the other Sensation, so I call this *Reflection*, the ideas it affords being such only as the mind gets by reflecting on its own operations within itself.

In our century, it was Jean Piaget who vigorously defended and expanded the notion of reflection. He lost no opportunity to distance himself from empiricists who denied the mind and its operations and wanted to reduce all knowing to a passive reception of objective 'sense data'. Yet, he should not have found it difficult to agree with Locke's division of ideas because it is not too different from his own division between figurative and operative knowledge. Similarly, I feel, Locke would have had a certain respect for Piaget's effort to set understanding at a distance and to make it the object of investigation. And both men, I have no doubt, would have agreed with Dewey about the risk of deceiving oneself by taking mental experiences as given. It is therefore with caution that I shall proceed to

discuss, in the pages that follow, first my own view of reflection, abstraction, representation, and the use of symbols, and then a tentative interpretation of Piaget's position. If, at times, I may sound assertive, I would beg the reader to keep in mind that I am fully aware of the fact that I am merely offering conjectures—but they are conjectures which I have found useful in constructing a model of mental operations.

REFLECTION

If someone, having just eaten an apple, takes a bite out of a second one, and is asked which of the two tasted sweeter, we should not be surprised that the person could give an answer. Indeed, we would take it for granted that under these circumstances any normal person could make a relevant judgment.

We cannot observe how such a judgment is made. But we can hypothesize some of the steps that seem necessary to make it. The sensations that accompanied the eating of the first apple would have to be remembered, at least until the question is heard.² Then they would have to be re-presented and compared (in regard to whatever the person called 'sweetness') with the sensations accompanying the later bite from the second apple. This re-presenting and comparing is a way of operating that is different from the processes of sensation that supplied the material for the comparison. Reflecting upon experiences is clearly not the same as having an experience.

A hundred years after Locke, Wilhelm von Humboldt wrote down a few aphorisms which, posthumously, his editors put under the heading 'About Thinking and Speaking'. The first three aphorisms deal with reflection:

1. The essence of thinking consists in reflecting, i.e., in distinguishing what thinks from what is being thought.
2. In order to reflect, the mind must stand still for a moment in its progressive activity, must grasp as a unit what was just presented, and thus posit it as object against itself.
3. The mind then compares the units, of which several can be created in that way, and separates and connects them according to its needs. (1907, p. 581)³

I know of no better description of the mysterious capability that allows us to step out of the stream of direct experience, to re-present a chunk of it, and to look at it as though it were direct experience, while remaining aware of the fact that it is not. I call it mysterious, because, although we can all do it as easily as flipping a switch, we have not even the beginnings of a model (least of all an 'information processing' model) that would suggest how it might be achieved.

'To grasp as a unit what was just presented' is to cut it out of the continuous experiential flow. In the literal sense of the term, this is a kind of abstraction—namely the simplest kind. Focused attention picks a chunk of experience, isolates it from what came before and from what follows, and treats it as a closed entity. For the mind, then, 'to posit it as object against itself', is to *re-present* it.

In the next two sections, I want to deal with abstraction and re-presentation one after the other.

ABSTRACTION

As Humboldt stated in his third aphorism, chunks of experience, once isolated, can be compared, separated, and connected. This makes possible further steps of abstraction, among them the kind that Piaget and many others have called ‘generalizing abstraction’. Because it seems crucial in all forms of naming and categorization, it has been discussed for a long time. To clarify the core of the notion, I once more return to Locke, because he produced a very simple and widely accepted description of the process:

This is called Abstraction, whereby ideas taken from particular beings become general representations of all the same kind; and their names general names, applicable to whatever exists conformable to such abstract ideas. (1690, Book II, Chapter X, §9)

Locke’s use of the words ‘being’ and ‘exist’ in this context caused Berkeley, who had a very different view of ‘existence’, to voice a sarcastic objection against his predecessor.

Whether others have this wonderful faculty of abstracting their ideas, they best can tell; for myself, I find indeed I have a faculty of imagining, or representing to myself, the ideas of those particular things I have perceived, and of variously compounding them. I can imagine a man with two heads, or the upper parts of a man joined to the body of a horse, I can consider the hand, the eye, the nose, each by itself abstracted or separated from the rest of the body. But then whatever hand or eye I imagine, it must have some particular shape and colour. (1710, Introduction, §10)

This passage is interesting for two reasons. Berkeley claims, much as did Humboldt, that we are able to represent to ourselves particular experiential items and that we are also able to segment them and to recombine the parts at will. Then however, he goes on to claim that whatever we re-present to ourselves must have the character of a particular—from which he concludes that we cannot have general ideas.

Both these claims concern re-presentation and are, I believe, perfectly valid. But what follows from them is that we are unable to re-present general ideas, not that we cannot have them. Berkeley, it seems, unwittingly trapped himself into this position about abstraction. At the beginning of his Treatise, he says among other things:

Thus, for example, a certain colour, taste, smell, figure and consistence having been observed to go together, are accounted one distinct thing, signified by the name apple; other collections of ideas constitute a stone, a tree, a book, and the like sensible things. (1710, §1)

Berkeley, of course, was aware of the fact that he would apply the name 'apple' not only to one unique 'thing', but to countless others that fitted his description in terms of 'colour, taste, smell, figure, and consistence', but to him this arose from the association of the word and it seems that he took the ensuing generalization simply for granted. Had he analyzed it the way he analyzed other conceptual operations, he might have changed his view about abstraction. I hope to make this clear with the help of an example.

A child growing up in a region where apples are red would necessarily and quite correctly associate the idea of redness with the name 'apple'. A distant relative arriving from another part of the country, bringing a basket of yellow apples, would cause a major perturbation for the child, who might want to insist that yellow things should not be called 'apples'. However, the social pressure of the family's usage of the word will soon force the child to accept the fact that the things people call 'apple' come in different colors. The child might then be told that apples can also be green, which would enable the child to recognize such a particular green thing as an apple the first time it is brought to the house.

Berkeley, I would say, was quite right when he maintained that every time we imagine an apple, it has to have a specific color, but he was wrong to claim that we could, therefore, not have a general idea in our heads that allows us to recognize as apples items that differ in some respects, but nevertheless belong to that class.

Hence I suggest that, *pace* Berkeley, we are quite able to abstract general ideas from experience and that we do this by substituting a kind of place-holder or variable for some of the properties in the sensory complex we have abstracted from our experiences of particular things. I see no reason why one should not call the resulting cognitive structure a concept. Such a structure is more specific with regard to some properties and less specific with regard to others, and it is precisely because of this relative indeterminacy that it enables us to recognize items that we have never seen before as exemplars of a familiar kind.

In short, in order to recognize several particular experiential items, in spite of differences they may manifest, as belonging to the same kind, we must have a concept that is flexible enough to allow for a certain variability. That is, instead of specific particulars it must contain variables. Yet it is clear that, in order to *imagine* for instance an apple, we have to decide what color it is to be, because we cannot possibly visualize it red or green or yellow at one and the same time. Berkeley, therefore, was right when he observed that whenever we re-present a concept to ourselves, we find that it is a particular thing and not a general idea. What he did not realize was that the abstraction necessary to recognize things of a kind, does not automatically turn into an image that can be re-presented.

The situation, however, is somewhat complicated by our ability to use symbols, but before considering this I want to deal with re-presentation.

RE-PRESENTATION

No act of mental re-presentation, which in this context of conceptual analysis means neither less nor more than the re-generation of a prior experience, would be

possible if the original generation of the experience had not left some mark to guide its reconstruction. In this requirement, representation is similar to recognition. Both often work hand in hand, that is, when one recognizes a Volkswagen though one can see only part of its back but is nevertheless able to visualize the whole. The ability to recognize a thing in one's perceptual field, however, does not necessarily bring with it the ability to re-present it spontaneously. We have all had occasion to notice this. Our experiential world contains many things which, although we recognize them when we see them, are not available to us when we want to visualize them. There are, for instance, people whom we would recognize as acquaintances when we meet them, but were we asked to describe them when they are not in our visual field, we would be unable to recall an adequate image of their appearance.

The fact that recognition developmentally precedes the ability to re-present an experiential item spontaneously, has been observed in many areas. It is probably best known and documented as the difference between what linguists call 'passive' and 'active' vocabulary. The difference is conspicuous in second-language learners but it is noticeable also in anyone's first language: a good many words one knows when one hears or reads them are not available when one is speaking or writing.

This lag suggests that having abstracted a concept that may serve to recognize and categorize a perceptual item is not sufficient to re-present the item to oneself in its absence. Piaget has always maintained that all forms of imaging and re-presenting are, in fact, acts of internalized imitation (Piaget, 1945).⁴ In this context, the metaphor of 'program' may be useful. A program is the fixed itinerary of an activity that can guide and govern the sequence of its re-enactment. But there are two points to be stressed. First, a program may specify the material on which to act, but it does not supply the material; second, a program may specify what acts are to be performed, but it supplies neither the acting agent nor the action.

The first of these limitations, I suggest, may account for the fact that to recognize an experiential item requires less effort than to re-present it spontaneously. This would be so, because in re-presentation not only a program of composition is needed, but also the specific sensory components, which must be expressly generated. In recognition, the perceiver merely has to isolate the sensory elements in the sensory manifold. As Berkeley observed, sensory elements are "not creatures of the will" (1710, §29). Because there are always vastly more sensory elements than the perceiving agent can attend to and use,⁵ recognition requires the attentional selecting, grouping, and coordinating of sensory material that fits the composition program of the item to be recognized. In re-presentation, on the other hand, some substitute for the sensory raw material must be generated. (As the example of the Volkswagen indicates, the re-generation of sensory material is much easier when parts of it are supplied by perception, a fact that was well known to the proponents of Gestalt psychology.)

A difference analogous to that between acting on actually present perceptual material, as opposed to acting on material that must itself be generated, arises from the second limiting feature I mentioned. With regard to the need for an acting agent, a program is similar to a map. If someone draws a simple map to show you

how to get to his house, he essentially indicates a potential path from a place you are presumed to know to the unknown location. The drawing of the path is a graphic representation of the turns that have to be made to accomplish that itinerary, but it does not and could not show what it is to move and what it is to turn right or left. Any user of the map, must supply the motion and the changes of direction with the focus of visual attention while reading the map. Only if one manages to abstract this sequence of motions from the reading activity, can one transform it into physical movement through the mapped region. (Note that this abstracting and transforming is by no means an easy task for those unaccustomed to map reading.)

A program, however, differs from a map in that it explicitly provides instructions about actions and implicitly indicates changes of location through the conventional sequence in which the instructions must be read. (But in the program, too, it is the user's focus of attention, while reading or implementing the program, that supplies the progressive motion.) Also, unlike a map, a program may contain embedded 'subroutines' for walking and turning (i.e., instructions how to act), but no matter how detailed these subroutines might be, they can contain only instructions to act, not the actions themselves. In other words, irrespective of how minutely a program's instructions have decomposed an activity, they remain static until some agent implements them and adds the dynamics.

In carrying out a program in an experiential situation, just as in following a map through an actual landscape, the sensory material in the agent's perceptual field can supply cues as to the action required at a given point of the procedure. In the representational mode, however, attention cannot focus on actual perceptual material and pick from it cues about what to do next, because the sensory material itself has to be re-presented. A re-presentation—at least when it is a spontaneous one—is wholly self-generated (which is one reason why it is usually easier to find one's way through a landscape than to draw a reliable map of it).

The increase of difficulty and the concomitant increase of effort involved in the production of conceptual structures when the required sensory material is not available in the present perceptual field, shows itself in all forms of re-presentation and especially in the re-enactment of abstracted programs of action. Any re-presentation, be it of an experiential 'thing' or of a program of actions or operations, requires some sensory material for its execution. That basic condition, I believe, is what confirmed Berkeley in his argument against the 'existence' of abstracted general ideas, for it is indeed the case that every time we re-present to ourselves such a general idea, it turns into a particular one because its implementation requires the kind of material from which it was abstracted.

This last condition could be reformulated by saying that there has to be some isomorphism between the present construct and what it is intended to reconstruct. Clearly, this isomorphism does not concern a 'thing-in-itself' but precisely those aspects one wants to or happens to focus on. As Silvio Ceccato remarked thirty years ago, what we visualize of objects in dreams is no more than is required by the context of the dream.⁶

More importantly, this selective isomorphism is the basis of graphic and schematic representations. They tend to supply such perceptual material as is required to bring forth in the perceiver the particular ways of operating that the maker of the graphic or schematic is aiming at. In this sense they are didactic, because they can help to focus the naive perceiver's attention on the particular operations that are deemed desirable. Hence, as I have suggested elsewhere (cf. Glasersfeld, 1987), they can be divided into iconic and symbolic representations, but neither kind should be confused with the mental re-presentations I am discussing here.

THE POWER OF SYMBOLS

Re-presentations can be activated by many things. Any element in the present stream of experience may bring forth the re-presentation of a past situation, state, activity, or other construct. This experiential fact was called *association* by Hume and used by Freud for his analyses of neuroses. The ability to associate is systematically exploited by language. To possess a word is to have associated it with a representation of which one believes that it is similar to the re-presentations the word brings forth in other users of the language. (Only naive linguists claim that these re-presentations are shared, in the sense that they are the same for all users of the given word.) In my terminology, a word is used as a symbol,⁷ only when it brings forth in the user an abstracted generalized re-presentation, not merely a response to a particular situation (cf. Glasersfeld, 1974a).

Several things, therefore, are indispensable for a word to function as a symbol: (1) the phonemes that compose the word in speech, or the graphic marks that constitute it in writing, must be recognized as that particular item of one's vocabulary. This ability to recognize, as I suggested earlier, is preliminary to the ability to re-present and produce the word spontaneously. (2) The word/symbol must be associated with a conceptual structure that was abstracted from experience and, at least to some extent, generalized. Here, again, the ability to recognize (i.e., to build up the conceptual structure from available perceptual material) precedes the ability to re-present the structure to oneself spontaneously.

Once a word has become operative as a symbol and calls forth the associated meaning as re-presentations of chunks of experience that have been isolated (abstracted) and to some extent generalized, its power can be further expanded. By this I mean that, as particular users of the word become more proficient, they no longer need to actually produce the associated conceptual structures as a completely implemented re-presentation, but can simply register the occurrence of the word as a kind of 'pointer' to be followed if needed at a later moment. I see this as analogous to the capability of recognizing objects on the basis of a partial perceptual construction. In the context of symbolic activities, this capability is both subtle and important. An example may help to clarify what I am trying to say.

If, in someone's account of a European journey, you read or hear the name 'Paris', you may register it as a pointer to a variety of experiential 'referents' with which you happen to have associated it—for example, a particular point on the

map of Europe, your first glimpse of the Eiffel Tower, the Mona Lisa in the Louvre—but if the account of the journey immediately moves to London, you would be unlikely to implement fully any one of them as an actual re-presentation. At any subsequent moment, however, if the context or the conversation required it, you could return to the mention of ‘Paris’ and develop one of the associated re-presentations.

I have chosen to call this function of symbols ‘pointing’ because it seemed best to suggest that words/symbols acquire the power to open or activate pathways to specific re-presentations without, however, obliging the proficient symbol user to produce the re-presentations there and then.

This function, incidentally, constitutes one of the central elements of our theory of children’s acquisition of the concept of number (Steffe et al., 1983). In this theory, the first manifestation of an abstract number concept is a demonstration that the subject knows, without carrying out a count, that a number word implies or points to the sequential one-to-one coordination of all the terms of the standard number word sequence, from ‘one’ up to the given word, to some countable items. Indeed, we believe that this is the reason why, as adults, we may assert that we know what, say, the numeral (symbol) ‘381,517’ means, in spite of the fact that we are unlikely to be able to re-present to ourselves an associated particular chunk of experience; we know what it means, because it points to a familiar counting procedure or other mathematical method of arriving at the point of the number line which it indicates.

In mathematics this form of symbolic implication is so common that it usually goes unnoticed. For instance, when one is told that the side of a pentagon is equal to half the radius of the circumscribed circle multiplied by $\sqrt{(10-\sqrt{25})}$, one does not have to draw the square roots to understand the statement—provided one knows the operations the symbols ‘point to’. The potential ability is sufficient, one does not have to carry out the indicated operations. Because it is so often taken for granted that mathematical expressions can be understood without carrying out the operations they symbolize, formalist mathematicians are sometimes carried away and declare that the manipulation of symbols constitutes mathematics.

PIAGET’S THEORY OF ABSTRACTION

Few, if any, thinkers in this century have used the notion of abstraction as often and insistently as did Piaget. Indeed, in his view, “All new knowledge presupposes an abstraction” (Piaget, 1974a, p. 89). But not all abstraction is the same. Piaget distinguished two main kinds, ‘empirical’ and ‘reflective’, and then subdivided the second. He has frequently explained the primary difference in seemingly simple terms, for example:

Empirical abstractions concern observables and reflective abstractions concern coordinations. (Piaget, 1977b, vol. II, p. 319)

One can thus distinguish two kinds of abstraction according to their exogenous or endogenous sources... (Piaget, 1974a, p. 81)

Anyone who has entered into the spirit of genetic epistemology will realize that the simplicity of these statements is deceptive. The expressions 'observables' and 'exogenous' are liable to be interpreted in a realist sense, as aspects or elements of an external reality. Given Piaget's theory of knowledge, however, this is not how they were intended. In fact, the quoted passages are followed by quite appropriate warnings. After the first, Piaget explains that no characteristic is in itself observable. Even in physics, he says, the measured magnitudes (mass, force, acceleration, etc.) are themselves constructed and are therefore results of inferences deriving from preceding abstractions (Piaget, 1977b, *ibid.*, p. 319). In the case of the second quotation, he adds a little later: "There can be no exogenous knowledge except that which is grasped as content, by way of forms which are endogenous in origin" (Piaget, 1974a, p. 91). This is not an immediately transparent formulation. As so often in Piaget's writings, one has to look elsewhere in his work for enlightenment.

FORM AND CONTENT

The distinction between form and content has a history as long as Western philosophy and the terms have been used in many different ways. Piaget's use of the distinction is complicated by the fact that he links it with his use of 'observables' (content) and ' coordinations' (forms). "The functions of form and content are relative, since every form becomes content for another that comprises it" (Piaget, 1977b, *ibid.*, p. 319). This will make sense, only if one recalls that, for Piaget, percepts, observables, and any knowledge of objects, are all the result of a subject's action and not externally caused effects registered by a passive receiver. In his theory, to perceive, to remember, to re-present, and to coordinate are all dynamic, in the sense that they are activities carried out by a subject that operates on internally available material and produces certain results.

A term such as 'exogenous', therefore, must not be interpreted as referring to a physical outside relative to a physical organism, but rather as referring to something that is external relative to the process in which it becomes involved.

Observation and re-presentation have two things in common: (1) they operate on items which, relative to the process at hand, are considered given. The present process takes them as elements and coordinates them as 'content' into a new 'form' or 'structure'; and (2) the resulting new products can be taken as initial 'givens' by a future process of structuring, relative to which they then become 'content'. Thus, once a process is achieved, its results may be considered 'observables' or 'exogenous' relative to a subsequent process of coordination or a higher level of analysis.

As Piaget saw, this might seem to lead to an infinite regress (*ibid.*, p. 306), but he put forth at least two arguments to counter this notion. One of them emerges from his conception of scientific analysis. Very early in his career, he saw this analysis as a cyclical program in which certain elements abstracted by one branch of science become the 'givens' for coordination and abstraction in another. In an early paper (Piaget, 1929) and almost forty years later in his 'classification of the

sciences' (Piaget, 1967c), he formulated this mutual interdependence of the scientific disciplines as a circle: Biology–Psychology–Mathematics–Physics, and looping back to Biology. Hence, from his perspective, there is no linear progression without end, but simply development of method and concepts in one discipline leading to novel conceptualization and coordination in another. The recent impact of the physics of molecules and particles on the conceptual framework of biology would seem a good example.

SCHEME THEORY

The second reason against an infinite regress of abstractions is grounded in the developmental basis of Genetic Epistemology and is directly relevant here. The child's cognitive career has an unquestionable beginning, a first stage during which the infant assimilates, or tries to assimilate, all experience to such fixed action patterns (reflexes) as it has at the start (Piaget, 1975, p. 180). Except for their initial fixedness, these action patterns function like the schemes which the child a little later begins to coordinate on the basis of experience.

Schemes are composed of three elements: (1) an initial experiential item or configuration (functionally linked to what the observer would categorize as 'trigger' or 'stimulus'); (2) an activity the subject has associated with it; and (3) a subsequent experience associated with the activity as its outcome or result. Schemes thus govern the subject's segmentation and differentiation of experience.

When a novel item ('novel' in the observer's judgement) is assimilated to the initiating element of a scheme, it triggers the associated activity. If the activity leads to the expected result, the acting subject in no way differentiates the item from those that functioned like it in the past. But if, for one reason or another, the activity does not lead to the expected result, this generates a perturbation, which could be described as either disappointment or pleasant surprise. In either case, the perturbation may focus the subject's attention on the configuration that triggered the activity this time, and it may then be discriminated from those past experiences where the activity functioned in the expected manner (cf. Piaget, 1974b, p. 264). If the failure of the scheme and the ensuing discrimination of the novel item or situation leads to the tightening of the criteria of assimilation that determine what can and what cannot be taken as a trigger for the particular scheme, this would constitute an accommodation of the initiating conceptual structure.⁸ Similarly, if the outcome was a pleasant surprise, this, too, may lead to an accommodation, in the sense that a new scheme will subsequently be triggered by the newly isolated experience.

In infancy, during the child's first two years, that is, in the sensory-motor period, all this is assumed to take place without awareness and conscious reflection. Yet, the fact that three or four-month-old infants assimilate items (which, to an observer, are not all the same) as triggers of a particular scheme, is sometimes described as the ability to generalize. Indeed, this is what animal psychologists, working with rats or monkeys, call 'stimulus generalization'. With children at a later stage, however, when reflection has begun to operate, the

discriminating of experiential items that do function in a given scheme from others that do not, constitutes a mechanism that functions as the source of empirical abstractions that are recognized as such by the acting subject. This, obviously, raises the question when and how the acting subject's awareness is involved.

One reason why that question seems quite urgent, is that the word 'reflection', ever since Locke introduced it into the human sciences, has implied a conscious mind that does the reflecting. A second reason is that in many places where Piaget draws the distinction between the 'figurative' and the 'operative', this tends to reinforce the notion that the operative (what Locke described as 'the ideas the mind gets by reflecting on its own operations') requires consciousness. As a consequence, it would be desirable to unravel when, in Piaget's theory of the cognitive development, the capability of conscious reflection arises.

Piaget himself, as I have said elsewhere (Glaserfeld, 1982), rarely makes explicit whether, in a given passage, he is interpreting what he is gathering from his observations (observer's point of view), or whether he is conjecturing an autonomous view from the observed subject's perspective (cf. Vuyk, 1981, vol. II). This difference seems crucial in building a model of mental operations and, therefore, to an understanding of his theory of abstraction and, especially, reflective abstraction. I shall return to this question of consciousness after the next section, where I try to lay out the kinds of abstraction Piaget has distinguished.

FOUR KINDS OF ABSTRACTION

The process Locke characterized by saying, 'whereby ideas taken from particular beings become general representations of all the same kind', falls under Piaget's term empirical abstraction. To isolate certain sensory properties of an experience and to maintain them as repeatable combinations, that is, isolating what is needed to recognize further instantiations of, say, apples, undoubtedly constitutes an empirical abstraction. But, as I suggested earlier, to have composed a concept that can serve to recognize (assimilate) items as suitable triggers of a particular scheme, does not automatically bring with it the ability to visualize such items spontaneously as re-presentations. Piaget makes an analogous point—incidentally, one of the few places where he mentions an empiricist connection:

But it is one thing to extract a character, *x*, from a set of objects and to classify them together on this basis alone, a process which we shall refer to as 'simple' abstraction and generalization (and which is invoked by classical empiricism), and quite another to recognise *x* in an object and to make use of it as an element of a different (non-perceptual) structure, a procedure which we shall refer to as 'constructive' abstraction and generalisation. (Piaget, 1969, p. 317)

The capability of spontaneous re-presentation (which is 'non-perceptual', too) develops in parallel with the acquisition of language and may lead to an initial, albeit limited form of awareness. Children at the age of three or four years, are not incapable of producing some pertinent answer when they are asked what a familiar

object is like or not like, even when the object is not in sight at the moment. This suggests that they are able not only to call forth an empirically abstracted representation but also to review it quite deliberately.

The notion of empirical abstraction covers a wider range of experience for Piaget than is envisioned in the passage I quoted from Locke. What Locke called ‘particular beings’ were for him ‘ideas’ supplied by the five senses. Because, in Piaget’s view, visual and tactual perception involve motion, it is not surprising that the internal sensations caused by the agent’s own motion (kinesthesia) belong to the ‘figurative’ and are therefore, for him, raw material for empirical abstractions in the form of motor patterns.⁹

That such abstracted motor patterns reach the level where they can be re-presented, you can check for yourself. Anyone who has some proficiency in activities such as running down stairs, serving in tennis, swinging for a drive in golf, or skiing down a slope, has no difficulty in re-presenting the involved movements without stirring a muscle. An interesting aspect in such ‘dry reruns’ of abstracted motor experiences is that they don’t require specific staircases, balls, or slopes. I mention this because it seems to me to be a clear demonstration of deliberate and therefore conscious re-presentation of something that needed no consciousness for its abstraction from actual experience. This difference is important also in Piaget’s subdivision of reflective abstractions to which we turn now.

From empirical abstractions, whose raw material is sensory-motor experience, Piaget, as I said earlier, distinguished three types of reflective abstraction. Unfortunately, the French labels Piaget chose for them are such that they are inevitably confused by literal translation into English.

The first ‘reflective’ type derives from a process Piaget calls ‘*réfléchissement*’, a word that is used in optics when something is being reflected, as for instance the sun’s rays on the face of the moon. In his theory of cognition, this term is used to indicate that an activity or mental operation (not a static combination of sensory elements) developed on one level is abstracted from that level of operating and applied to a higher one, where Piaget then considers it to be a ‘*réfléchissement*’. (Moessinger and Poulin-Dubois, 1981, have translated this as ‘projection’, which captures something of the original sense.) But Piaget stresses that a second characteristic is required:

Reflective abstraction always involves two inseparable features: a ‘*réfléchissement*’ in the sense of the projection of something borrowed from a preceding level onto a higher one, and a ‘*réflexion*’ in the sense of a (more or less conscious) cognitive reconstruction or reorganization of what has been transferred. (Piaget, 1975, p. 41)

At the beginning of the first of his two volumes on reflective abstraction (Piaget, 1977b), the two features are again mentioned:

Reflective abstraction, with its two components of ‘*réfléchissement*’ and ‘*réflexion*’, can be observed at all stages: from the sensory-motor levels on, the infant is able, in order to solve a new problem, to borrow certain

coordinations from already constructed structures and to reorganize them in function of new givens. We do not know, in these cases whether the subject becomes aware of any part of this. (Piaget, 1977b, vol. I, p. 6)

In the same passage he immediately goes on to describe the second type of reflective abstraction:

In contrast, at the later stages, when reflection is the work of thought, one must also distinguish thought as a process of construction and thought as a process of retroactive thematization. The latter becomes a reflecting on reflection; and in this case we shall speak of ‘abstraction réfléchie’ (reflected abstraction) or ‘pensée réflexive’ (reflective thought). (Ibid.)

Since the present participle of the verb ‘*réfléchir*’, from which both the nouns *réfléchissement* and *reflexion* are formed, is ‘*réfléchissante*’, Piaget used ‘abstraction réfléchissante’ as a generic term for both types. It is therefore not surprising that in most English translations the distinction was lost when the expression ‘reflective abstraction’ was introduced as the standard term.

The situation is further confounded by the fact that Piaget distinguished a third type of reflective abstraction which he called ‘pseudo-empirical’. When children are able to re-present certain things to themselves but are not yet fully on the level of concrete operations, it happens that the subjects, by leaning constantly on their perceivable results, can carry out certain constructions which, later on, become purely deductive (e.g., using an abacus or the like for the first numerical operations). In this case we shall speak of ‘pseudo-empirical abstraction’ because, in spite of the fact that these results are read off material objects as though they were empirical abstractions, the perceived properties are actually introduced into these objects by the subject’s activities. (Piaget, 1977b, vol. I, p. 6).

To recapitulate, Piaget distinguishes four kinds of abstraction. One is called ‘empirical’ because it abstracts sensory-motor properties from experiential situations. The first of the three ‘reflective’ ones, projects and reorganizes on another level a coordination or pattern of the subject’s own activities or operations. The next is similar in that it also involves patterns of activities or operations, but it includes the subject’s awareness of what has been abstracted and is therefore called ‘reflected abstraction’. The last is called ‘pseudo-empirical’ because, like empirical abstractions, it can take place only if suitable sensory-motor material is available.

THE QUESTION OF AWARENESS

One of the two main results of the research carried out by Piaget and his collaborators on the attainment of awareness, he summarized as follows in *La prise de conscience*¹⁰:

... action by itself constitutes an autonomous knowledge of considerable power, for while it is only ‘know-how’ and not knowledge that is conscious of itself in the sense of conceptualized understanding, it nevertheless constitutes the source of the latter, because the attainment of consciousness

nearly always lags quite noticeably behind this initial knowledge which is remarkably efficacious even though it does not know itself. (Piaget, 1974b, p. 275)

The fact that conscious conceptualized knowledge of a given situation developmentally lags behind the knowledge of how to act in the situation, is commonplace on the sensory-motor level. In my view, as I mentioned earlier, this is analogous to the temporal lag of the ability to re-present a given item relative to the ability to recognize it. But the ability spontaneously to re-present to oneself a sensory-motor image of, say, an apple, still falls short of what Piaget in the above passage called ‘conceptualized understanding’. This would involve awareness of the characteristics inherent in the concept of apple or whatever one is re-presenting to oneself, and this kind of awareness constitutes a higher level of mental functioning.

This further step requires a good deal more of what Locke called the mind’s ‘art and pains to set (something) at a distance and make it its own object’. A familiar motor pattern is once more a good example: we may be well able to re-present to ourselves a tennis stroke or a golf swing, but few, if any, would claim to have a ‘conceptualized understanding’ of the sequence of elementary motor acts that are involved in such an abstraction of a delicately coordinated activity. Yet it is clear that, insofar as such understanding is possible, it can be built up only as a ‘retroactive thematization’, that is, after the whole pattern has been empirically abstracted from the experience of enacting it.

In Piaget’s theory, the situation is similar in the first type of reflective abstraction: he maintains that it, too, may or may not involve the subject’s awareness.

Throughout history, thinkers have used thought structures without having grasped them consciously. A classic example: Aristotle used the logic of relations, yet ignored it entirely in the construction of his own logic. (Piaget & Garcia, 1983, p. 37)

In other words, one can be quite aware of what one is cognitively operating on, without being aware of the operations one is carrying out.

As for the second type, ‘reflective thought’ or ‘reflected abstraction’, it is the only one about which Piaget makes an explicit statement concerning awareness:

Finally, we call the result of a reflective abstraction ‘reflected’ abstraction, once it has become conscious, and we do this *independently of its level*. (Piaget, 1977b, vol. II, p. 303; my emphasis)

When one comes to this statement in Piaget’s summary at the end of the second volume on the specific topic of reflective abstraction, it becomes clear that the sequence in which he usually discusses the three types is a little misleading, because it is neither a developmental nor a logical sequence. What he rightly calls ‘reflective thought’ and lists as the second of three types, describes a cognitive phenomenon that is much more sophisticated than reflective abstractions of type

one or type three and, moreover, is relevant also as a further development of empirical abstraction.

I would suggest that the two meanings of the word 'reflection' be assigned in the following way to Piaget's classification of abstractions: it should be interpreted as projection and adjusted organization on another operational level in the case of reflective abstraction type one and pseudo-empirical abstraction; and it should be taken as conscious thought in the case of reflective abstraction type two (also called 'reflected').

In his two volumes *La prise de conscience* (1974b) and *Réussir et comprendre* (1974c), there is a wealth of observational material from which Piaget and his collaborators infer that consciousness appears hesitantly in small steps each of which conceptualizes a more or less specific way of operating. Like Humboldt, Piaget takes the mind's ability to step out of the experiential flow for granted, but he then endeavors to map when and under what conditions the subject's awareness of its own operating sets in; and he tries to establish how action evolves in its relation to the conceptualization which characterizes the attainment of consciousness (Piaget, 1974b, p. 275ff). In the subsequent volume, he provides an excellent definition of what it is that awareness contributes:

To succeed is to comprehend in action a given situation to a degree sufficient to attain the proposed goals: to understand is to master in thought the same situations to the point that one can resolve the problems they pose with regard to the why and the how of the links one has established and used in one's actions. (Piaget, 1974c, p. 237)

The cumulative result of the minute investigations contained in these two volumes enabled Piaget to come up with an extremely sophisticated description of the mutual interaction between the construction of successful schemes and the construction of abstracted understandings, an interaction that eventually leads to accommodations and to finding solutions to problems in the re-presentational mode, that is, without having to have run into them on the level of sensory-motor experience.

In this context, one further thing must be added. In the earlier sections, I discussed the fact that re-presentation follows upon recognition and that the 'pointing' function of symbols follows as the result of familiarity with the symbols' power to bring forth re-presentations that are based on empirical abstractions. As the examples I gave of abstracted motor patterns should make clear, symbols can be used, simply to point to such patterns, in which case the re-presentation of action can be curtailed, provided the subject has consciously conceptualized the action and knows how to re-present it.

I now want to emphasize that this pointing function of symbols makes possible a way of mental operating that requires conscious conceptualization and, as a result, gives more power to the symbols. Once reflective thought can be applied to the kind of abstraction Piaget ascribed to Aristotle (cf. passage quoted above), there will be awareness not only of what is being operated on but also of the operations that are being carried out. Piaget suggested this in an earlier context:

A form is indissociable from its content in perception but can be manipulated independently of its content in the realm of operations, in which even forms devoid of content can be constructed and manipulated. ... logico-mathematical operations allow the construction of arrangements which are independent of content ... pure forms ... simply *based on symbols*. (Piaget, 1969, p. 288; my emphasis)

In my terms this means, symbols can be associated with operations and, once the operations have become quite familiar, the symbols can be used to point to them without the need to produce an actual re-presentation of carrying them out. If this is accepted as a working hypothesis, we have a model for a mathematical activity that was very well characterized by Juan Caramuel,¹¹ twenty-five years before Locke published his *Essay*:

When I hear or read a phrase such as “*The Saracen army was eight times larger than the Venetian one, yet a quarter of its men fell on the battlefield, a quarter were taken prisoner, and half took to flight*”, I may admire the noble effort of the Venetians and I can also understand the proportions, without determining a single number. If someone asked me how many Turks there were, how many were killed, how many captured, how many fled, I could not answer unless one of the indeterminate numbers had been determined. . . .

Thus the need arose to add to common arithmetic, which deals with the determinate numbers, another to deal with the indeterminate numbers. (Caramuel, 1670, p. 37; original emphasis)

In Europe, Caramuel says, this ‘other’ arithmetic, which deals with abstractions that are “more abstract than the abstract concept of number”, became known as algebra. Given the model of abstraction and reflection I have discussed in these pages, it is not difficult to see what this further abstraction resides in. To produce an actual re-presentation of the operative pattern abstracted from the arithmetical operation of, say, division, specific numbers are needed. This is analogous to the need of specific properties when the re-presentation of, say, an apple is to be produced. But there is a difference: The properties required to form an apple re-presentation are sensory properties, whereas the numbers needed to re-present an ‘operative pattern’ in arithmetic are themselves abstractions from mental operations and therefore re-presentable only with the help of some sensory material. Yet, once symbols have been associated with the abstracted operative pattern, these symbols, thanks to their power of functioning as pointers, can be understood, without the actual production of the associated re-presentation—provided the user knows how to produce it when the numerical material is available.

CONCLUSION

Abstraction, re-presentation, reflection, and conscious conceptualization interact on various levels of mental operating. In the course of these processes, what was

produced by one cycle of operations, can be taken as ‘given content’ by the next one, which may then coordinate it to create a new ‘form’, a new structure; and any such structure can be consciously conceptualized and associated with a symbol. The structure that then functions as the symbol’s meaning for the particular cognizing subject, may have gone through several cycles of abstraction and reorganization. This is one reason why the conventional view of language is misleading. In my experience, the notion that word/symbols have fixed meanings that are shared by every user of the language, breaks down in any conversation that attempts an interaction on the level of concepts, that is, attempts to go beyond a simple exchange of soothing familiar sounds.

In analyses like those I have tried to lay out in this chapter, one chooses the words that one considers the most adequate to establish the similarities, differences, and relationships one has in mind. But the meanings of whatever words one chooses are one’s own, and there is no way of presenting them to a reader for inspection. This, of course, is the very same situation I find myself in, vis-à-vis the writings of Piaget. There is no way of discovering what he had in mind—not even by reading him in French. All I—or anyone—could do, is to ‘interpret’, to construct and reconstruct until a satisfactory degree of coherence is achieved among the conceptual structures one has built up on the basis of the read text.

This situation, I keep reiterating, is no different from the situation we are in, vis-à-vis our non-linguistic experience, that is, the experience of what we like to call ‘the world’. What matters there, is that the conceptual structures we abstract turn out to be suitable in the pursuit of our goals; and if they do suit our purposes, that they can be brought into some kind of harmony with one another. This is the same, whether the goals are on the level of sensory-motor experience or of reflective thought. From this perspective, the test of anyone’s account that purports to interpret direct experience or the writings of another, must be whether or not this account brings forth in the reader a network of conceptualizations and reflective thought that he or she finds coherent and useful.

PHILOSOPHICAL POSTSCRIPT

It may be time for a professional philosopher to reevaluate the opposition between empiricism and rationalism. The rift has been exaggerated by an often ill-informed tradition in the course of the last hundred years, and the polarization has led to utter mindlessness on the one side and to various kinds of solipsism on the other. Yet, if we return to Locke, from the partially Kantian position of a constructivist such as Piaget, we may be able to reformulate the difference.

24. *The Original of all our Knowledge.* –In time the mind comes to reflect on its own operations about ideas got by sensation, and thereby stores itself with a new set of ideas, which I call ideas of reflection. These are the impressions that are made on our senses by outward objects that are extrinsic to the mind; and its own operations, proceeding from powers intrinsic and proper to itself, which, when reflected on by itself, become also objects of

contemplation—are, as I have said, the original of all knowledge. (Locke, 1690, Book II, Chapter I, §24)¹²

With one modification, this statement fits well into my interpretation of Piaget's analysis of abstractions. The modification concerns, of course, the "outward objects that are extrinsic to the mind". In Piaget's view, exogenous and endogenous do not refer to an inside and an outside of the organism, but are relative to the mental process that is going on at the moment. The 'internal' construct that is formed by the coordination of sensory-motor elements on one level, becomes 'external' material for the coordination of operations on the next higher level; and the only thing Piaget assumes as a given starting-point for this otherwise closed but spiraling process, is the presence of a few fixed action patterns at the beginning of the infant's cognitive development.

Both Locke's and Piaget's model of the cognizing organism acknowledge the senses and the operations of the mind as the two sources of ideas. Locke believed that the sensory source of ideas, the 'impressions' generated by 'outward objects', provided the mind with some sort of picture of an outside world. Piaget saw perception as the result of the subject's actions and mental operations aimed at providing, not a picture of, but an adaptive fit into the structure of that outer world. The functional primacy of the two sources, consequently, is assigned differently: Piaget posits the active mind that organizes sensation and perception as primary, whereas Locke, especially later in his work, tends to emphasize the passive reception of impressions by the senses. The difference, however, takes on an altogether changed character, once we consider that the concept of knowledge is not the same for both thinkers. For Locke it still involved the notion of Truth as correspondence to an independent outside world; for Piaget, in contrast, it has the biologist's meaning of functional fit or viability as the indispensable condition of organic survival.

The difference, therefore could be characterized by saying that classical empiricism accepts without question the static notion of being, whereas constructivist rationalism accepts without question the dynamic notion of living.

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NOTES

- ¹ Locke divided this work into Chapters, Books, and numbered paragraphs.
- ² Memory, as Heinz von Foerster (1965) pointed out, cannot be a fixed record (because the capacity of heads, even on the molecular level, is simply not large enough); hence, it must be thought of as dynamic, i.e., as a mechanism that reconstructs rather than stores.
- ³ A first English translation of Humboldt's aphorisms was published by Rotenstreich (1974). The slightly different translations given here are mine.
- ⁴ It is crucial to keep in mind that Piaget emphatically stated that knowledge could not be a copy or picture of an external reality; hence, for him, 'imitation' did not mean producing a replica of an object outside the subject's experiential field, but rather the re-generation of an externalized experience.
- ⁵ Cf. William James (1892): "One of the most extraordinary facts of our life is that, although we are besieged at every moment by impressions from our whole sensory surface, we notice so very small a part of them" (p. 227).
- ⁶ Ceccato said this several times in our discussions on the operations that constitute 'meaning' (1947-1952), when I was the translator for his journal *Methodos*.
- ⁷ Note that my use of the word 'symbol' is not the same as Piaget's for whom symbols had to have an iconic relation to their referents.
- ⁸ Needless to say, the perturbation may simply lead to a retrial or to a modification of the activity; the latter, if successful, would also constitute an accommodation.
- ⁹ Having introduced an idea from Ceccato's operational analyses into Piaget's model, I today believe that the motion necessary in perception need not be physical, but can often be replaced by the motion of the perceiver's focus of attention (cf. Glasersfeld, 1981a).
- ¹⁰ The title of this volume, as Leslie Smith (1981), one of the few conscientious interpreters of Piaget, pointed out, was mistranslated as 'The grasp of consciousness' and should have been rendered as 'the onset' or 'attainment of consciousness'.
- ¹¹ I owe knowledge of Caramuel's work to my late friend Paolo Terzi, who immediately recognized the value of the 17th century Latin treatise, when it was accidentally found in the library of Vigevano. Caramuel, a Spanish nobleman, architect, mathematician, and philosopher of science, had been 'exiled' as bishop to that small Lombard city, because he had had several disagreements with the Vatican.
- ¹² It may be helpful to remember that the first sentence of Kant's introduction to the first *Critique of pure reason* (1781) reads: "Experience is undoubtedly the first product that our intelligence brings forth, by operating on the material of sensory impressions" (p. 17).

REPRESENTATION AND DEDUCTION

... for the roses had the look of flowers that are looked at. (Thomas S. Eliot, 1914, p. 10)

The epigraph was chosen from a poet, to make clear from the outset that my paper is intended as a contemplative stroll through theory and not as a report on empirical research. We are all concerned with teaching, and poets, I'm sure you will agree, have had some success in that area. They may have achieved it unintentionally, but their lack of didactic ambition does not seem to have impeded the learning of those who wanted to learn, nor has it diminished their desire to learn. I have no statistical evidence for these assertions, but I nevertheless intend to push further in the direction they indicate. Poets know, perhaps better than others, that readers or listeners cannot be given ready-made thoughts, images, and ideas. They can only be given words. Being given words, however, they will inevitably bring forth thoughts and images of their own; and by presenting particular combinations of words, one can, at least to a modest extent, guide the conceptual construction of the meaning which, eventually, the readers or listeners will believe to have found in the text.

Poets also know (only too well) that it is no use trying to tell a listener or reader that his or her interpretation is 'wrong'. Paul Valéry said:

Once published, a text is like an appliance of which anyone can make use the way he likes and according to his means; it is not sure that the builder could use it better than others. Besides, he knows well what he wanted to make, and that knowledge always interferes with his perception of what he has made. (1933, p. 1507)

Since teachers of mathematics, as a rule, know well what they are explaining, that knowledge invariably interferes with their own perception of their explanation. Consciously or unconsciously they take for granted that certain things are 'self-evident', and they forget that what seems evident in mathematics is always contingent upon the habit of performing specific mathematical operations.

As seasoned users of language, we all tend to develop an unwarranted faith in the efficacy of linguistic communication. We act as though it could be taken for granted that the words we utter will automatically call forth in the listener the particular concepts and relations we intend to 'express'. We cling to the illusion that speech 'conveys' ideas or mental representations. But words, be they spoken or written, do not convey anything. They can only call forth what is already there, and they can stimulate new combinations. This should become clear every time we

test the representations our words have called forth in a listener; but we don't see it because of our unwarranted presuppositions concerning the process of communication.

One misapprehension stems from the general notion of 'representation'. As that term is used in psychology and cognitive development, it is ambiguous in more than one way. First, like many words ending in '-ion', 'representation' can indicate either an activity or its result. This ambiguity rarely creates difficulties. Far more serious is the epistemological ambiguity to which the word gives rise. It creates an unwholesome conceptual confusion.

The distinction I want to make clear concerns two concepts which, for instance in German, are expressed by two words, *Darstellung* and *Vorstellung*; both are usually rendered in English by 'representation'. The first designates an item that corresponds in an iconic sense to another item, an original to which it refers. The second designates a conceptual construct that has no explicit reference to something else of which it could be considered a replica or picture. (In fact, *Vorstellung* would be better translated into English as 'idea' or 'conception'.)

Thus, if one uses the word in the second sense, it would help to spell it 're-resentation'. The hyphenated 're' could be taken to indicate repetition of something one has experienced before. This would lessen the illusion that mental representations are replicas or images of objects in some real world. It would help to focus attention on the fact that what one re-presents to oneself is never an independent external entity but rather the re-play of a conceptual item one has derived from experience by means of some sort of abstraction (cf. Piaget, 1945, p. 237).

The ability to re-present to oneself prior constructs is an essential part of all cognitive activities. It comes into play when you ask yourself whether the soufflé you are eating now is as exquisite as the one you had in Dijon twenty years ago. Whenever you compare sensory experiences, and one of them is not in your actual perceptual field, that experience must be re-presented. As one gets older, one realizes that the memory from which one re-plays such past sensory experiences cannot always be trusted. Soufflés (and other sinful experiences) from one's distant past tend to seem sweeter than present ones. But this nostalgic tendency is not what I want to discuss here.

The ability to re-present is just as crucial in the use of symbols. The so-called 'semantic nexus' that ties a symbol to what it is supposed to stand for, ceases to function when the symbol user is not able to re-present the symbol's meaning. Memory, clearly, plays no less a part in the symbolic domain than in that of sensory experience. Irrespective of the particular position you may have adopted concerning the foundations of mathematics, you will all agree that symbols such as '+', '-', '×', and '÷' refer to operations and can, in fact, be interpreted as imperatives (add!, subtract!, multiply!, divide!). To obey any such imperative, one must not only know the operation it refers to, but also how to carry it out; one must know how to re-play the symbolized operation with whatever material happens to be at hand. That is to say, if operator-signs are to function as symbols, the operations to which they refer must have been abstracted by the symbol user from

the sensory-motor material with which they were implemented in that symbol user's own prior experience. (There are, of course, several levels of abstraction, but at the bottom there has to be sensory-motor material.)

The semantic nexus between an operator-symbol and the abstracted operation it designates is no less indispensable in logic than it is in mathematics. Quine (1976) speaks of "the inseparability of the truths of logic from the meanings of the logical vocabulary" (p. 109). Logical truth, of course, refers to the reliability of deductive inferences that can be derived from the chosen premises; it does not pertain to the experiential foundation of either premises or conclusions. If a syllogism were formed with the premises 'All socialists are evil' and 'Snoopy is a socialist' it would as logically lead to the conclusion that 'Snoopy is evil' as the traditional syllogism leads to the conclusion that Socrates is mortal. The logical 'truth' of a deduction is not impaired by the experiential falseness of the premises. Although the logic of the syllogism is in no way tied to what seems likely or unlikely in the thinker's experiential world, following the rules and carrying out the operations that are called forth by the use of words belonging to the 'logical vocabulary' is nevertheless an activity and, as such, requires an active, thinking agent. Hume (1739-40, Book I, Part IV, Chapter I) saw this, and concluded that deduction, because it involved a psychological process, could not be as infallible as classical logicians like to believe. If this introduction of doubt were legitimate, doubt would eventually infest also the realm of mathematical operations. To discuss it may therefore not be an idle exercise—especially if, as I believe, Hume's notion can be tied to the theory of re-presentation.

As far as deductive logic is concerned, what the premises say should always be explicitly posited rather than taken as statements of fact. Their relation to the experiential world is irrelevant. What matters is that they be taken *as though* they were unquestionable, as hypotheses which one accepts for the time being, and that their hypothetical status should always be carried over to the conclusion. In other words, we should always explicitly say:

If all men are mortal, and If Socrates is a man, Then Socrates is mortal.

This emphasizes two things: first, that one is dealing with assumptions whose experiential validity one has decided not to question for the moment; and, second, that the logical certainty one attributes to the conclusion pertains to the operations that are called forth by the logical terms 'if', 'all', 'and', and 'then'. These two aspects are the basis of our faith in the infallibility of deductive procedures.

John Stuart Mill, in an attempt to subvert faith in the syllogism, argued that, in order truthfully to formulate the premise that all men are mortal, one should have to examine all members of the class called 'men' with respect to their mortality. If, having done this, no exception to the rule had been found, one would know that Socrates is mortal, because, being a man, he must have been tested for mortality. If, on the other hand, he had not been tested, this could only mean that either he was not considered a 'man', or that the use of the term 'all' in the premise is unwarranted. This is a neat argument, but it shows that Mill did not see the premises of the syllogism as deliberate assumptions but as statements of

experiential fact. Once this is understood, the argument no longer goes against the syllogism but against the misconception that deductive inferences should automatically be 'true' in the experiential domain.

There may, however, be other problems. If the premises of syllogisms are understood as deliberately hypothetical conceptual structures (which one agrees not to question), one may still want to examine the deductive procedure, a procedure that involves several steps. Having constructed the premises, one must call up the logical operations designated by the tokens of the logical vocabulary and re-play these operations with the re-presentations of the premised conceptual structures. That is to say, in order to come to a conclusion, the conceptual construct created for the major premise must have been maintained unchanged, at least long enough to be available for re-presentation when one has created the conceptual construct for the minor premise and is ready to proceed with the logical operations that relate the two premises so as to produce the conclusion.

Whether or not one believes with Kant that the deductive operations called forth by logical terms are part of the inherent, a priori repertoire of the human mind, it seems plausible that, rather than being created each time anew, they are re-played, much like pre-programmed subroutines, when the associated symbol or sequence of symbols gains the agent's attention in an appropriate context (cf. Wittgenstein, 1969, §522/523). If this is the case, some form of memory would be required for the performing of logical operations, and since memory would have to be considered a psychological phenomenon, one might be tempted to invoke Hume's doubt.

The question of memory arises even more clearly in connection with the hypothetical conceptual structures that are generated in response to the not specifically logical components of the premises, that is, the hypothetical conceptual structures to which the logical operations must be applied. All deductive procedures require that we trust our ability to maintain, and re-present as they were, the conceptual structures and the operational routines we intend to use. If we doubt this ability, all logic goes by the board. We are not inclined that way. It would be as disruptive as doubting the reliability of memory and all the other electronic devices in a computer.

However, we may still question how we acquire logical operations. Professional philosophers usually dismiss any consideration of the developmental aspects of thought as 'genetic fallacy' and pretend that logicians and other users of logical operations do not have to construct the required procedures but have them ready-made in their minds even if they do not always use them. Like Piaget, I find this an absurd contention. Instead, I would suggest that it is precisely the experiential success of inductively derived rules that provides both the occasions and the motivation for the abstraction of the specific logical operations that are then associated with symbols and used without reference to experience.

From that perspective, it seems clear that, in the construction of the syllogistic procedure, the components of the premises that are not the specifically logical terms must be interpretable by the active agent in a way that makes sense in the context of that agent's experience. It seems likely that we come to make the

necessary reflective abstractions when we apply rules that work, rather than rules that are countermanded by experience. If we have never formulated a tentative rule of the kind ‘all roses I have seen, smelled sweet’, we would not be tempted to say: “this flower looks like a rose—therefore it will smell sweet”. In other words, if we have had no success with inductive inferences, we are unlikely to proceed to deductive ones.

To conclude, let me try to apply this line of thought to the basic understanding of numbers and how they interact. A child can no doubt learn by heart expressions such as ‘ $5 + 8 = 13$ ’. However, in order to understand them, she must be able to re-present the meanings of the involved symbols. As in the syllogism, the parts of such numerical expressions involve assumptions. ‘5’ means that one assumes a plurality of countable items which, if they were counted (i.e., if number words were coordinated with them one-to-one), they would use up the number words from ‘one’ to ‘five’. The ‘+’, then, signifies that a second plurality of items which, by itself, would use up the number words from ‘one’ to ‘eight’, is to be counted with the number words that follow upon ‘five’ (cf. Steffe et al., 1983). Children may re-present these pluralities and the counting activity in many different ways. The sensory-motor material they use to implement the abstracted patterns is irrelevant. What matters is that they have abstracted these patterns and can re-play them in whatever context they might be needed. For I would claim that only if they have acquired a solid facility in the generation of this kind of re-presentation can they possibly enter into the garden of mathematical delights.

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A CONSTRUCTIVIST APPROACH TO EXPERIENTIAL FOUNDATIONS OF MATHEMATICAL CONCEPTS

Mathematics is the science of acts without things—and through this, of things one can define by acts. (Paul Valéry, 1935, p. 811)

This is a revised version of a paper which, as an anonymous reviewer guessed, was written some time ago; in fact I wrote it in 1990/91 for the *2nd International Conference on the History and Philosophy of Science in Science Teaching*, Queen's University, Kingston, Ontario, 1992. Much has happened since then in the philosophy of mathematics, especially regarding the 'naturalization' of the highly abstract concepts that were at the core of the debate on the reality of mathematical objects. As I understand it, naturalization in that discipline is the attempt to illuminate the foundations of mathematics by mathematical rather than philosophical thinking and it leads to the dismantling of the Platonist notion that mathematical objects exist in an absolute sense. I fervently agree with this dismantling, but my approach is on a much lower level of abstraction and focuses on how the most elementary concepts, such as unit, plurality, number, point, line, and plane could be derived from ordinary experience. I have added a postscript with references to and brief comments on publications by Brian Rotman, Penelope Maddy, and George Lakoff as samples of recent voices.

INTRODUCTION

In a popular lecture, given at Heidelberg in 1870, Hermann von Helmholtz said that it was the relation of geometry to the theory of cognition that emboldened him to speak of geometrical subjects (in Newman, 1956, vol. 1, p. 638). It is in precisely that spirit that I venture into the domain of mathematical thinking—not as a practitioner of that specific art but as a student of conceptual construction who also has an interest in education. My purpose is not to discuss mathematics as it may appear to mathematicians exercising their craft, but rather to suggest a way to think of the conceptual origin of *some* basic building blocks without which mathematics as we know it could not have developed. The constructivist approach puts in question the notion of universal conceptual 'objects' and their ontological derivation and, consequently, argues against Platonic or Chomskyan innatism or any other metaphysical foundationalism.

I realize that some may consider any investigation of conceptual development an intrusion of psychologism, but even philosophers of mathematics speak of the derivation of notions:

The ideas, now in the minds of contemporary mathematicians, lie very remote from any notions which can be immediately derived by perception through the senses; unless indeed it be perception stimulated and guided by antecedent mathematical knowledge. (Whitehead, 1956, p. 393)

How ideas are derived is, after all, a legitimate question for cognitive psychology, and to conjecture paths that might lead from the senses to mathematical abstractions seemed a tempting enterprise, because awareness of some experiential building blocks could help to humanize a subject that all too often seems forbidding to students. I was encouraged to pursue that question by the published evidence of wide-spread dissatisfaction with the traditional dogma among philosophers and mathematicians themselves (Davis & Hersh, 1980; Lakatos, 1976; Lorenzen, 1974; Mittelstrass, 1987; Quine, 1969; Tymoczko, 1986a, b; Wittenberg, 1968). Since the ontological foundations of mathematics have again been put into question during recent decades, and since more and more often it is acknowledged that mathematics is the product of the human mind,¹ the approach from the point of view of mental operations abstracted from experience should no longer be considered inadmissible. Indeed, once one relinquishes Plato's notion that all ideas are prefigured in every newborn's head, one cannot avoid asking how they could possibly be built up.

One might object that such an approach would be an incestuous undertaking because it obviously starts with some, albeit rudimentary, ideas of what the building blocks might be. To this I would answer that the very same pertains to all epistemological investigations, because questions about human knowledge are inevitably asked and tentatively answered by a human knower. This was inherent in Vico's (1710) slogan '*verum ipsum factum*' (the true is what is made) and it was independently and more explicitly formulated by Kant, when he wrote "reason can grasp only what she herself has produced according to her design" (1787, p. XIII). My purpose, therefore, is to isolate *possible* preliminary steps of the construction. But first I want to show my route of approach.

In his 'proposals for reviving the philosophy of mathematics', Reuben Hersh writes:

What has to be done in the philosophy of mathematics is to explicate (from the outside, as part of general human culture, rather than from the inside, within mathematical terms) what mathematicians are doing. (1986, p. 22)

I am not a mathematician, and my remarks are therefore not in mathematical terms. But it should also be quite clear that I am not offering them as part of general human culture, because the radical constructivist orientation from which they spring is certainly *not* general. In my view, the part of human culture that concerns questions of knowledge and knowing, not specifically in mathematics, but in the

entire experiential field, suffers from precisely the same ambivalence and hypocrisy that Hersh imputes to the philosophy of mathematics.

When Hersh writes, a few sentences after the quoted passage, that such an explication would present “the kind of truth that is obvious once it is said, but up to then was perhaps too obvious for anyone to bother saying” (ibid.), he manifests faith in philosophical perspicacity far greater than the perspicacity shown by our general culture in the course of the two thousand five hundred years since epistemology began. My constructivist orientation is *radical* because it proposes to cut the cognizing activity and its results loose from the traditional dependence on an assumed ontology. It is an attempt to do without the notion of truth as a representation of an experiencer-independent reality, material or metaphysical (cf. Glaserfeld, 1989). Hence, the approach I am expounding here may upset not only ‘Platonist’ mathematicians but all who are philosophically or emotionally tied to some form of realism. My intention, however, is simply to contribute to a discussion that is still wide open.

MATHEMATICS AND COMMUNICATION

Concerning criticism in mathematical matters, Whitehead explained that “there are always three processes to be kept perfectly distinct in our minds” (1956, p. 395). The way Whitehead has formulated and explicated these three processes provides a good basis for laying out some features of the constructivist approach.² We must first scan the purely mathematical reasoning to make sure that there are no mere slips in it—no casual illogicalities due to mental failure. Any mathematician knows from bitter experience that in first elaborating a train of reasoning, it is very easy to commit a slight error which yet makes all the difference. But when a piece of mathematics has been revised, and has been before the expert world for some time, the chance of casual error is almost negligible (p. 395). It seems difficult not to agree with this statement, since it describes a checking procedure with which we are quite familiar (e.g., when someone is hanging a picture, drawing a map, writing a computer program, and so forth). On second thought, however, we may notice that Whitehead refers to an expert’s check of another person’s ‘purely mathematical reasoning’, which he then calls ‘a piece of mathematics’. This may prompt us to ask where the expert finds the entity that is to be checked. If it was a piece of mathematical *reasoning*, it must have been generated by someone’s thinking. Since we cannot read minds, access to another’s thinking or reasoning requires an act of communication. In other words, before a piece of reasoning can be checked, it must be formulated in some language or symbols that are known to both the author and the expert who is to do the checking. (From the constructivist point of view, communication is itself problematic, and I shall deal with it in the context of Whitehead’s second process, where it is even more relevant.)

For formalists, there should not be much of a communication problem. They take the presented symbols as they find them, and check whether or not they have been combined according to generally accepted rules. Formalists are concerned with syntax, not with conceptual semantics. The author’s acts of *reasoning* would

not be questioned, because—although formalists do not usually say this explicitly—from their perspective, symbols have to be perceived but need no *interpretation*, and the correctness or error of a mathematical expression depends exclusively on its formal compliance with the rules of the chosen symbol system.

That this is not a very satisfactory approach to questions of the mathematical underground, has been remarked by many critics during recent decades. Hersh has expressed the objection in a very general way: “Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later” (1986, p. 19). That the score ought to conform to the rules of the scoring system, therefore, is simply a precondition to any judgment concerning what the score might represent on the conceptual level. Mathematical symbols, of course, are far more complex and layered than musical notation and some of the conceptual ‘music’ they are intended to signify presumably arises on the higher levels of symbolization. In what follows, however, I want to focus on the very lowest level, the level on which non-mathematical experiences provide material for the abstraction of the most elementary mathematical building blocks.

Having discussed the possibility of ‘mere slips’, Whitehead turns to the starting-points of mathematical reasoning. He writes:

The next process is to make quite certain of all the abstract conditions which have been presupposed to hold. This is the determination of the abstract premises from which the mathematical reasoning proceeds. This is a matter of considerable difficulty. In the past quite remarkable oversights have been made, and have been accepted by generations of the greatest mathematicians. The chief danger is that of oversight, namely, tacitly to introduce some condition, which it is natural for us to presuppose, but which in fact need not always be holding. (pp. 395-396)

The presupposition of unwarranted conditions is something radical constructivism claims to have unearthed in several areas that have no obvious connection with mathematics. One area, however, that *is* relevant for the discussion of thinking and, as philosophers of mathematics have recently found, is indispensable for any critique of mathematical reasoning, is the kind of social interaction we call linguistic or symbolic communication (cf. Davis & Hersh, 1980; Tymoczko, 1986a).

Where communication is concerned, we habitually—and hence mostly tacitly—operate on the presupposition that others who use language or symbols that we readily recognize as such, are using them with the same meanings that *we* have come to attribute to them. This assumption is made habitually, because without it, most if not all of our everyday linguistic and symbolic interactions would be futile. Indeed, we are constantly reinforced to assume that our meanings are shared by others, because by and large our ordinary communicatory interactions work remarkably well. But the bulk of our ordinary communicating is about two experiential areas. Either it concerns sensory-motor objects, where perceptual feedback helps us to avoid gross misinterpretation; or it concerns emotions, and in the emotional sphere, where meanings are notoriously vague, the margin for

interpretation is so wide that we are rarely compelled to consider feedback that upsets the pleasant generic feeling of understanding or being understood.

In contrast, when we are communicating within the relatively systematized domain of a science, we are dealing to a large extent with abstract concepts and relations. In this area, such feedback as we do receive about the other's *reasoning* usually springs from inferences we draw from the technical context, *as we see it*, or from the other's actions, *as we observe them*. Whether the respective concepts are actually the same, cannot be ascertained, but insofar as scientific training imposes conventions of thinking, a certain degree of compatibility can be assumed.

SOCIAL ADAPTATION AND COMPATIBILITY

For Whitehead, almost twenty years after *Principia Mathematica*, there was no doubt that there was a fixed set of logical rules that formed the solid basis of mathematics. And the rules of logic were taken to be *a priori* and therefore not only unquestionable but also inevitably inherent in every thinker's rational procedures.

Like the methodology of Ceccato's Italian operationalist school,³ radical constructivism is an attempt to do without the assumption of *a priori* categories or rules. Categories are seen as the results of mental construction (as in Piaget's theory, in the case of space, time, and causality⁴); the sort of rules that make possible the repetition and checking of *logical* procedures are based on the coordination of specific mental operations with specific symbols. This coordination has to be accomplished by every single thinking subject, and the 'intersubjectivity' that makes possible the communication of logical procedures can be achieved only through the individuals' social interaction and mutual adaptation of their subjective coordinations (of mental operations and symbols). This is what Maturana (1980) has called a 'consensual domain' generated by the 'coordination of the coordinations of actions'. In this view, then, the *meaning* of both natural language and mathematical symbols is not a matter of 'reference' in terms of independently existing entities, but rather of subjective mental operations which, in the course of social interaction in experiential situations, achieve a modicum of intersubjective compatibility.

After his exposition of the three critical processes, Whitehead made the very important general remark: "The trouble is not with what the author does say, but with what he does not say. Also it is not with what he knows he has assumed, but with what he has unconsciously assumed" (p. 396). This becomes particularly relevant when he discusses the third process.

This third process of criticism is that of verifying that our abstract postulates hold for the particular case in question. It is in respect to this process of verification for the particular case that all the trouble arises. In some simple instances, such as the counting of forty apples, we can with a little care arrive at practical certainty. But in general, with more complex instances, complete certainty is unattainable. Volumes, libraries of volumes, have been written on the subject. It is the battleground of rival philosophers. There are two distinct

questions involved. There are particular definite things observed, and we have to make sure that the relations between these things really do obey certain definite exact abstract conditions. There is great room for error here. The exact observational methods of science are all contrivances for limiting these erroneous conclusions as to direct matters of fact. (p. 396)

Whitehead then discusses the problem of ascertaining that an experiential object is actually of ‘the same sort’, so that one can ascribe to it a condition that was abstracted from a prior sample. “The theory of Induction”, he says, “is the despair of philosophy—and yet all our activities are based upon it”.

The problem of induction which, as Whitehead defines it here, is the question whether one could justify the *generalization* of an idea that was abstracted from a particular sample of experiences, concerns the entire domain of science, not specifically mathematics. Here, I therefore merely emphasize that, from the constructivist point of view, ‘particular definite things observed’ and ‘direct matters of fact’ are generated in ways that differ from the conventional realist account implied by this passage from Whitehead. The ‘practical certainty’ in the case of the forty apples, however, falls within the realm of experiences that are crucial to the thesis I want to develop, because according to it, the certainty does not spring from the counted ‘objective’ things, but from the mental operations of the counter (cf. for instance Piaget, 1977b, p. 71).

I have mentioned that, from the constructivist point of view, there are problems in discussing an individual’s mathematical reasoning, because, in order to be discussed, criticized, or assessed in any way, this reasoning must first be uttered or written and then *interpreted* by the critics or evaluators. To this I now want to add that whatever Whitehead intended by the expression ‘direct matters of fact’ is not something that is an unquestionable *given*, but rather the result of an individual’s *interpretation* of experience which, to that individual, *appears to be compatible* with the interpretations of other individuals. Compatibility—and this cannot be stressed too much—is not the same as identity. The distinction is important, because it alters the notion of ‘understanding’, of ‘shared’ ideas and conceptual structures, and thus also the notion of ‘accepting a proof’. When I agree with what another says or does, it means no less, but also no more, than that I interpret the statement or the action in a way that manifests no discrepancies from what I might say or do under the given circumstances *as I see them*. This leaves room for uncounted discrepancies that did not happen to surface; and we all know how often, after a feeling of agreement, a feeling of harmony and complete understanding, some further interaction brings out a discrepancy that had remained hidden in all that went before.

Tymoczko (1986b) maintains that the type of criticism that is necessary for the development of proofs is essentially social: “Without criticism, proof-ideas cannot develop into proofs” (p. 48). In other words, when some reasoning, presented in speech or writing, is claimed to constitute a proof, this communication has to be interpreted and criticized by others. Then the critique has to be interpreted and answered by the author, and only when the critics interpretation of the answer is

judged adequate by them, will the presented reasoning be promoted to the status of proof. No matter how many iterations this procedure might go to, it is clear that it can never guarantee the identity of concepts and conceptual relations used by the different thinkers involved. It can at best lead to apparent compatibility—and this, of course, is quite sufficient for the *practice* of mathematics (or any other domain of human cooperation).

But if compatibility is all that can be achieved, we cannot found mathematics in an ontological realm of ideas presumed to exist independently of any thinking subject. Yet Lakatos was probably right when he remarked: “It will take more than the paradoxes and Gödel’s results to prompt philosophers to take the empirical aspects of mathematics seriously” (1986, p. 44). From the outside, however, witnessing the crumbling of what were considered the foundations of mathematics, it seems reasonable to ask what the most elementary building blocks could be that might serve as a basis for the constitution of the mysterious structures or ‘objects’ that mathematics develops. To pursue that quest requires an empirical investigation, where ‘empirical’ has its original meaning and refers to *experience*. But clearly it will not be sensory experience that matters, but the experience of mental operations. As Hersh (1986) put it, “mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects)” (p. 22).

If we do not want to believe that ideas are innate or God-given, but the result of subjective thinkers’ conceptual activity, we have to devise a model of how elementary mathematical ideas could be constructed—and such a model will be plausible only if the raw material it uses is itself not mathematical.

TO BEGIN AT THE BEGINNING

To have the idea of counting, one needs the experience of handling coins or blocks or pebbles. To have the idea of angle, one needs the experience of drawing straight lines that cross, on paper or in a sand box. (Reuben Hersh, 1986, p. 24)

Unless we are able to conceive of something that is unitary, in the sense that we distinguish it as a discrete item in our experiential field and are able to ‘recognize’ other items *like* it, we cannot have a plurality. If we have no plurality, we have no occasion to count—and if we did not count, it is unlikely that we should ever have arithmetic and mathematics, because without counting there would be no numbers. Hence, if we want to get some inkling as to how arithmetic arises, we may have to begin with the concepts of unit and plurality, as well as the activity of counting which generates the concept of number. The ensuing development was described by Paul Lorenzen (1974):

The foundation of arithmetic is the pre-arithmetical praxis: the use of counting-signs (e.g., |, ||, |||, ||||, ...) in counting collections (heaps, herds, groups, complexes, ...); using counting-signs rather than the collections

themselves, to make comparisons of amounts; adding and subtracting counting-signs (instead of certain operations with the collections). (p. 199)

Rotman (1987, p. 8) uses the same notation of counting signs, but both he and Lorenzen seemed to take the concepts of unit and collection for granted. Yet, the ‘pre-arithmetical praxis’ obviously does not begin with counting-signs. Before any collections can be counted and coordinated with signs that can be used in their stead, experiential items must be gathered in collections such as heaps or herds. To do this, we must distinguish more than one experiential item, that is, a *plurality*, such that each of the individual items satisfies whatever conditions govern membership in the particular heap, herd, or collection we want to form.

The first task, then, is the distinction of individually discrete ‘things’ in our experiential field. To normal adult humans, who are experienced managers of a more or less familiar environment, it may seem absurd to suggest that the segmentation of their experiential world into discrete things should not be an ontological given. But even the most orthodox epistemologists, at least since the days of John Locke, have discarded this commonsense notion. Many modern scientists, for example, Mach (1910, p. 42) and Bridgman (1961, p. 46), explicitly stated this, and Albert Einstein (1936) formulated one of the simplest, uncompromising descriptions of how we come to furnish our world with discrete things:

I believe that the first step in the setting of a ‘real external world’ is the formation of the concept of bodily objects and of bodily objects of various kinds. Out of the multitude of our sense experiences we take, mentally and arbitrarily, certain repeatedly occurring complexes of sense impressions (partly in conjunction with sense impressions which are interpreted as signs for sense experiences of others), and we correlate to them a concept—the concept of the bodily object. Considered logically this concept is not identical with the totality of sense impressions referred to; but it is a free creation of the human (or animal) mind. (p. 291)

Piaget (1937) provided a minute analysis of how object concepts might be constructed by the very young child, and from Edmund Husserl we have a suggestion that is particularly relevant to the present context. In his *Philosophie der Arithmetik* (1887), Husserl proposed that the mental operation that unites different sense impressions into the concept of a ‘thing’ is similar to the operation that unites abstract units into the concept of a number (pp. 157-168). I accept this hypothesis, but want to point out that, in order to have several units that can be united to form a number, one must have a concept of plurality. I therefore want to unravel the steps involved in the initial development and fill in some of the details that seem necessary.

Compounds of sensory impressions presumably acquire their first stability, as Einstein suggested, through repetition. Brouwer (1949) proposed that the perceiving subject’s self-directed attention “performs identifications of different sensations and of different complexes of sensations, and in this way, in a dawning

atmosphere of forethought, creates *iterative complexes of sensations*" (p. 1235; original emphasis). The stability of such a sensory compound manifests itself in the subject's ability to 'recognize' it when it is produced again. Children clearly show this in the early stages of language acquisition. Once they have isolated a group of sense impressions and have associated them with, say, the word 'cup', they may toddle to the kitchen table and, pointing with their finger, say 'cup', then point to another and say 'cup', and repeat this procedure for every cup they happen to perceive on the table. Psycholinguists call this phenomenon 'labeling' and it provides good evidence that some kind of structure has been formed which allows children to utter the associated word whenever their perceptual mechanisms produce sensory signals that can be fitted into that particular structure.

THE ABSTRACTION OF PLURALITY

However, to say that a child that does this has a concept of *plurality*, would be reading too much into the episode. In fact, it usually takes at least a month or two before the child will use the plural 'cups' instead of acknowledging individual cups singly. This delay cannot be explained by the simple fact that the child has to learn a different word (i.e., the plural form), because in order to use the new word, the child also has to learn to isolate a different experiential situation. The plural 'cups' must be associated with a perceptual situation that contains *more than one* cup—and this 'more than one' is *not* a perceptual fact. The inference that more than one cup is on a table is not based on the sensory impressions isolated as cups, but on the *awareness* that one has repeated the same operations of isolating and recognizing within certain boundaries of space and time. The conception of a plurality, therefore, is the result of a repetition of mental operations that accompany the sensory impressions but are themselves not sensory. And the basis on which the conceptual structure called 'set' can be created is laid only when, in a further step of abstraction, the operations that generated a plurality are seen as an operational pattern *without* considering the sensory items that constituted the collection.⁵

Consequently, the notion of a plurality of things, for which language supplies the plural form of the things' name, should not be considered a mathematical concept. Though it results from mental operations, it is tied to sensory experience and does not require an abstract concept of unit. Indeed, Husserl makes clear that the ordinary meaning of the word 'one' (used in opposition to a plurality, as in 'more than one') must be distinguished from the *numerical* concept of 'one', which designates an abstract unit (1887, p. 128ff).

That the abstract units required in arithmetic are not quite the same as the units constituted by the discrete objects in our experiential world, was indicated also by Frege (1884, p. 58), when he said that the things we number must be distinguishable, whereas the units of arithmetic are not, because they are conceptual and have to be identical in every instance.

Brouwer suggested that the fact that attention can be directed, enables the mind to produce the complexes of sensory elements that we perceive as 'things', and that

this ability derives from the inherent character of consciousness which, in his view, was not a steady state but an oscillating function (1949, p. 1235). A similar idea was proposed quite independently, by Silvio Ceccato (1966), who posited a pulsating attentional mechanism that generates patterns of focused and unfocused pulses which can then be used to segment sensory material into iterable conceptual structures. I have further developed this approach in a hypothetical model of the construction of the concepts of unit and number (Glaserfeld, 1981a). The model leans on Piaget's notion of reflective abstraction⁶ and offers at least an hypothetical skeleton of mental operations that lead from the inception of discrete sensory things to pluralities, collections, arithmetic units, set, and number. Here, however, I want to concentrate, not on the mechanisms of abstraction, but on experiential elements that may serve as its raw material. In what follows, I shall make the case that counting provides the most plausible basis for the abstraction of the concept of 'numerosity' (the cardinal aspect of number).

COUNTING AND NUMBER

If we want to agree with what Hersh says in the quotation I have placed at the beginning of the section 'To begin at the beginning' above, it will be necessary to make quite clear what we mean by 'counting'. The word has been used for diverse manifestations that range from a toddler's meaningless recitation of a few number words to the function of a gadget that indicates radioactivity. From the constructivist point of view, counting is a very specific, complex activity (cf. Steffe et al., 1983). According to our definition, it has three components: A conventional number word sequence, a plurality of unitary sensory items (perceived or visualized), and the one-to-one coordination of successive number words and the items in the collection.

Imagine an ordinary, nonphilosophical observer watching a mason who utters the standard sequence of number words as he points to the bricks lying beside him. If 'eighteen' turns out to be the last number word to which a brick could be coordinated, it should be quite clear to the observer *why* the mason might now announce that there are eighteen bricks in the counted collection. If the observer has been attentive and neither missed, nor found fault with, any of the steps in the mason's procedure, she herself has indeed come to the same result. Nevertheless, if one asked either of them how they *know* that there are eighteen bricks, the chances are that they would both answer: "Well, I just counted them!" It is unlikely that either of them would explain in sufficient detail the procedure that was carried out and why the last number word used could be taken to indicate the numerosity of the collection.

We have all learned to count in early childhood, and we take for granted that the last number word of a count tells us *how many* items there were involved. We are, indeed, so accustomed to this, that we do not consciously think, whenever we hear or read a number word, that it entails a count of as many discrete units as there are number words in the conventional sequence leading up to it from 'one'. Yet, if we did not know this in some way, the number word could have no meaning for us.

The fact is that number words have become symbols for us, and as such they symbolize the counting procedure that leads up to them, *without our having to carry out* that procedure or even having to think of it.⁷ Even in the case of numbers that are higher than we could ever actually reach by counting, the tacit knowledge that there is a procedure by means of which, theoretically, we could reach them, constitutes the first (but by no means the only) characteristic of *number* as an abstract concept.

In short, I submit that the three elementary concepts of arithmetic—unit, set, and number—are abstractions, not from physical objects or other sensory material, but from mental operations that thinking subjects must carry out themselves. At the beginning, ontogenetically speaking, these operations develop as corollaries of actions which, in order to be performed, require sensory-motor material. This material need not be the same for all thinking subjects, it merely provides the occasion. However, once patterns of mental operations have been abstracted, they become *mathematical* concepts through association with symbols that can ‘point’⁸ to them without invoking their actual execution.

The concept of unit is abstracted from the perceptual operation of combining various sense impressions to form a ‘thing’; the concept of set is derived by abstracting the plurality of abstract units from a collection of things (i.e., considering an experientially bounded plurality but not the sensory items that were used to generate it); the concept of number arises when number words or numerals have become symbols that tacitly point to a possible count that leads up to them.

I want to emphasize that this analysis concerns the basic inception of the concepts, and that in the vast domain of mathematics each of them can be indefinitely enriched by the addition of further abstractions. Besides, there is the whole area of geometry to which I now want to turn.

FROM ACTION TO ABSTRACTION

The understanding is a wholly active power of the human being; all its ideas and concepts are but its creation, ... External things are only occasions that cause the working of the understanding ... the product of its action are ideas and concepts. (Immanuel Kant, 1787, Vol. VII, p. 71)

In the beginning, geometry is usually presented as a matter of points, lines, and planes. In ordinary language, we have no difficulty in finding experiential objects to which we can apply these words. Most of these objects, however, do not satisfy the mathematician’s requirements. Hence there are mathematical definitions, or rather, expressions that purport to serve that purpose. But even mathematicians themselves are not always pleased with them and therefore add more technical formulations, based not on experience but on a specific mathematical frame of reference, such as a system of orthogonal coordinates. Thus my *Mathematical Dictionary* (James & James, 1959) has the following entry for ‘point’:

- (1). An element of geometry which has position but no extension.

- (2). An element of geometry defined by its coordinates, such as the point (1,3). (p. 274)

Although it is a long time ago, I can still remember our teacher, before we had heard anything about coordinates, making a small mark on the blackboard and saying, as he turned to us: "This is a point". Then he hesitated for a moment, looked back at the mark, and added: "Of course, a geometrical point has no extension". This left us wondering about grains of sand, specks of dust, and other smallest items, but we remained perplexed because all of them still had *some* extension. After a few days the perplexity was forgotten. We had learned to make points with our pencils, and all that mattered was that they were not noticeably wider than the lines we drew.⁹

The obstacle here is that, logically, it is impossible to move smoothly from small and smaller to 'no extension', and yet hold on to a something that could be discriminated; and it is equally impossible to move from few and fewer to 'none', and yet hold on to the notion of plurality. Hence, what is needed is another approach. When we first meet 'zero', we can see it as the number word to use when all countable items have been taken away. And there is an analogous approach to 'point' in a visual experience that most will have had in one way or another. Imagine, for instance, sailing away, on a perfectly smooth lake, from a small floating object, say, a bottle. It gets smaller and smaller, and suddenly you cannot see it any longer, though you are still looking at the point where it was. The bottle is gone, and it would seem more adequate to identify the point with the focus of your attention. This gets rid of the problem of size, because it is never the focus of attention that has a size, but only the *things* one is focusing on.¹⁰

Hence I propose to think of 'point' as the very center of the area in the focus of attention. In the visual field, then, it would be the center of an item we are focusing on. If that item is so small that we cannot distinguish a center from the circumference, we have an item that can *represent* our concept of point, but it is not itself a point, because we can still imagine that it *has* a center, even if we cannot see it. But then this center turns into the vanishing point, a conceptual construct that derives from movement and attention.

Another way to approach the concept of point was suggested by Ceccato in conversations we had around 1950: Think of a form of cheese, he said, and the way one cuts it by pulling a wire through it. (This, of course, was in Italy, where large forms of cheese are always cut in this way.) The first cut, say a vertical one, gives you a plane. If you cut again vertically, intersecting the first plane, the two cuts give you a vertical line. And if now you cut horizontally, the intersection of the three cuts gives you a point. What you have to focus on, of course, is not the wire, nor the space it leaves, but the movements, because in movements we feel direction but no lateral extension.¹¹

To my mind, both these approaches are more adequate than merely *saying* that a point has no extension. They come closer to describing what one can *do* to arrive at the concept that has no sensory instantiation.

In the case of 'line', there is a reputable precedent. In his *Critique of pure reason*, Kant says, in order to experience a line, one must *draw* it (1787, p. 138). But he said it in German, and translation, as so often, obscures the original meaning. The English verb 'to draw' is used indiscriminately for producing images with a pencil and for what horses do to a cart. In German the two activities require different verbs. Kant used *ziehen*, which means 'to drag' or 'to pull' and does not refer to a graphic activity except in the case of lines. Since he spaced the word (to emphasize it), he had in mind the physical motion.¹²

In the same vein, Brunschvicg (1912) says of the straight line:

The elementary operation that is to furnish the simplest image is the stroke (*trait*). The hand places itself somewhere, it stops somewhere; from the point of departure to the point of arrival, the mind has not become aware of a division or a change in the movement accomplished by the hand. Hence there is no reason to suspect that the path linking the two points might not be a uniform and unique line, a straight segment that could serve to measure the distance. In fact, we know by what roundabout way geometry was led to question the evidence that seems to support the uniqueness of the straight line between two points; and we understand that it was the ease and certainty (of the act) that enabled the mind to cling to what is given by intuition. (p. 503)

Intuition, here, I suggest is precisely what Kant meant when he said *Anschauung*, that is, the view of an experience upon which we are reflecting. The line, then, is a reflective abstraction from a uniform movement we make. To this I add, that this movement need not be that of a hand or other visible object, but it can be the movement of our attention in whatever field we happen to be considering. And the straightness of an object can, in practice, be checked by shifting the focus of attention in uniform motion, as cabinet makers do when they look along the edge of a board to check that it is not warped.

A SECOND DIMENSION

Having found an experiential basis for points and lines, we may follow the definition of 'plane' in James and James (1959): "A surface such that a straight line joining any two of its points lies entirely in the surface" (p. 273). Implicit in this criterion is the requirement to look in at least two directions, which is equivalent to *drawing* more than one line. Hence the axiom that a plane is defined by three points. But each of these defining elements itself involves only one direction and therefore provides no immediate experience of the two-dimensionality of surfaces or planes.

It has also been suggested that a plane can be constructed by moving a line sideways. This is interesting, because in order to follow this movement, the focus of attention has to be widened to cover at least a certain stretch of the line. A practical equivalent would be to move one's hand on a surface, and feeling no change either in the tactile pressure of fingers and palm or in the direction of movement. In both cases there is an expansion of the focus of attention in order to

monitor more than one point. And this expansion is the opposite of the shrinking in the example of the bottle on the lake.

Confrey (1990) proposed that the way an object increases or decreases in size, as we move towards or away from it, provides an experiential basis for both exponential change and geometric similarity. I fully accept this idea and want to stress that, in the present context, its most relevant aspect is the expansion, respectively contraction, of the area covered by our attention. This movement provides experiential situations from which, in the expanding direction, the two-dimensionality of the plane can be abstracted, whereas the shrinking direction may lead to the abstraction of a concept of point.

These brief suggestions are the merest beginning of an analysis of the conceptual foundations of geometry. Other experiential situations can be found that may serve as raw material for the abstraction of the traditional ‘basic elements’, and none is unique in the sense that it could not be replaced by another. But as with the concepts of unit, plurality, and number, I believe that the minute analysis of elementary actions and operations that might occasion their abstraction is a direction of research that is well worth pursuing. If students, at the time when geometry is introduced, were offered experiences of this kind, they might come to understand that the lines drawn on paper and the physical models of bodies they are shown are merely occasions for mental operations that have to be actively carried out in order to abstract the basic concepts of geometry.

A SOURCE OF CERTAINTY

Mathematics, like theology and all free creations of the Mind, obeys the inexorable laws of the imaginary. (Gian-Carlo Rota, 1980, p. xviii)

In the preceding sections I have argued that common non-mathematical activities, such as isolating objects in the visual or tactual field, coordinating operations while they are being carried out, and generating a line by a continuous uniform movement, are the experiential raw material that provides the thinking subject with opportunities to abstract elementary mathematical concepts. If one accepts this view, one is faced with the puzzling question how such obviously fallible actions can lead to the certainty that mathematical reasoning seems to afford.

The puzzle is not unlike the one that arises if we write the traditional textbook syllogism with a first premise that we assume to be false—for instance, ‘All men are immortal’. If we proceed with ‘Socrates is a man’, the conclusion that Socrates is immortal will be just as certain and *logically* ‘true’ as the opposite conclusion, which we get when we start with the more plausible first premise that asserts the mortality of all humans.

This puzzle disappears if it is made clear that the premises of a syllogism must be considered as *hypotheses* and should be preceded by ‘if’. Their factual relation to the experiential world is irrelevant for the formal functioning of logic. Considering them to be ‘as though’ propositions, makes sure that, for the time being and during the subsequent steps of the procedure, one is not going to

question them. The steps of that procedure are, on the one hand, the specific mental operations designated by terms such as ‘all’, ‘some’, ‘no’, ‘is a’, ‘then’, etc., *and*, on the other hand, the operation of combining the two premises. Assuming that these operations are carried out in the customary way, the certainty of the conclusion springs from the fact that the situations specified by the premises are *posited* and, therefore, not to be questioned during the course of the procedure.¹³

Viewed in this way, the syllogism also becomes immune to a criticism which, I believe, was first brought up by John Stuart Mill (1843, III.2). Because he was mainly concerned with the logical uncertainty of inductive generalization, he remarked that, in order to be justified in saying that *all* men are mortal, one should have examined all members of the class called ‘man’. If Socrates is rightly considered a member of this class, one must have come across him during the examination and one does not need the conclusion, to know that he is mortal. If, however, one has not examined Socrates, then either the second premise or the conclusion is false. When an ‘if’ is placed before the premises, this voids the above argument because it obviates the examination of all members of the class involved; and what the conclusion affirms is made unquestionable, because this now derives from what was hypothetically posited and is therefore not dependent on an examination of actual experiences.

THE HYPOTHETICAL TRICK

Yet there remains a question. How do we come to feel certain that the conclusion tells us something that was contained in the premises? This goes to the core of the deductive procedure and is crucial for my thesis. If the major premise is of the form ‘all X are B’, it implies that there is a *bounded collection* of Xs, either experiential or hypothesized. But collections are the result of mental operations, and to form a collection, we clearly must first have formed a plurality. In turn, to form a plurality we need to conceive of discrete unitary items that have some attribute, let us say A, in common. If we now consider a discrete unitary item and find that it has the attribute A, we may say: “Ah, here is another X” (and we may proceed to examine whether it, too, is B, and thus fits our hypothesis). But note that, to say ‘another’, we must *remember* that at some earlier moment we attributed A to some item(s). Similarly, to conclude in the syllogism that Socrates is mortal, we must *remember* that we previously formed a collection called ‘man’ and attributed mortality to it, because this is the basis on which we now feel certain that, if Socrates can be considered a member of that collection, he must be mortal.

I believe it was Euler, who first used circles to indicate the bounded collections involved in propositions such as ‘Some A are B’ and ‘all A are B’.¹⁴ And he went on to show how the syllogistic procedure could be visualized in this manner. The mystery of logic, purported to be so difficult to approach, he says, immediately strikes the eye if one uses these figures (Euler, 1770, letter 103). And he is right. Two concentric circles do convey the notion of containment with great force—but at the moment of perceiving this symbolic containment, one still has to remember that the outer circle is intended to stand for the major premise and the inner circle

for the minor. The circles help to make palpable the relation but not *what* is being related. Hence, no matter how we look at it, the judgment of *certainty* involves faith in the flawless functioning of the particular kind of memory we use in carrying out the syllogistic procedure.

The ‘certainty’ in arithmetic is analogous in that it, too, depends on mental operations and not on a fit with the experiential world. If someone tells me that there are seven oranges in the kitchen, three on the table and four on the windowsill, I do not have to accept this as an unquestionable ‘truth’—even if I believe that he or she is using the number words the way I myself would use them. There might have been an error, either in recognizing oranges or in counting. But I cannot question the simple statement that $3 + 4 = 7$. Unpacked, this statement means: You count a collection and come to ‘3’, then you count another collection and come to ‘4’; now you can consider the two collections as one collection, and if you count it, you will come to ‘7’. Provided one’s procedure follows the standard number word sequence and coordinates its terms to countable items in the standard fashion, one is going to arrive at the standard result every time. Because the operations involved have become habitual and we are not aware of carrying them out, we get the impression that there is something preordained about their results, something that could not be otherwise. As Spencer Brown said about the existence of the universe: “It comes through a very clever trick. It depends on an elaborate procedure for forgetting just what it was we did to make it how we find it” (in Keys, 1972, p. 31).

Hence, there is the involvement of memory. In order to know (when you have finished counting the composite collection and came to the result ‘seven’) that 7 is the sum of a collection of 3 and a collection of 4, you must *remember* that you counted these two collections before you combined them. As in the case of the syllogism, a graphic representation of the procedure, say ‘prearithmetical signs’ like those Lorenzen suggested for counting, may help to visualize the procedure—but in order to tie the graphic signs to the process of addition, you have to remember the meaning that was attributed to them at the outset.

This leads to the conclusion that one can do neither logic nor mathematics without *doing* things which, themselves are not specifically mathematical. Depending on the kind of mathematical result aimed at, there will be activities such as isolating discrete perceptual or visualized items, moving a limb or the focus of attention, attributing meanings to signs or symbols, considering explicitly or implicitly limited contexts, and remembering the conceptual commitments that have been made during these mental operations. The certainty of the results, then, springs on the one hand from the fact that one operates in a hypothetical mode and therefore obliges oneself not to question what one has hypothesized; and on the other hand, on implicit faith in one’s memory of meanings attributed, of operations carried out, and of the results they produced.¹⁵

CONCLUSION

If what I have outlined is a viable approach, there are several things that will have to be considered when the foundations of mathematics are discussed. First, they cannot be discussed without the use of language. From the radical constructivist point of view, there is no neat distinction between private and public language, because all meaning of signs and symbols, including the linguistic kind, is built up by individuals on the basis of their own subjective experiences of isolating objects, events, and the relations among them. No doubt everyone's meanings are modified and adapted in the course of interaction with other speakers, but the result of such adaptation is *compatibility*, not identity. And the compatibility achieved is relative to the particular interactions an individual has participated in. The expression 'shared meaning' is a deceptive fiction because sameness can never be ascertained.

Second, some (and perhaps all) of the indispensable elements in mathematical thinking are conceptual constructs that were abstracted from operations carried out with sensory material, operations that are involved in segmenting and ordering experience long before we enter into the realm of mathematics.¹⁶

It seems to me that these are two good reasons for considering mathematics to be an 'empirical' enterprise. Lakatos and others (e.g., Davis & Hersh, Tymoczko) have called it 'quasi-empirical' because it does not directly deal with physical objects. To a radical constructivist, however, physical objects, too, are conceptual constructs abstracted from a way of experiencing that imposes structure on an essentially amorphous sensory manifold. Mathematics deals with constructs that no longer contain sensory or motor material because they are abstracted from mental operations carried out with that material; but this does not make them any less experiential—and 'empirical', after all, is but another word for 'experiential'. The poet/mathematician Paul Valéry has said this with uncommon elegance in the epigraph I placed at the beginning.

POSTSCRIPT

Penelope Maddy (1997) proposed an approach "that turns away from metaphysics and towards mathematics" (p. 233). What concerns her is not the justification or ontological reality of the relevant concepts but how mathematicians have formed them. The formation she describes takes place on a level of abstraction beyond the very first one that uses sensory experiences as raw material (which is the topic of my paper). Set theory has been so successful, she claims, because it has created its own conceptual ontology that does not require the Platonic ontology philosophers are mainly unwilling to relinquish. But as Quine (1969) put it: "The old epistemology aspired to contain, in a sense, natural science; it would construct it somehow from sense data. Epistemology on its new setting, conversely, is contained in natural science, as a chapter of psychology" (p. 83). As I understand it, Maddy's suggestion is that foundations of mathematics have to be found within set theory.

Encouraged by the popular success of *Metaphors we live by* (Lakoff & Johnson, 1980), George Lakoff (Lakoff & Núñez, 2000) embarked on the exploration of the

metaphors that, he believes, form the source of mathematics. The meaning of mathematical terms, he claims, consists in conceptual metaphors. Each conceptual metaphor is: “a unidirectional mapping from entities in one conceptual domain to corresponding entities in another conceptual domain” (Lakoff & Núñez, 2000, p. 42). From my point of view, this is a highly misleading statement. In my world, metaphors can be defined as the attempt to transfer a property assumed to be characteristic of one type of experiential item to an item that is usually considered not to have that property (e.g., if I say: ‘My brother in law is a gorilla’, it is up to you to discover from the context which of a gorilla’s properties I am attributing to my brother in law). But this is not the only kind of metaphor Lakoff has in mind. He also wants to call metaphor any relational pattern that has been abstracted from an experiential situation. This happens to be exactly what Peirce called an ‘abstractive observation’; and Piaget some thirty years later called it ‘reflective abstraction’, showing how patterns of physical action could lead to patterns of mental operating. Piaget is mentioned in a marginal context only, Peirce not at all, and, what is even more astonishing, there is no reference to Brouwer, who explicitly linked the origin of number to an experiential situation (cf. Section ‘To begin at the beginning’ above). Lakoff and Núñez’s *Where mathematics comes from* (2000) suggests a great number of applications of his theory of metaphor to mathematical concepts. The usefulness of this move will have to be judged by mathematicians. However, Lakoff says practically nothing about the elementary concepts which I deal with in my article. He takes discrete countable things and pluralities for granted and his account of counting makes no mention of a conventional number-word sequence, without which, I would claim, no concept of number can be formed.

Brian Rotman, judging by what he wrote in *Mathematics as sign* (2000), would agree with what I wrote in Section ‘Counting and number’, above. “Numbers”, he says, “appear as soon as there is a subject who counts” (p. 38). He justifies this in the preceding paragraph:

However possible it is for them (i.e., whole numbers) to be individually instantiated, exemplified, ostensibly indicated in particular, physically present, pluralities such as piles of stones, collection of marks, fingers, and so on, numbers do not arise, nor can they be characterized, as single entities in isolation from one another: they form an ordered sequence, a *progression*. It seems impossible to imagine what it means for ‘things’ to be the elements of this progression except in terms of their production through the process of counting. (Ibid.)

As a general maxim he states that “A mathematical assertion is a *production*, a foretelling of the result of performing certain actions upon signs. ... Thus, for example, the assertion ‘ $2 + 3 = 3 + 2$ ’ predicts that if the Subject concatenates 1 1 with 1 1 1, the result will be identical to his concatenating 1 1 1 with 1 1” (p. 16).

Following Deleuze and Guattari (1994), Rotman distinguishes ordinals and cardinals in a way that fits my thinking and seems enlightening to me: ordinals are rhythmic, directional in terms of serial continuation, and hence akin to melody;

cardinals, in contrast, seem a parallel presentation, a harmony. I see this as matching the notion that ordinals are formed by focusing on the repetition of counting acts, whereas cardinals arise from reflecting a counted plurality as a unit (cf. Rotman, 2000, p. 146).

Much more, I am sure, has been written during the last fifteen years that would merit a comment; but my article was never intended as a review of the field.

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NOTES

- ¹ e.g., “Mathematical objects are invented or created by humans” (Hersh, 1986, pp. 22-23) and particularly his reference to Piaget: “One cannot overestimate the importance of his central insight: that mathematical intuitions are not absorbed from nature by passive observation, but rather are created by the experience of active manipulation of objects and symbols” (p. 26).
- ² I have separated and numbered the three parts, although they form a consecutive passage in Whitehead’s essay.
- ³ An interdisciplinary research group founded by Silvio Ceccato in the 1940s, which published the journal *Methodos* and was later incorporated as the Center for Cybernetics of Milan University.
- ⁴ cf. Piaget (1937), *La construction du réel chez l’enfant* (and half a dozen other volumes whose titles refer to these concepts).
- ⁵ The explanations of the term ‘set’ one finds in textbooks (e.g., that sets are somewhat like a jury or the signs of the zodiac) do not diminish the obscurity, because they omit the crucial fact that such collections are formed on the basis of an extrinsic consideration that is quite unmathematical and that the concept of ‘set’ requires one to take the units of the collection and deprive them of whatever attributes they might have beyond being considered units.
- ⁶ Cf. Piaget’s two 1977 volumes *Recherches sur l’abstraction réfléchissante*; also Beth and Piaget, 1961.
- ⁷ A full description of this pointing function can be found in Glasersfeld, 1991.
- ⁸ I am avoiding the word ‘refer’ because of its usual connotation of reference to real-world objects.
- ⁹ I might add that perplexities of this kind arose also at other times during our mathematics instruction, especially when we came to differential calculus and integration. As the perplexities mounted, more members of the class concluded that mathematics is incomprehensible and not much fun to pursue. This was a pity and could have been avoided, if only it had been made clear from the beginning that geometry is conceptual and that the world we consider external and physical does not provide geometrical entities but perceptual situations from which we may abstract them.
- ¹⁰ The same, incidentally, goes for ‘location’. The visual focus of attention has no location, except relative to things one has isolated in the visual field. But this would lead to a consideration of the concept of ‘space’, which lies beyond the scope of this discussion.
- ¹¹ Ceccato later discarded this explanation because it did not reduce the concept’s structure to moments of attention (Ceccato, 1966, p. 500). However, as an experiential scenario that might lead to the abstraction of the concept it is still a good example.
- ¹² To an English reader, ‘to draw a line’ is most likely to suggest the appearance of a mark on paper or on some other surface. But Kant’s emphasis shows that he wanted to draw attention to the action of the hand.

CHAPTER 19

- ¹³ I was delighted to discover, on rereading Beth's 1961 essay, that this idea was contained in a passage he quoted from a French philosopher of the 1920s, which I do not remember having read before:

To deduce is to construct. One demonstrates only hypothetical judgments; one demonstrates that one thing is the consequence of another. For that purpose one uses the hypothesis to construct the consequence. The conclusion is necessary, ...They (the premises) are propositions that have been admitted beforehand, either by virtue of preceding demonstrations or as definitions or postulates. (Goblot, 1922, as quoted in Beth & Piaget, 1961, p. 21).

If not explicitly hypothetical, the propositions used as premises may be inductive inferences or results of prior deductions from inductive inferences; and since inductive inferences cannot be considered logically certain, they are in this context equivalent to hypotheses. Also, it should be stressed that certainty, in this context, concerns the derivation of the conclusion from the premises and not the traditional notion of truth relative to a 'real' world.

- ¹⁴ Cf. Euler, 1770, Letters 102-108 (February 14, to March 7, 1761).
- ¹⁵ It seems to me that this faith in human memory is not essentially different from the confidence we are expected to have in the functioning of a computer that has carried out a 'proof' that would take a human longer than a life time. (cf. Tymoczko, 1979).
- ¹⁶ About the nonverbal mental images that mathematicians make use of, Einstein said: in his case, they were "visual and some of muscular type" (in Hadamard, 1954, p. 143).

THE CONCEPTUAL CONSTRUCTION OF TIME

Before Kant, one might say, we were in time; now time is in us. (Arthur Schopenhauer, 1851, p. 81)

Let me say at once, the tentative and rather speculative ideas I am presenting here are not intended to answer the question of what time *is*, in an ontological sense. I am interested in how the thinking mind might come to have a *concept* of time. This concept has been a problem from the beginning of Western philosophy. It was implicit in the irreconcilable conflict between Parmenides' notion of an eternal, changeless world of being and Heraclitus' world of unceasing flux. The history of modern science, and especially biology—as Stephen Gould so convincingly argues—manifests the dichotomy expressed by the contrasting metaphors of 'Time's arrow and Time's cycle', which he took as title for his book. As Gould (1987) remarks:

We often try to cram our complex world into the confines of what human reason can grasp, by collapsing the hyperspace of true conceptual complexity into a single line, and then labeling the ends of the line with names construed as polar opposites ... (p. 191)

What human reason grasps, it grasps by conceptualizing, and then relating the results in its thinking. I therefore want to change Gould's statement and say, it is the hyperspace of experience that we collapse into our concepts. In this sense, knowledge is always our own construction.

Jean Piaget has shown us that it is more fruitful to ask how we construct the notions and conceptual schemes that enable us to deal with the experiential world than to engage in the ultimately undecidable metaphysical debates about the nature of reality.

As you all know, Piaget developed a theory of cognition that is in many ways compatible with Kant's approach to reason. But he vigorously opposed Kant's notion that the ideas of space and time are given a priori. Consequently, he developed a model to show the possibility of constructing a concept of time that is neither innate nor, as evolutionary epistemologists claim, predicated on adaptation to a universe presumed to be temporal in itself. The section on time in Piaget's fundamental work, *La construction du réel chez l'enfant*, ends with two important conclusions:

L'enfant, devenant capable d'évoquer des souvenirs non liés à la perception directe, parvient par cela même à les situer dans un temps qui englobe toute l'histoire de son univers.¹

... la durée propre est située par rapport à celle des choses, ce qui rend possible à la fois l'ordination des moments du temps et leur mesure en relation avec les points de repère extérieurs.² (1937, p. 306)

In his inimitable way, Piaget showed in this section of the book, how he developed these conclusions by minutely observing his children, Laurent and Jacqueline. As in all construction of conceptual models, Piaget begins by observing cognizing organisms and then conjectures and analyzes their experiences. This is necessarily speculative and at best hypothetical. First, because in each and every one of us, rational reflection begins much later than the construction of the first basic schemes that enabled us somehow to order and provisionally systematize experience and observations. Second, because when we eventually begin to reflect upon conceptual operations, we are already well accustomed to the use of language and its metaphors. Even the most meticulous scientist cannot be aware of all the implications of the words he or she uses in an analysis. Common expressions such as 'history' and 'duration' implicitly involve the notion of *continuity*, and we therefore tend to think of time as a *flow*. It is an image we tacitly accepted when we first heard it said that 'time goes by' or '*le temps passe*'; and the image persists regardless of whether we visualize time as an arrow or as a cycle.

I believe that the image of time moving and 'going by' is misleading. What goes by are our experiences.

We know that while we are experiencing one thing, we cannot experience another. Experiences, therefore, are manifestly sequential. But they do not move like boats floating down a river. There is no moving substrate that carries them along. Each one is superseded by another—just as are the words when you are reading or, indeed, listening to me now. There are no intervals filled with the flowing of time. Some items are retained in memory, others not. If there are gaps in retrospect, we know that they were filled with other experiences, or sleep.

A conceptual model of the operations that generate a particular piece of knowledge must not take any image for granted. I repeat—I am focusing on *knowing*, not on the metaphysical question of what things *are*.

From my point of view, the passages I have quoted from Piaget provide an excellent description of the general process, but they do not cover certain details. There is one point which, although Piaget touches upon it in the context of 'object permanence' (*ibid.*, p. 75), I have not found explained anywhere in his writings: it is the construction of the kind of *continuity* that is implicit in concepts such as history, duration, and flow.

All authors I have read agree that the relation of continuity is a crucial component of the concept of time. It, too, must be analyzed in terms of sensorimotor experience, mental operations performed with this material, and reflective abstraction from these operations.

The model I am proposing uses elements which, although not those actually used by Piaget, could be seen as compatible with his. My conceptual analyses are built on the presupposition that, because experience is inherently sequential, abstract concepts begin with a reflection upon sequences constituted by moments of attention.

The idea to characterize events by sequences of static situations, like the single frames of a film, was formulated by Silvio Ceccato at the Cybernetics Center in Milan in the 1950s, while we were working on a project to make the meaning of verbs accessible to a computer.³ Only recently did I discover that Henri Bergson (1907), half a century earlier, suggested that ordinary knowledge was cinematographical (p. 331). Later, however, I was told that he discarded that notion and adopted the metaphor of a tune. This seems very appropriate. Although the sounds of the notes are separate, the continuity (and, by implication, duration) arises because each is modified by the preceding one.

In a tune, there is, indeed, a connection constructed with the help of the successive interaction and modification of sounds. But this is an empirical abstraction from the recurrence of individual notes and its functions in the manner of links in a chain. I would call it *sensorimotor connection*, because it does not bestow continuity on other notes or other elements. It links changes and creates a succession. Piaget describes this in the context of experiments with moving objects (Piaget, 1927, p. 67). Although this provides suitable material for the construction of a concept of time, it does not itself involve that concept.⁴

What I am pursuing is, I think, close to what the mathematician William Rowan Hamilton called 'pure time' and described in his *Mathematical Papers* (1832) as:

... distinguished on the one hand from all actual Outward Chronology (or collections of recorded events and phenomenal marks and measures), and on the other hand from all Dynamical Science (or reasonings and results from the notion of cause and effect). (In Hankins, 1976, p. 343)

The frame metaphor of cinematography helps to illuminate a relation that remains implicit in the patterns of change such as musical tunes. In a succession of separate frames, connections between two or more frames are not *given*. They may be suggested by the quality of their content and thus give rise to an empirical abstraction. But such an abstraction merely links a specific succession of static frames, such as successive individual sounds that compose a tune. The frames are static, and only an active mind can supply a relational concept beyond the simple succession.

A further act of reflection is needed to separate the pattern of connectedness from the specific material, so that it may become available as a general concept of continuity and duration.

Piaget (1937) indeed mentioned the need of such an act of reflective abstraction in a passage in which he explained how an object finally acquires its 'permanence':

En effet, par le fait même qu'il entre dans le système des représentations et des relations abstraites ou indirectes, l'objet acquiert, pour la conscience du sujet, un nouveau et ultime degré de liberté: il est conçu comme demeurant

identique à lui même quels que soient ses déplacements invisibles ou la complexité des écrans qui le masquent. (p. 75)⁵

It is the ability to re-present the object to oneself when it is not actually available in the perceptual field, that leads to the conception of a maintained identity. Piaget provides an important hint when he locates the origin of this ability in the child's experience of a moving object that is temporarily hidden by a screen. In that situation, the child tracks the object visually, continues the tracking motion of her eyes when the object disappears behind a visual obstacle, and picks it up again when it reappears. This has been demonstrated experimentally and it shows that the connection between the object before and after its disappearance is provided by the continuous motion the child carries out with her eyes. But this, too, is still a sensorimotor continuity. It is not unlike the continuity of a tune, where the preceding notes, because they continue to reverberate, are linked to the following ones.

In the construction of permanence, it is essential that the object, when it is perceived *again*, is taken to be the self-same individual as before—not merely an object that happens to be like the former one.

Such an individual identity is often tacitly assumed when there are no sensorimotor elements, such as visual tracking, or the reverberation of a sound, to provide an experiential continuity. In these cases it is a purely conceptual construction, and the use of language tends to obscure it. The word 'same' in English (and '*le même*' in French) is a case in point. If we say to our friend Tom: "You are wearing the same shirt Jack is wearing", we are speaking of two shirts; if we say: "You are wearing the same shirt you wore yesterday", we have only one shirt in mind.

In Piaget's terms, both instances could be described as *assimilation*. The results, however, are different, and the difference opens the path to two divergent conceptual constructions. In the first case, Tom and Jack may or may not both be in our visual field. It is irrelevant, because the judgment of sameness is based on the comparison of two separate sensory impressions which, in themselves, entail no permanence. We merely have to be able to re-present to ourselves the former shirt when we see the second. Indeed, this re-presentation can serve as prototype of a class, if we meet other people with shirts that we find 'the same' with regard to the characteristics we have retained from the earlier experience.

In the second case, however, we regard the shirt, as Piaget says, as *demeurant identique à lui même*, that is, as the self-same individual that we encountered before. This individual identity has to be stretched and preserved throughout the interval between our first perception of the shirt and the perception we are assimilating to it now. For us, this interval between the two perceptions was filled with a succession of other experiences (perhaps even a night's sleep). Hence there is no sensorimotor continuity whatever. The shirt's preservation of identity, therefore, extends through a domain *outside* the field of our experience. I have elsewhere (chapter 16, this book) suggested that the creation of this domain eventually serves as foundation for what we think of as 'being' and what philosophers call ontological reality.

Here, however, we are concerned with the concept of time, and in this context I want to stress that, although the imaginary domain where objects can preserve their individual identities provides the opportunity for the construction of time, it does not by itself constitute it. An additional operation has to be carried out, and I return to the example of the shirt to explain it.

When we assimilate Tom's shirt to the one we saw him wear yesterday, we attribute individual identity to it and assume that it is the self-same shirt. The assumption may of course be wrong. Tom might reply that he has half a dozen of these shirts. This would compel us to conceive of a class instead of an identity. But if our assumption of identity is not contradicted, we have to think of a single shirt, one and the same individual that has a continuous connection with the one we saw yesterday. Yet, we have no continuous sensorimotor elements to warrant such a connection, and therefore have to construct it as a continuity *outside* our own field of experience. We have to think of it as a link that is separate but, as it were, parallel to the succession of experiences we have had in the interval between the two shirt-perceptions. We remember the succession of our actual experiences as continuous and having a sequential order, and we can now project the pattern of sequentiality on the imaginary line that preserved the shirt's individual identity. By this projection we generate a sequentiality without events, an *abstracted flow*. This is what Hamilton (1832) called 'pure time'. In fact, he gave an excellent description of this operation of projection:

... we form the *nearest approach* to the idea of time when we think of one order as the mental basis of another, and consider the latter arrangement, which in this view resembles the course of events, as reducible to a mental dependence on the former arrangement which corresponds to the course of time. (In Hankins, 1976, p. 347)

Wittgenstein, incidentally, expressed the same idea in his earliest work, the *Tractatus logico-philosophicus*, which he wrote during the First World War and which, although he later discarded parts of it, contains a great many valuable intuitions. He there wrote: "The description of the temporal sequence of events is only possible if we support ourselves on another process" (1933, §6.361).

Let me end by emphasizing once more that what I have presented here in no way contradicts Piaget's expositions. It is nothing but a slight amplification which, in my view, strengthens his position against Kant's *a priori* and highlights his account of the construction of reality.

ACKNOWLEDGMENT

My thinking was greatly expanded by reading Thomas L. Hankins' wonderfully lucid analysis of Hamilton's ideas (cf. Hankins, 1976).

NOTES

¹ English translation: "As the child becomes able to recall memories unrelated to direct perception, it succeeds in this way to situate them in a time that comprises the entire history of its universe".

CHAPTER 20

- ² English translation: "... the child's own duration is placed in relation to the duration of things, which makes possible at once the ordering of moments of time and their measurement in relation to external reference points".
- ³ We used this method successfully in many semantic analyses, but Ceccato did not publish it until much later; cf. Ceccato and Zonta, 1980.
- ⁴ See my 'Notes on the concept of change' (chapter 16, this book).
- ⁵ English translation: "Indeed, by the very fact that it enters into the system of representations and abstract, or indirect, relations, the object acquires an ultimate degree of freedom in the subject's consciousness: It is now conceived as remaining identical in spite of all its displacements and the complexity of the screens that mask it".

ANTICIPATION IN THE CONSTRUCTIVIST THEORY OF COGNITION

I am not a computer scientist and I do not speak the languages of Quantum Computation, Hyperincursion, or Cellular Automata. But a couple of weeks ago I read some of the prose sections of Robert Rosen's *Anticipatory Systems*, and it gave me the hope that the 'anticipation' referred to in the title of this conference would not be altogether different from the anticipations we depend on in managing and planning our ordinary lives as human beings. It is a topic I have been concerned with throughout the many years that I battled against the mindless excesses of the behaviorist doctrine in psychology. I was involved with languages and conceptual semantics, and I was among those outsiders who thought that, when people speak, they mostly have a purpose and are concerned with the effect their words will have. This view was generally considered to be unscientific, and I am therefore very happy that 'anticipatory systems' have now become a subject for open discussion among *hard* scientists.

Since most of you are probably deeply immersed in specialized research of a more or less technical nature, you may not have had occasion to consider that anticipation would have to be a fundamental building block of any theory of psychology that merits to be called cognitive. And you may not have had reason to wonder why the discipline that is now called Cognitive Science still has not moved very far from the input/output, stimulus/response paradigm. But although everyone now agrees that intelligence and intelligent behavior are the business of a mind, few are ready to concede much autonomy to that rather indefinite entity.

I do not intend to bore you with a survey of a situation that, to me, seems quite dismal. Instead, I shall focus on the one theory that, in spite of all sorts of shortcomings, is in my view the most promising basis for further development, and consequently the most interesting for people involved in the construction of autonomous models.

The theory I am going to talk about is the one Jean Piaget called Genetic Epistemology. The name was not chosen at random. He wanted to make clear that he intended to analyze knowledge as it developed in the growing human mind, and not, as philosophers usually have done, as something that exists in its own right, independent of the human knower. The name should have warned psychologists that Piaget's theory was not merely a theory of cognitive development, but also constituted a radically different approach to the problems of knowledge. However, especially in the English-speaking world, Piaget was mostly considered as a child

psychologists, and his readers disregarded his break with traditional Western epistemology.

This neglect was unfortunate. Piaget actually provided a theoretical model of the human cognitive activity that is more complete and at the same time contains fewer metaphysical assumptions than those of most philosophers.

Towards the end of his working life, which lasted over more than six decades, he said:

The search for the mechanisms of biological adaptation and the analysis of that higher form of adaptation which is scientific thought [and its] epistemological interpretation have always been my central aim. (Piaget, in Gruber & Vonèche, 1977, p. XII)

The term ‘adaptation’ is the salient point. In many of his writings (he published over 80 books and several hundred articles) he reiterates that what we call knowledge cannot be a representation of an observer-independent reality. And every now and then, as in the passage I quoted, he says that the human activity of knowing is the highest form of adaptation. But he rarely put the two statements together—and this may have made it easier for both his followers and his critics to ignore the revolutionary conceptual change his theory was demanding.

If you consider that in the context of the Darwinian theory of evolution, ‘to be adapted’ means to survive by avoiding constraints, it becomes clear that, for Piaget, ‘to know’ does not involve acquiring a picture of the world around us. Instead, it concerns the discovery of paths of action and of thought that are open to us, paths that are viable in the face of experience.

The passage I quoted also indicates that there is more than one level of adaptation. On the sensorimotor level of perception and bodily action, it is avoidance of physical perturbation and the possibility of survival that matter. On the level of thought we are concerned with concepts, their connections, with theories and explanations. All these are only indirectly linked to the practice of living. On this higher level, viability is determined by the attainment of goals and the elimination of conceptual contradictions.

To understand Piaget’s theoretical scaffolding it is indeed indispensable to remember that he began as a biologist. He knew full well that the biological, phylogenetic adaptation of organisms to their environment was not an activity carried out by individuals or by a species. It was the result of natural selection, and natural selection does nothing but eliminate those specimens that do not possess the physical properties and the behavioral capabilities that are necessary to survive under the conditions of the present environment. All organisms that are equipped with senses and the ability to remember sensory experiences can, of course, to some extent individually increase their chances of survival by practical learning. Traditionally this was considered a separate domain, and it was explained by association. Piaget, however, who focused on human development, connected it to the biological principle of the reflex.

In most textbooks of behavioral biology, reflexes are described as automatic reactions to a stimulus. Piaget took into account two features that are usually not

mentioned. The first was that the existence of heritable reflexes could be explained only by the fact that a fixed reaction, acquired through an accidental mutation, produced a result that gave the individuals who had it, an edge in the struggle for survival.

It is important to see that the specific property or capability that constitutes the evolutionary advantage has to be incorporated in the genome before the conditions arise relative to which it is considered adapted. Remaining aware of the role of its result, Piaget thought of a reflex, as consisting of three elements:



The addition of ‘expectation’ sprang from the second observation Piaget had made, namely that most if not all the reflexes manifested by the human infant disappear or are modified during the course of maturation. The ‘rooting reflex’, for instance, that causes the baby to turn its head and to begin to suck when something touches its cheek, goes into remission soon after nourishment through a nipple is replaced by the use of cups and spoons.

Piaget also found that new ‘fixed action patterns’ can be developed. Such acquired reflexive behaviors are an integral part of our adult living. Among them are the way we move our feet when we go up or down stairs, the innumerable actions and reactions that have to become automatic if we want to be good at a sport, and, of course, the rituals of greeting an acquaintance and of small talk at a cocktail party. There are also reflexes that may lead to disaster—for example the way we stamp our foot on the brake pedal when an unexpected obstacle appears before us on the road.

An acquired reflex that impressed me much when I was young, was the one developed by the adolescent men and women of societies that prescribed skirts for females and trousers for males. In a sitting position, these women would unconsciously spread their skirt when something was thrown to them, whereas the men would clamp their knees together. (In those days, this was still used in the strictly male monasteries of Greece and Macedonia, in order to detect female intruders. Today, they have presumably thought of another test.)

Anyway, the more sophisticated view of the reflex enabled Piaget to take the tripartite pattern of perceived situation, action, and result as the basis for what he called ‘action scheme’. It provided a powerful model for a form of practical learning on the sensorimotor level that was the same, in principle, for animals and humans.

Studies of animal behavior had shown that even the most primitive organisms tend to move towards situations that in the past provided agreeable experiences rather than towards those that proved unpleasant or painful. Humberto Maturana (1970a) has characterized this by saying:

A living system, due to its circular organization, is an inductive system and functions always in a predictive manner: what happened once will occur

again. Its organization (genetic and otherwise) is conservative and repeats only that which works. (p. 15-16)

This was not intended to imply that primitive living organisms actually formulate expectations or make predictions. It was a sophisticated observer's way of describing their behavior. The pattern of learning, however, is the same as in Piaget's scheme theory, and once we impute to an organism the capability of reflecting upon its experiences, we can say that the principle of induction arises in its own thinking.

This principle has its logical foundation in what David Hume called the 'supposition that the future will resemble the past' (1748, Essay 3, Part 2). Having observed that, in past experience, situation A was usually followed by the unpleasant situation B, an organism that believed that this would be the case also in the future, could now make it its business to avoid situation A. Together with its inverse (when situation B is a pleasant one, and A therefore leads to pursuit rather than avoidance), this is perhaps the first manifestation of anticipatory behavior.

To be successful, however, both pursuit and avoidance have to be directed by more or less continuous sensory feedback, and this, too, involves a specific form of anticipation. In their seminal 1943 paper, Rosenblueth, Wiener, and Bigelow wrote:

The purpose of voluntary acts is not a matter of arbitrary interpretation but a physiological fact. When we perform a voluntary action, what we select voluntarily is a specific purpose, not a specific movement. (p. 19)

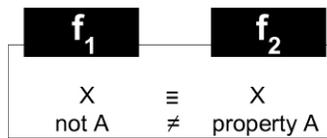
In their discussion of purposeful behavior, they used the example of bringing a glass of water to one's mouth in order to drink. The term 'negative feedback', they explained, signifies that "the behavior of an object is controlled by the margin of error at which the object stands at a given time with reference to a relatively specific goal" (ibid.). Such goal-directed behavior, however, has another indispensable component. In order to 'control' the margin of error indicated by the feedback—in the given example this would be to reduce the distance that separates the glass from one's mouth—the acting subject must decide to act in a way that will reduce the error. And nothing but inductive inferences from past experience can enable the subject to choose a suitable way of acting.

Let us look at the example more closely. I am thirsty, and there is a glass of water in front of me on the table. From past experience I have learned (by induction and abstraction) that water is a means to quench my thirst. This is the 'voluntary purpose' I have chosen at the moment. In other words, I am anticipating that water will do again what it did in the past. But to achieve my purpose, I have to drink the water. There, again, I am relying on past experience, in the sense that I carry out the 'specific movements' which I expect (anticipate) to bring the glass to my lips. It is these movements that are controlled and guided by negative feedback.

When I reflect upon this sequence of decisions and actions, it becomes clear that the notion of causality plays an important role in the event. All my decisions to carry out specific actions are based on the expectation that they will bring about a change towards the desired goal.

The connections between causes and the changes they are supposed to produce as their effects, have forever been the subject of scientific investigation. The concepts of voluntary purpose and that of goal, however, were branded as remnants of teleological superstition and therefore considered inadmissible in the domain of science. The advent of cybernetics and the successful construction of goal-directed mechanisms has demonstrated that the proscription was unwarranted. Yet, today, there are still a good many scientists who have not fully appreciated the theoretical revolution. It therefore seems worth while to provide an analysis of the conceptual situation.

Many years ago, Silvio Ceccato, the first persistent practitioner of Bridgman’s operational analysis, devised a graphic method of mapping complex concepts by means of a sequence of frames (I have borrowed the term ‘frame’ from cinematography; cf. Glasersfeld, 1974b). Because no single observation can lead to the conclusion that something has changed, we need a sequence of at least two frames showing something that acquires a difference. Consequently, the mapping has the following form:



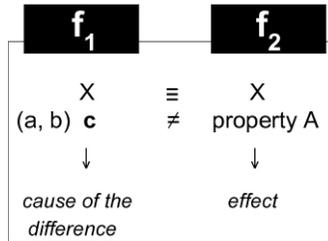
where ‘X’ represents an item that is considered to be the same individual in both frames (indicated by the identity symbol ‘ \equiv ’). In short, we maintain an item’s individual identity throughout two or more observational frames, and, at the same time, we claim that in the later frame it has gained a property ‘A’ that it did not have in the earlier one (or we claim that it lost a property it had before).

The condition of identity may seem too obvious to mention, but analytically it is important to make it explicit, because of the ambiguity of the expression ‘the same’. In English we say, “This is the same man who asked directions at the airport”, and we mean that it is the same individual; but we might also say to a new acquaintance, “Oh, we are driving the same car—I, too, have an old Beetle!” and now we are speaking of two cars. In the first case, we could add, “Look, the man has changed—he’s had a haircut!”. In the second case, we cannot speak of change although our car is blue, and the other’s yellow.

In French, the ambiguity of ‘le même’ is analogous, and in German and Italian, although two words would be available to mark the conceptual difference, their use is quite indiscriminate. In fact, the situation in all these languages is worse, because common usage has modified the meaning of ‘identical’ so that it can refer to the similarity or equivalence of two objects as well as to the individual identity of one.

Without the conception of change there would be no use for the notion of causation. It arises the moment we ask why a change has taken place. I have suggested that this question most likely springs from the fact that we attribute a new property (or the loss of a property) to an item which we nevertheless want to consider one and the same individual. This has the appearance of a contradiction,

and we are looking for a way to resolve it. Because the frames indicating a change represent sequential observations, such a reason has to be found in the earlier one. We therefore examine what else could have been perceived in frame 1. We may compare the experience to others we remember, in which X remained unchanged throughout several frames, and try to find something that was present this time, but not present when X did not change. Or we may behave like scientists and replicate the situation of f_1 , adding one by one new elements a, b, c, ..., to isolate something we can hold responsible for the change. If we find one, we can map it as follows:



If we really are scientists, we will run all sorts of experiments in order to construct a theoretical model that shows how the element 'c' effects the change. If we are successful, we will proudly add 'c' to the tools we use in attempts to modify the world.

In everyday living, we are not so meticulous. If we find that some element was present two or three times when a given X changed in a desirable way, we are likely to assume that it is the cause, and we will use that element in the hope that it will bring about the desired change. Even if it doesn't, it may take a number of failures to discourage us. If someone provides a metaphysical reason why it should work, failures do not seem to matter at all.

I am not sure, but I think it was the literary critic Cyril Connolly who made a startling observation in this regard. Insurance companies, he said, have the most sophisticated questionnaires to assess the risks involved in issuing a life insurance policy to an applicant. The questionnaires are based on meticulous research of mortality statistics. Connolly was struck by the fact that the questionnaires never ask the question "Do you pray?". And he wondered why people continued to pray for survival in all sorts of crisis, when the greatest experts of mortality clearly had found no evidence that it had an effect.

All this involves anticipation. The use of a cause-effect link in order to bring about a change is based on the belief that, since the cause has produced its effect in the past, it will produce it in the future. We project an established experiential connection into the domain of experiences we have not yet had. Hume has explained how we establish such connections: the repeated observation that the two items happened in temporal contiguity led us to infer and formulate a rule that says, if A happens, B will follow. Therefore, if we want B to happen, we try to generate A. In other words, we have a purpose and we act in a way which, we believe, will attain it.

The psychological establishment, which, from the 20s of this century until well into the 70s, was dominated in the United States by the dogma of behaviorism, considered purpose a mentalistic superstition. "Careless references to purpose are still to be found in both physics and biology, but good practice has no place for them", Skinner still wrote in 1971 (p. 8).

Natural scientists in physics, chemistry, and astronomy had found no reason to engage in thoughts about purpose in their disciplines, and they relegated it summarily to the realm of teleology. As Ernest Nagel (1965) put it:

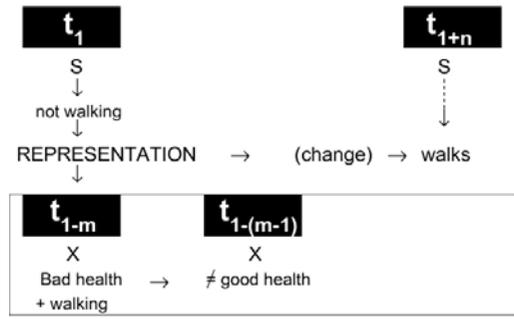
Perhaps the chief reason why most contemporary natural scientists disown teleology, and are disturbed by the use of teleological language in the natural sciences, is that the notion is equated with the belief that future events are active agents in their own realization. Such a belief is undoubtedly a species of superstition. (p. 24)

To believe that the future affects the present is no doubt a superstition, but to declare that purpose and goal-directed action must be discarded because they are teleological notions is no better. It shows an abysmal ignorance of the difference between empirical and metaphysical teleology.

I have suggested elsewhere that Aristotle, who provided the most valuable analysis of the concepts of causation, was well aware of the ambiguity. In his exposition, it becomes clear that what he called 'final' cause, that is, the embodiment of a telos or goal, had two quite distinct applications (Glaserfeld, 1990a). On the one hand, he saw the religious metaphysical belief that there was a telos, an ultimate, perfect state of the universe that draws the progress of the world we know towards itself. On the other, there was a second notion of the final cause, which he exemplified by saying that people go for walks for the sake of their health (in McKeon, 1947). This was a practical explanatory principle for which there is, indeed, an overwhelming amount of empirical evidence.

In this practical manifestation of finality, no actual future state is involved, but a mental re-presentation of a state that has been experienced as the result of a particular action. Even in Aristotle's day, bright people had noticed that those who regularly took some physical exercise such as walking, had a better chance of staying healthy. They had observed this often enough to consider it a reliable rule. Given that they had Olympic games and were interested in the performances of athletes, they probably also had some plausible theory of why exercise made one feel better. Consequently, they were confident in believing that going for walks was an efficient cause that had the effect of maintaining and even improving your health. People who felt that their physical fitness was deteriorating could, therefore, reasonably decide to use walking as a tool to bring about a beneficial change in their condition.

If I use the method of sequential frames to map this conceptual situation and give temporal indices to the individual frames, I get the following diagram:



‘S’ is the person who wants to improve his/her health. Believing that in the past [t_{1-m} to $t_{1-(m-1)}$], other people ‘X’ have caused their health to improve by walking, the person decides to take a walk.

Thus, the cause of S’s change of state, from not walking to walking, does not lie in the future. Instead, the operative cause of the person’s change of state is a rule that was empirically derived in the past by means of an inductive inference from observations. It is a rule that is based on the notion of efficient cause, and the person is anticipating that it will be efficacious also in the future.

There is nothing mysterious or superstitious about this way of proceeding and it certainly is not unscientific. In fact, it is no different from what microbiologists do when they place a preparation under the microscope for the sake of seeing it enlarged; or from what astronomers do when they carefully program the mechanism of the telescope to track the star they want to observe. But there is nothing particularly scientific about these ways of proceeding either. They are analogous to what we as ordinary people do all day long. We turn a door handle and expect the door will open, we put a seed into soil and anticipate a flower, and every time we flip a switch, we anticipate a specific effect. None of these maneuvers works every time, but success is frequent enough to maintain our faith in the viability of the respective causal connections.

Yet, we may call all these anticipatory procedures teleological, because they involve goals which, by definition, lie in the future for the individual actor. When I planned to take a particular train from Brussels, my goal was to be in Liège at a future moment. But it was not this future goal that would cause me to arrive there. Instead it was the expectation that the train scheduled to go to Liège would do once more what it had done often enough to warrant my anticipation that it would do it again.

CONCLUSION

One of the slogans of Skinner’s behaviorism was: “Behavior is shaped and maintained by its consequences” (1971, p. 18). Hence it would be affected by what happens after it. But there is no mysterious time reversal—only a specious description of what actually goes on. And Skinner was wrong to condemn teleology, insofar as he was speaking of intelligent organisms. Their behavior is

shaped by the consequences their actions had in the past, and that is precisely what constitutes the ‘empirical’ teleology I have discussed above. As Maturana said, we “function in a predictive manner: what occurred once will occur again”. In other words, we can learn from our experience and abstract regularities and rules regarding what we can expect to follow upon certain acts or events.

The mysterious feature is our ability to reflect on past experiences, to abstract specific regularities from them, and to project these as predictions into the future. This pattern covers an enormous variety of our behaviors. I would suggest that there are three related but slightly different kinds of anticipation involved.

1. Anticipation in the form of implicit expectations that are a prerequisite in many actions. For instance, the preparation and control of the next step when we are walking down stairs in the dark; this does not require the prior abstraction of cause-effect rules, but it does require familiarity with specific correlations of actions in prior experience.

2. Anticipation as the expectation of a specific future event, based on the observation of a present situation; this is a prediction derived from the deliberate abstraction from actions and the consequences they had in past experience.

With regard to these two forms of anticipation, one may say that robots and other artificial mechanisms are today able to simulate them, if not actually, at least theoretically.

3. But there is a third form: Anticipation of a desired event, situation, or goal, and the attempt to attain it by generating its cause. This, too, is based on the abstraction of regularities from repeated correlations in past experience, and it is not considered scientific without a conceptual model that explains the cause-effect connection. But even where we have such a model, its simulation presents a problem. When we, the human subjects, pursue a goal and attempt to attain it by using regularities we have abstracted from our past experience, we ourselves have chosen the goal because we desired it. In contrast, Deep Blue, the chess program that can beat a human master and has in its repertoire thousands of cause-effect connections, does not know why it is playing or why it should be desirable to win.

The first two forms of anticipation I have listed are sufficient to say that, without them, every step we take would be a step into terra incognita. We would fall into precipices, crack our heads against walls, and only by accident would we find something to eat. In short, we may conclude that without anticipatory systems, there would be no life to speak of on this planet.

The third form, that involves the choice of goals, has an ominous significance. If we act in the belief that the future consequences of our actions will be similar to what they were in the past, we ought to be careful about the goals we choose. For whenever we attain them, these goal states may have further consequences, and they will be our responsibility. Therefore, if we don’t heed the ecological predictions scientists compute today, we may have nothing to anticipate tomorrow.

A CONSTRUCTIVE APPROACH TO ‘UNIVERSALS’

But it is beyond the power of human capacity to frame and retain distinct ideas of all the particular things we meet with: every bird and beast men saw; every tree and plant that affected the senses, could not find a place in the most capacious understanding. (John Locke, 1690, Book III, Chapter III, §2)

During the Middle Ages, much of the debate about universals concerned the question whether or not they could be considered to ‘exist’. According to their point of view, the discussants could be separated at least roughly into four schools.

1. **Platonic Realists**, who characterized their position by the slogan *universalia ante res*; they believed that we have general concepts *before* we experience things.
2. **Aristotelian Realists**, whose slogan was *universalia in rebus sunt*, because they considered them inseparably inherent in things.
3. **Nominalists**, who maintained that words are the only universals, because they can be applied to a variety of things.
4. **Conceptualists**, for whom universals arose *post res*, that is, after the experience of things because they were generated by abstractive thinking from particulars.

All four schools have their problems.

According to 1, all the universals that cover inventions and newfangled things, such as windmills, chastity belts, highways, quarks, and credit cards, would have to have ‘existed’ at the very beginning. Of course, if you believe with Plato that God supplies every newborn with the full complement of necessary ideas, there is no problem, because God, being omniscient, knows all human inventions before they are made. But for modern thinkers this is not a congenial model.

The Aristotelian notion (2) that the universals are already in the particular things does not answer the crucial question of how they manage to become *general*, given that we never meet them *except in particulars*.

Nominalism (3) still does not explain how it comes about that we find more or less large groups of particulars that can be subsumed under one universal word.

From my constructivist perspective, number 4, the conceptualist approach, is the most interesting; but in the form it was expounded by the empiricists, who partially relied on the nominalist’s assumption, it is not quite satisfactory either. I shall therefore lay out a modified brief version of a historically much longer conceptual development that I consider more in keeping with recent ideas.

The abstractive thinking, that produces universal ideas, was well described, for instance, by Descartes:

If, as has often to be done, one thing be deduced from a number of things, we must remove from the ideas of things whatever does not require present attention, so that the remaining features may be the more readily retained in memory. (1701, Rule XII, p. 56)

The epigraph I have placed at the beginning of this paper is Locke's compelling statement of the *need* for such abstractions. I would suggest that most, if not all, would agree with this statement irrespective of which of the four approaches to universals they subscribe to. Even Berkeley, who vigorously fought against the notion of 'abstract general ideas' that were supposed to have *existence* in their own right, freely admitted the generalizing use of words (Berkeley, 1710, Introduction, §12).

In fact, the whole debate about universals is not about whether or not, say, the name 'triangle' can be applied to an infinite variety of three-cornered shapes, but rather about whether or not the abstraction it designates actually represents an observer-independent entity. Berkeley makes this quite explicit and it is therefore useful to quote him at some length:

It is, I know, a point much insisted on, that all knowledge and demonstration are about universal notions, to which I fully agree: but then it doth not appear to me that those notions are formed by abstraction in the manner premised—*universality*, so far as I can comprehend, not consisting in the absolute, positive nature or conception of anything, but in the relation it bears to the particulars signified or represented by it; by virtue whereof it is that things, names, or notions, being in their own nature *particular*, are rendered *universal*. Thus, when I demonstrate any proposition concerning triangles, it is to be supposed that I have in view the universal idea of a triangle; which ought not to be understood as if I could frame an idea of a triangle which was neither equilateral, nor scalenon, equicrural; but only that the particular triangle I consider, whether of this or that sort it matters not, doth equally stand for and represent all rectilinear triangles whatsoever, and is in that sense *universal*. (*ibid.*, §15)

There follows more than a page in which Berkeley tries to substantiate (not very successfully, I think) that, having demonstrated that the sum of the angles in one type of triangle is equal to two right angles, he can extend this finding to all types of triangle, because the length of the sides and the size of the angles do not play any role in the proof. Apparently he himself was not satisfied with his argumentation, for in 1732 he added the following passage to his last edition of the *Principles*:

And here it must be acknowledged that a man may consider a figure merely triangular, without attending to the particular qualities of the angles, or relations of the sides. So far he may abstract: but this will never prove that he can frame an abstract, general, inconsistent idea of a triangle. (1732 addition to 1710, §16)

To assume such a partial abstraction seems indispensable because we all are able to *recognize* as triangles, figures whose sides and angles are not all the same. In this respect, then, Berkeley was a nominalist. A name, he says, though it is the name of a particular, can play the part of a sign, and thus become general (1710, Introduction, §12).

But this leaves open the question of how it comes about that different particulars can be designated by the same sign. For realists, Platonic or Aristotelian, this is no problem because they take it for granted that things-in-themselves manifest similarities that make it possible to sort them into 'natural kinds'. For nominalists and conceptualists, however, this can hardly be a forgone conclusion, because in their model there is no room for generality outside language.

I do not want to say that Berkeley did not manage to wriggle out of this dilemma; but it is by no means clear to me how he did it. In my view he created the problem himself, when he wrote, in the context of universals: "But my conceiving or imagining power does not extend beyond the possibility of real existence or perception" (1710, Principles, §5). For the reader, the statement is confusing because the 'or' makes it ambiguous. One could interpret it as though existence and perception were different affairs; but in Berkeley's model it is exclusively *being perceived* that generates existence (*esse est percipi*). For Berkeley himself, the statement is a trap, because it links 'conceiving' and 'imagining.' For me these are distinct capabilities which, although they sometimes manifest themselves simultaneously, do not coincide on the level of operations. We are able to conceive of many things whose composition is 'abstract', because in principle they lie beyond perception. We have, for instance, a concept of negative numbers although we do not perceive them; we can apply the concept of 'lie' without specifying a truth that is being countermanded; and we can, indeed, form a concept of triangle without imagining specific side lengths and specific angles.

This last example can serve as a useful illustration. Let us suppose you are at the entrance to a field and you begin to walk in a straight line; at a certain point you make a turn to the right, walk another stretch straight, make a second turn to the right, and return in a straight line to the entrance. You know that your walk has described a triangle, irrespective of the length of the straights and the size of the angles. You performed a program of construction that operationally determines and thus defines a triangle although instead of specific values for sides and angles it contains *variables*. The point is that, even in perception, we do not find ready-made triangles but have to build them up from smaller elements.

Berkeley was clearly very close to this insight, when he wrote that one "may consider a figure merely triangular, without attending to the particular qualities of the angles, or relations of the sides". But he was blocked by two conventional notions that prevented him from further elaborating on his statement. Although he had realized that items such as space, time, and number were "things of the mind", he apparently maintained the traditional belief that both concepts and mental images were static and unitary. This was the first blocking notion. The second originated in the indubitable experience that it is impossible to *imagine* a triangle without substituting specific values for the variables in the above definition. Indeed, the operational definition can serve for the *recognition* of triangles,

because the missing values are supplied by the very act of perception. But it is impossible to visualize a triangle that should be equilateral, isosceles, and scalene at the same time.

I have elsewhere shown that the ability to recognize something is but a precursor of the ability to imagine that same thing (Glaserfeld, 1979, 1995). Daily experience provides innumerable examples of this. We all have many acquaintances of whom we are unable to produce a mental image, although we recognize them when we meet them in the street; similarly, for all of us there are words of our language (and many more of a second language) whose meaning we know when we hear or read them, but they are nevertheless unavailable to us when we are speaking or writing.

This difference between concept and mental image does, I believe, open a fertile new path in the debate on universals, and the constructivist approach provides a plausible explanation.¹ If one relinquishes the usual assumption that concepts must be conscious, static entities, it becomes possible to show their construction with the help of an empirical example. Lettvin, Maturana, Pitts, and McCulloch (1959) have demonstrated that the nervous system of a frog discovers a possible prey whenever four different fibers of the optical nerve conduct impulses in a certain sequence to the ganglion cells which, in a manner of speaking constitute the frog's center of command. The single impulses are reactions to stimuli which can be described respectively as: (1) local light-dark contrast, (2) convexity of a small dark object, (3) movement of a dark shape, and (4) sudden darkening of the local visual field. Given this arrangement, 1, 2, and 3, react to anything that behaves like a small insect on a light background. Jointly they trigger the action with which the frog captures its prey (according to the species of frog, jumping and snapping or shooting out the tongue). Impulse 4 serves to impede the action whenever a sudden darkening signals the possibility of a looming danger to the frog. Thus one can say that the coordination of 1, 2, and 3, constitutes the matrix that enables the frog to recognize its food—but it also leads to the fact that the frog will react to a small moving shadow or a rolling black pellet as though it were a bug. We are therefore dealing with a construct of the frog's nervous system, and it does not give the frog a picture of reality but merely a sufficiently successful method for finding nourishment in its environment. Were the frog in a position consciously to reflect on its way of operating, one might add that, for it, the program of the three impulses constitutes the operational definition of an edible insect. However, in order to speak of a mental image, one would have to ascribe to the frog the additional ability to implement the program deliberately when the relevant nerve fibers do not supply the required impulses.

It is interesting to note that fulfillment of this last condition is also the criterion on the basis of which Piaget ascribed the achievement of 'object permanence' to children who were about two years old. To recognize an object was not sufficient; the child also had to be able to imagine the object when it was not in the present field of experience. It is characteristic of innumerable developmental psychologists that they claimed to have demonstrated object permanence in much younger

children, cats, and other animals, although their tests in no way tested the ability to imagine an absent object.

Returning to the problem of universals, I would summarize the constructivist approach in the following manner:

Programs of construction in which characteristic elements are coordinated in such a way that they serve to cut pieces out of the flow of experience and to recognize them as belonging to a class, I would call 'recognition matrices' or 'recognition concepts'. Consequently I would suggest to limit the term 'universal' to those concepts that are accessible to a subject's conscious reflection and which the subject can also deliberately call up as mental images.

Of neither the recognition concepts nor of the universal ones could one say that they 'exist', as long as one intends the term 'to exist' as referring to some form of independent *being* in space and time. Both types of concept are repeatable programs and as such have no subsistence in the flow of mental operations and therefore no duration or substance.² They do not belong to the furniture of an independent world.

To conclude, I want to add that, from my constructivist position, no ontological affirmations can be made and therefore there can be no claim that one describes a 'reality'. All that is ever intended is to suggest models of how one might think about certain things.

NOTES

- ¹ The difference I am making here corresponds in part, but only in part, to what Kant explains as the opposition of 'pure' and 'empirical' *Anschauungen*; cf. *Kritik der reinen Vernunft*, B742-744. (The term *Anschauung* is usually translated as 'intuitions', but this does not render the German meaning that is closer to 'view').
- ² Cf. John Dewey's statement that an operation is grasped in thought as a relation which "is independent of the instances in which it is overtly exemplified, although its meaning is found only in the *possibility* of these actualizations" (1960, p.163).

PART IV

COMMENTS

EXPERIENCES OF ARTIFACTS

People's Appropriations / Objects' 'Affordances'

To a constructivist, knowledge is not a mere commodity to be transmitted—emitted at one end, encoded, stored, and reapplied at the other—nor is it information, sitting ‘out there’ and waiting to be uncovered. Instead, knowledge is (derived from) experience, and actively constructed and re-constructed by subjects in interaction with their worlds. In Piaget’s words (1937): “Intelligence organizes the world by organizing itself” (p. 311). In Ernst von Glasersfeld’s terms: “The world we live in ... is always and necessarily the world as we conceptualize it (this book, chapter 1, p. 9).

One of the implications of adopting a constructivist stance is the idea, dear to radical constructivists, that ‘reality’ as we know it “lies forever on the other side of our experiential interface” (ibid., p. 7) and that “knowledge does not [and cannot] represent a [true] picture of the ‘real’ world but provides structure and organization to experience” (ibid., p. 8). In other words, “What determines the value of conceptual structures is their experiential adequacy, their goodness of fit with experience, their viability as means for the solving of problems—among which is, of course, the never-ending problem of consistent organization that we call understanding” (ibid., p. 9).

A second implication is a strong call to abandon the *conduit metaphor* of human communication for what Reddy refers to as the *tool-maker’s paradigm* (Reddy, 1993). At the core of the *conduit metaphor* lays the deeply rooted notion that meaning resides in words, and that words are carriers of meaning (containers), to be conveyed (like on a conveyer belt) between speakers and listeners. The *tool-maker’s paradigm*, by contrast, emphasizes that meaning does not reside in words, texts, or artifacts. Instead, “it is subjective in origin and resides in a subject’s head, not in the word which, because of an association, has the power to call up, in each of us, our own subjective representation” (Glasersfeld, this book, chapter 1, p. 10).¹

In my tribute to Ernst von Glasersfeld, I wish address some of the paradoxes that arise if one adopts a non-critical radical constructivist stance to account for creative people’s interactions with—and through—(hu)man-made artifacts, in particular as they engage in the process of ‘world-making’, to use Goodman’s expression (1978), or designing in a broad sense .

CONSTRUCTIVISM AND THE PRACTICE OF DESIGN

Designing (*progettare* in Italian) can be seen as the flipside of reflective abstraction: an iterative process of mindful concretization, or materialization of ideas (*concrétisation réfléchie* in French).² To design is to give form, or expression, to inner feelings and ideas, thus projecting them outwards, making them tangible.

In the practice of design, the purpose is not to represent what is out there (or model how things are) but to imagine what is not (or envision how things could be) and to bring into existence what is imagined. Creators are fabricators of possibilities embodied: They both make and make-up things! Important here is the notion that a designer's projections emerge through a conversation with—and through—their own and other people's externalizations. The nature and quality of this conversation are a key to all forms of learning, and paramount to intelligence itself. In Schön's terms, learning *is* designing, and designing *is* a conversation with—and through—artifacts (Schön, 1983, p. 76).

In what follows, I explore why treating our own externalizations *as-if* they had an existence, beyond our immediate rapports with them, is a viable mental attribution—even if we know that we cannot know their whereabouts. In substance, expecting our creations to have integrity (lending them autonomy, permanence, identity) while remaining mindful of their qualities (letting them 'speak' and celebrating them for what they are, at the risk of over-interpreting) are two facets of cognitive adaptation, as defined by Piaget.³ Their function is to elevate human transactions (between me/not me) beyond blind projections, or pure assimilation, with its unfortunate consequence: reducing anything that is other to a mirror-of-me (over assimilation).

As Ernst von Glasersfeld remarks: Vico saw long ago that "facts are made by us and our way of experiencing, rather than given by an independently existing objective world. But that does not mean that we can make them as we like. They are viable facts as long they do not clash with experience, as long as they remain tenable in the sense that they continue to do what we expect them to do" (this book, chapter 1, p. 9).

Along with Piaget, I think that attributing, or projecting, meaning in the objects of our experience is a pillar of cognitive adaptation.⁴ In contrast to Piaget, I put more emphasis on the accommodative pole of human adaptation—which does not come without complications within a constructivist framework.⁵

I explore some of the pragmatic, ecological, and epistemological consequences that arise if one opens this Pandora box. I wonder in particular: 1. How can designers take responsibility for the qualities of their creations if they assume—I caricature the constructivist stance—that people will use them as Rorschach stains any way? 2. Beyond setting limits to our actions and/or clash with our expectations, what qualities does it take for an artifact be able to call up, in each of us, our own subjective representation?

PEOPLE AS WORLD-MAKERS AND WORLD READERS

People are 'world-makers'

People spend a great deal of their time carving out their niches—virtual and physical—so that they fit their needs, support their purposes, and augment their potential. They build cities, homes, and furniture, they invent computers and airplanes, and they create alphabets, geometries, and musical scores. Young and old, they give form or expression to their ideas. People are also busy keeping track of their experience and leaving traces behind. They mark their grounds to find their ways, and they use the marks they leave behind as anchors to orient themselves. Newcomers to a culture are left to live with the marks traced by others.

Early on, children learn to set the stages and to build the props, which enable them to best explore, enact, and ultimately work through many intriguing or captivating events. As they reach their second birthday (with the apparition of the symbolic function), most children engage in pretend or fantasy play. As they recast their experience on a make believe-ground (do *as if*), young pretenders bring their feelings and ideas to life and, by the same token, make their dreams come true. This act of projection enables them to revisit personal experience *as if* it was not-theirs, and, through this act of distancing, engage in a dialogue that brings about greater intimacy and deeper understanding.

People are 'world-readers'

People 'read' meaning into existing forms, be they their own or those produced by others, and they do so in creative ways. Readers, in other words, are in no way passive consumers. Instead, they engage designed artifacts by reconstructing them through the lens of their interests and experience.

In the same way that designers are readers (they constantly reedit their drafts), readers are constructors. As Bordwell mentions about film audiences: "The artwork sets limits on what the spectator does. But within these limits, the viewer literally *recasts the play*" (Bordwell, 1986, p. 30). Viewers impose their order upon things by rearranging or replacing clues, by filling in blanks or 'creating phantoms', by ignoring clues, and by forcing causal-temporal connections. In Piaget's words, world-readers assimilate incoming signals (in this case, a narrative unfolding) to feed their views (they interpret them through the lens of previously reconstructed experience) and they accommodate their views only in so far as some unexpected puzzlements or surprises are called upon by the materials.

ARTIFACTS AS INTERLOCUTORS

Artifacts set limits to people's reconstructions

They do so by opening up greater or lesser mental elbowroom (*Spielraum* in German) for personal interpretation. Indeed, to suggest that readers *recast the plot* does not entail that they do so from scratch. Instead, people compose with what is

offered by the materials—one may say with what the materials evoke—to improvise their part.

For a text, like any other artifact, to capture our imagination, it has to embark us in a journey filled with wonders and surprises. At the same time, it needs to provide the right balance of freedom and structure to allow for personal reconstruction. A setting that gives out all its secrets before you even start wondering is disengaging because it excludes you as a partner. A setting that offers no resistance at all is disengaging because it is predictable (too malleable, it mirrors). Both will not have much evocative, or holding power. Objects (*Gegenstaende* in German) stand against our tendency to blindly assimilate them.

Learning as a conversation with artifacts

People learn by switching roles from being producers to being critics, from being actors to being audiences, from holding the stage to moving into the background. People also zone in and out of situations to change their stance. In other words, no matter how embedded we are in a situation there comes a time when we distance ourselves to look at things from afar. Putting on a critic's hat and shifting perspectives enable us to engage our own creations *as-if* they had been produced by 'another' or existed independently, and then, reengage them again (Ackermann, 1996).

As designers converse with their externalizations, they usually start a dialogue with a whole range of interlocutors, imaginary or 'real', to whom they address their work and from whom they borrow, or draw inspiration (Valéry, 1940). As soon as an idea takes shape, or form, it gains both a physical and a social existence (Habraken, 1985). It is not exaggerated to say that while interacting with the world, a creator's mind moves both in-and-out of its own expressions (from production to critic), and back-and-forth from itself (to include viewpoints of others).

MINDS ENGAGE FORMS AND FORMS ENGAGE MINDS: RADICAL CONSTRUCTIVISM IN QUESTION

Strengths

Radical constructivism offers a solid framework to discuss how meaning is constructed and interpreted through form.

Based on the assumption that any organism, to be viable, needs to carve a niche within a world too big to ensure anyone's survival, radical constructivists remind us that if the world appears to us as we see it, it is not because it is made that way, but because we are made the way we are (Watzlawick, 1984)! In other words, reality takes on un-recognizable facets when captured through the prisms of different creatures. Conversely, what we think of as 'shared realities' feel shared because they are produced and recognized by minds of a similar kind.

Ernst von Glasersfeld makes a compelling case for why taken-for-shared stabilities are not true to reality but akin to the organism's survival and growth.

Early on, human infants make sense of their experience by building *stabilities* in the world. These stabilities, or cognitive invariants as Piaget refers to them, are constructs of the mind. Object permanency, conservation of object-size, and the creation of a coordinate system to situate objects and movements in space, are examples of cognitive invariants built by humans in interaction with their world. Creating invariants is a needed self-orienting device invented by intelligent organism to find their ways, or survive (Ackermann, 2004).

Limitations

While offering a solid framework to discuss how meaning is constructed and interpreted through form, radical constructivism leaves partially unanswered the question of how forms engage minds. At the cost of caricaturing, let me put it this way: fellow constructivists seem to ignore that, once launched, an artifact takes on a life of its own, thus transcending both the author's intentions and any singular act of interpretation.

As fellow humans, we share (or think we share) enough common ground that we sometimes forget that “what we know depends on how we came to know it!” (Watzlawick, 1984, p. 9). We then act *as-if* our invented realities had always been there and we rely on them—and refer to them—as tangible and shareable entities (objects to think with, and relate through).

In other words, as they interact with people and things, intelligent beings tend to exaggerate both what they have in common and how stable our world is. While this amnesia is a problem to constructivists, it is also a viable strategy when it comes to designing artifacts for others to live or learn in: Lending autonomy and existence, we posit, sharpens our sensitivity to—and respect of—their qualities, independent of their author's intentions and our immediate relation with them.

EVOCATIVE POWER OF (HU)MAN-MADE ARTIFACTS

Objects' affordances

Norman (1988) introduced the term ‘affordance’ to refer to an object's ability to signal its potential uses. Borrowing from Gibson, he used the word to gauge the qualities of everyday object. To Norman, a poorly designed object is one that fails to signal its built-in affordances. As an example, a lamp that does not tell you how to switch it on, a doorknob that remains ambiguous as to whether it wants to be pushed or pulled, or a panhandle that confuses you: “hold me/do not hold me” (i.e., I burn you if you do, I let you down if you do not).

The concept of affordance, while limited to uses, provides a first step toward understanding how forms engage minds. It highlights that indeed human-made artifacts signal potential usages through their built-in features, or embedded qualities.

Genius loci!

In *Poetics of Space*, the French philosopher Gaston Bachelard further explores why humans can be deeply moved by what he calls *felicitous* objects or places, and why felicitous objects and places can, in turn, be said to *reverberate* atmospheres, or ambiances, in ways that capture human imagination (Bachelard, 1964). To Bachelard, felicitous objects and spaces (things able to transport us) cannot and should not be characterized by functionality alone. Instead, they attract us because they have become topographies of our intimate being. As such, they ‘speak a language’ that enters in resonance with felt human aspirations.

Even a doorknob, to take a previous example, could become a felicitous object if it did not just call up our urge to ‘push or pull to enter’. It too could retain our attention, suspend our breath and, why not, slow down our steps, by conveying something that speaks to, or evokes, our experience of passage between places. Everyday objects could speak a language much more un-tangible and rich, in resonance with our being and aspirations. Ideally, designers could endow objects with the ability to speak such language.

To take a less trivial example, think of the differences between different types of shutters, such as a persienne, a venitienne, and a jalousie. All are shutters yet each calls upon different feelings along the continuums of being/seen, privacy/public, light and darkness. My European friends usually prefer French windows to American guillotines not just because windows can be cleaned and shutters keep light and thieves out, but because the opening of both windows and shutters are along a vertical axe (left and right from the center), allowing arm movements that enter in syntony with breathing in and out, which procures a sense of delight as one opens or closes, letting light and air in.

Bachelard offers important insights into what it takes for (hu)man-made artifacts to enter in syntony with our needs as sensory-motor creatures (smooth to our senses), to tackle our expectations as rational beings (functional), and to ‘speak to’ our aspirations as artists and poets (poetics). He explores the phenomenological touch-points between people’s experiences of intimate things, and thing’s capacity to inspire and ‘transport’ people.

TO CONCLUDE

Constructivism, especially in its radical form, emerges from the growing awareness that any so-called reality is—in the most immediate and concrete sense—the construction of those who believe they have discovered and investigated it. What is supposedly found is an invention whose inventor is unaware of his act of invention, who considers it as something that exists independently of him; the invention then becomes the basis of his world view and actions. (Watzlawick, 1984, p. 10)

This being said, not all ‘constructed realities’ are equally engaging, as partners or habitats. Some feel delightful while others leave us cold. Some please most of us, while others attract only a few.

Like a good conversationalist, a well-designed object is one that ‘knows’ how many surprises and wonders contribute to capture people’s imagination. Like a narrative text, it offers “a dynamic system of competing and mutually blocking retardatory pattern” (Sternberg, 1978, p.177). In Shklovsky’s words, “a narrative text is less like an elevator than a spiral staircase which, littered with toys, dog leashes, and open umbrellas, impedes our progress” (in Bordwell, 1985, p. 38).

From a pragmatic-ecological standpoint, it seems essential for designers of human-made artifacts (from educational software to playgrounds and books) to take responsibility for their products by not assuming—I caricature the constructivist’s stance—that learners will use them as Rorschach stains anyway (projective test). Designers need to acknowledge that their products will survive after them, and that it is ultimately the built artifact, rather than the builder’s intentions, that becomes part of other people’s cultural heritage. It is ‘its’ qualities that will persist to evoke or signal potential uses to newcomers who encounter them for the first time. Denying the power of places and things to impact people can breed a culture of ‘not caring’.

Surely, designers cannot predict how their creations will be appropriated. What designers can, however, is be attentive to the idea that, once conceived, their creations are no longer a mere extension of themselves. Instead, they come to exist as separate entities. They gain both autonomy and an identity as ‘invented realities’, an integral part of the cultural landscape in which other newcomers will live and grow.

People read into artifacts because of who they are, and because artifacts offer clues. Like archeological sites or eroded landscapes, they are marked by their history, and they embody the knowledge or collective experience that went into their being (both reflecting and transcending builders’ and readers’ intents and aspirations). The constructivist’s nightmare may well come true! Yes, human-made artifacts can call upon certain experiences and uses, and discourage others. And they sometime impose their logic, or constraints, beyond the builder’s intents and any particular act of interpretation.

From an epistemological standpoint, it seems equally important for learning researchers and educators to rethink the role of accommodation in cognitive adaptation. To Piaget, we have seen, intelligence is adaptation, and adaptation is the ability to maintain the maximum of what is acquired while opening up to the maximum of novelty. Adaptation, in other words, calls for a balance between openness and closure, assimilation and accommodation.

In Piaget’s own words: “Assimilation is by its very nature conservative, in the sense that its primary function is to make the unfamiliar familiar, to reduce the new to the old” (Piaget, 1954, pp. 352-353). Accommodation, by contrast, shakes and de-crystalizes existing schemes so that they fit to the demands of the environment. Its primary function is to make what is familiar unfamiliar again, and to question the old by listening to the new.

Circular reaction was a term introduced by Baldwin, and used by Piaget, to refer to infants' ability to modify their activity in relation to an unexpected effect, and actively reproduce some interesting events, which were at first discovered by chance. The concept of circular reaction is a concept of major importance to psychologists. According to Flavell, the principle's value "lies in the fact that it is the device *par excellence* for making new adaptations, and of course new adaptations are the heart and soul of intellectual development at any stage" (Flavell, 1963, p. 93).

Radical constructivism, I suggest, gives more weight to the assimilatory pole of human adaptation. While this is an important contribution, especially in an intellectual climate mostly dominated by innatist or empiricist views, it is useful to remember that an organism that solely assimilates is not viable. Such an organism fails because it is insensitive to variations in the environment that may help it adjust its conceptual structures. Such an organism closes up to surface perturbations for 'blindness' to others and inconsideration for odds. Conversely, an organism that solely accommodates is at risk of disrupting its own momentary equilibrium, for an excess of openness to external solicitations. An act in which assimilation and accommodation are in balance constitutes a truly adapted act!

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NOTES

¹ The tool-maker's paradigm, according to Reddy (1993), is a far better candidate than the "conduit" metaphor to understand how meaning is built and read into form. Words are not like little wagons, which transport meaning back and forth between a sender and a receiver (who load and unload them at each end). They are more like a misty landscape, or an obscure blueprint, the contents and textures of which are filled-in by the interlocutors. The tool-maker's paradigm further suggests that people's minds are like secret gardens surrounded by big walls. In each garden, the terrain and resources available are partially shared and partly unique. Inhabitants communicate among themselves by exchanging notes (blueprints, questions, suggestions) which they annotate and reedit to come to closer understanding.

The term 'concrétisation réfléchie' was coined by G. Célièrier (1992), who noted that Piaget had given it less thought than to 'abstraction réfléchissante'.

- ³ To Piaget, human cognition derives from biological adaptation, yet biological adaptation is not sufficient to define human cognition. Psychological adaptation allows infants to modify their own internal states, to enlarge their field of experience, and to construct an increasingly sophisticated set of strategies to deal with ever more complex situations. Psychological adaptation implies the organism's ability to expand its activity over greater and greater distance—in space and time—and over more and more complex detours.
- ⁴ Piaget broke down the biological notion of assimilation into 3 distinct forms. 1) *Reproductive assimilation* asserts that when an organism has a structure available, there is a tendency to exercise it. Its adaptive advantage is to consolidate activity through repetition. The need that triggers the behaviour is the very consolidation of the scheme itself, rather than any external stimuli. 2) *Generalizing assimilation* asserts that when an organism has a structure available, there is a tendency to exercise it in all sorts of different situations. Its adaptive advantage is to expand the experiential field. And 3) *Recognitory assimilation* asserts that when an organism has a structure available, there is a tendency to incorporate the objects acted upon into the structure of the scheme. In other words, an object that has been sucked and/or touched and/or seen is recognized as a 'thing' to suck and/or to touch and/or to see. The adaptive advantage of recognitory assimilation is attribution of meaning in a very primitive sense: objects can be 'tagged' by use, and 'gauged' according to how they fit or resist usages. The 'thing to__' gains both permanency and identifying features as a result of its consistent answers to given interventions.
- ⁵ Accommodation is the adaptive principle complementary to assimilation. It accounts for the adjustments of a scheme necessary to assure effectively the success of an action. A baby who sees the bottle and wants to reach it has to learn how to do so if the bottle is presented in a slightly different position. The adjustments necessary to succeed, despite variations in the context, are referred to as accommodation. The overall function of accommodation is to provide the baby with the possibility to adjust her/his pre-existing schemes so that they fit to the demands of the environment.

KNOWLEDGE AS REPRESENTATION

Nothing in my remarks below stands in opposition to the theses developed or held by Glaserfeld. I agree with him that “Knowledge pertains to the way in which we organize the world of our experience” and that constructivism “drops the requirement that knowledge be ‘true’ in the sense that it matches an objective reality. All it requires of knowledge is that it be viable, in that it fits into the world of the knower’s experience” (this book, chapter 8, p. 97). Moreover, “no matter how viable ... the solution of a problem might be, it can never be regarded as the only possible solution” (Ibid). Furthermore, I fully agree when he emphasizes the omnipresence of interpretations that construct the structures through which we organize our experience into a world.

Thus, to the extent that I am able to place myself in his position, I will have the impression of viewing the same world as he. And, as far as I am able to judge, the world of Glaserfeld is basically that of epistemology, even when it is presented in the form of a criticism of epistemology. But his is not a disembodied epistemology: “Once one accepts the view that all knowledge is a cognizing subject’s construction, that subject regains such autonomy as it can find within the constraints of an unknowable world. And with autonomy comes responsibility. What we know largely determines how we act. Consequently, if we want to act responsibly, we shall have to take responsibility also for the way we see the world” (Ibid., p. 98).

This being said, the place from which I view my world is different from Glaserfeld’s. As particularly concerns the starting point of my own reflection process, I see it as occurring in the shape of a check with which I was paid for my research at the University of Maryland’s physics department, in the midst of the Vietnam War era. Upon examining the particulars of the check, I was quite surprised to discover that the Pentagon was paying for my work, even though I was doing basic research having no foreseeable future application. As the result of this impromptu discovery, my perspective on the sciences underwent a drastic shift. It was at that moment that the sociopolitical dimension of the construction of knowledge began to become a part of my view of the world. Conversely, the practice of science no longer consisted in merely working in some office or laboratory, but instead grew out of a commitment that was fraught with risk. Specifically, it occurred to me that in the same way that what is left out of a medical file is likely to be forgotten when the time comes to opt for a particular therapy, so too whatever is not accounted for in a scientific representation will not warrant consideration for decision-making purposes in the social arena (concerning, for example, public health policies, funding applications, investment

choices, etc.). From that time on, radical constructivism equated, in my mind, with the social construction of the world.

My remarks are thus intended as a contribution to radical constructivism from the perspective of what this school of thought stands to gain in the way of insights from the sociology of the sciences and technologies. It is from this point of view that I will examine some important concepts used in this book, especially those of ‘representation’ and ‘fact’. I will also deal with the concept of standardization and with what I call the metaphysics of engineers.

REHABILITATING REPRESENTATIONS AND REPRESENTATIONAL KNOWLEDGE

Glaserfeld reserves some of his sharpest criticism for the concept of representation. In most people’s view, a representation is an image of reality that should be as similar as possible to the ‘real thing’. According to this conception, the existence of this image functions approximately the same way that Platonic ideas do, and the objective of all knowledge is thus to approximate this ‘reality’. Further, from this point of view, a drawing is more suitable than a caricature, a detailed map more so than a general map, numerical measurements more so than qualitative results, and so on.

Glaserfeld and others have showed the shortcomings of such an approach, so I will not expand on the issue here. However, it should be noted that, in many people’s minds, to speak of knowledge and truth amounts to drawing on just such an understanding of the notion of representation. And, from this perspective, every scientific practice is ultimately perceived as questing after the unique true representation—that is, the reflection or photograph of the world as it actually exists.

One may, however, resort to a very different notion of representation—a notion used, moreover, in mathematics, physics, political science, geography and in a number of other disciplines as well in various manners of handling everyday situations. In each such approach, representation is viewed in terms of one thing being substituted for another thing (but not for a ‘thing in itself’). Thus a representation occurs ‘in lieu of’ (as a substitute for) our experience, much as whenever a person is said to be representing the minister at an event or gathering (that is, ‘acting in the minister’s stead’).

Likewise, the members of Parliament are said to represent the people (keeping in mind that the term ‘the people’ immediately brings another representation into play). However, the representation referred to here does not mean that the MPs look *like* the people, but instead that if the function of representation goes off properly, consulting the MPs can be considered to be a suitable substitute for listening to the people. And thus is laid the basis of government through ‘representation’. The representation will be held valid to the extent that meeting a representative or representatives will amount to meeting those who are represented (at the least as far as the subject (topic) of the representation is concerned).

Or, to borrow another analogy, a roadmap works as the representation of a given country. Here again, this does not mean that the map looks *like* the country. Obviously, the sheet of paper commonly referred to as a map bears no such resemblance. But it does mean that the map can be used as the basis for a variety of discussions concerning the country under question. To speak of a map as a substitute or representation is to say that it stands in for places occurring over an area or territory. Thus it is possible to chart an itinerary for a given area on a map (a representation) without having set foot in the area at any time previously. Whenever, as a result of this charting process, a person is able to find him- or herself about in previously unfamiliar territory, it can be said that the map has represented the area or land suitably.

However, as Glaserfeld would surely emphasize, one would be mistaken in concluding that there is, on the one hand, ‘the true country’ and, on the other, all the more or less accurate representations (maps) of this same country. This is because that which is referred to as the country already constitutes a representation: one is always immersed in the realm of representations. The value of a map, moreover, has to be measured in relation to the type of project that one is pursuing. A tourist map will hardly be of much use to a geologist, nor will a geological map hold much interest for a tourist. Similarly, the equation of a particle’s movement is a representation of this movement (which is itself a representation of our experience). Indeed, for a whole series of situations, it is possible to work outward from the equation rather than from the observations; when that is the case, the equation can be said to represent the trajectory fittingly.

A patient’s medical file is also a representation. For, assuming that the file is given to someone who can read or decipher it, the file’s user will be able to participate meaningfully in a discussion concerning the therapy to be adopted even if he or she has never met the patient before. An essential component of this process consists in standardizing the concepts and the mode of representation relied on and shared—that is, in adopting a convention concerning a way using things and words. Without standardization, a representation loses much of its relevance. It has only a single, private use. Moreover, a theory is generally not qualified as being ‘scientific’ until it has become standard within the scientific community concerned.

Representation thus conceived of as a substitute fills a social function that is quite different from that performed by the representation-as-reflection that Glaserfeld has quite justifiably criticized. Furthermore, representation viewed from this angle offers a close parallel with the theory of representations current among mathematicians whenever they, for example, use co-ordinates to represent the points on a plane. Mathematicians tellingly refer to this ‘theory of representations’ as ‘mapping theory’.

Not all knowledge is representational, however. Such is the case, for example, of all our cognizing reflexes, or of how our immunity system ‘recognizes’ a virus. This sort of knowledge does not bring into play any representation. On the other hand, all scientific knowledge does belong to the category of representational knowledge: scientists construct representations that they would like to be adequate

for taking action. Representational knowledge functions like the roadmaps and medical files referred to above; they are a guide to actions in that the courses of action decided upon work exclusively from or on the basis of representations. The purpose of scientific work is to invent or devise representations that are appropriate to or commensurate with the situation at hand. From this perspective, however, representations by no means serve to produce an image of a preexisting reality. In short, the key question is no longer “How should I see the world so as to come closer to reality?” but “How might I devise a representation that enables one to act?” or “Which representations make sense in my situation?”. It is no longer a question of discovering a representation, but of inventing or devising one.

A system of representations enables human beings to develop the ‘world’ that will become theirs. This being said, there is no additional requirement whereby this world of their inventing must somehow strive to imitate a so-called world-in-itself. In short, this figured world constitutes a representation of our possibilities.

By replacing the concept of representation at the centre of intellectual work, it becomes easier to understand two requirements of scientific research. First, such representations as may be produced must also undergo testing or trialing, as with every other technique. Secondly, scientific conceptualization can be said to have reached completion only once the processes of socialization and standardization are well in place.

The testing of representations does not require much in the way of comments: if a current set of representations does not provide a useful basis for action, scientists will have to invent more powerful ones. The goal, however, is not to find representations that are ostensibly ‘truer’ than others. What is required from a representation is that it subsequently prove effective in relation to the project that it was devised to serve. Glasersfeld emphasizes adaptation as an essential component of any worthwhile representation. Or, as I would put it, a representation is worthwhile if it allows people to maneuver as ably as possible in the situations they are confronted with. So saying, the political dimension of knowledge emerges once again. Knowledge is power to the extent that one holds a set of representations that are deemed to be commensurate or congruent with the projects being pursued. From that point of view, knowledge appears as a technology: it is a material object (a representation: a map, a file, a sound, a written document, etc.) that, if used ably, makes it possible to carry out a project.

Standardization often gets bad press in the teaching world. Strong emphasis has been placed on the role of the subject in the construction of knowledge—justifiably so—yet, at the same time, there is a risk of forgetting or ignoring the vital role played by the use of fully standardized concepts (and thus fully standardized representations). Standardization certainly qualifies as a form of socialization whenever the latter is defined in terms of a process enabling us to share a language, understand each other and live together. It is also worth noting that scientific disciplines come about as the result of this process of standardization. Indeed, the standardization of language can be seen as a prototype of all other forms of standardization, for if the words we use were not standardized (or normed), a tower of Babel would arise out of the multiple worlds of our devising.

The standardization of concepts comes with its advantages and drawbacks. Among the advantages, it is important to emphasize the process of universalization: from the moment a concept has become standardized, it becomes usable everywhere such standardization has been accepted (or imposed). Universality of this kind also comes at a price, because the sciences in their current form tend to be universal along the lines of the English language. English became universal thanks to the British gunboats of the 19th century and the military and economic empire of the United States during the 20th. Similarly, it is due to the pressure of the Western powers and their interests that scientific language came to be the only objectively universal language. Standardization is never established in a purely rational way; it also results from social and at times brutal forces.

In short, from this point of view, the production of knowledge is also the production and legitimization of representations of possible actions. Knowledge can, in this light, be viewed in terms of standardized and socialized representations whose effectiveness or utility for the purposes of action has been tested.

WHAT ARE THESE THINGS CALLED 'FACTS'?

In light of the preceding, I will reconsider the concept of 'fact'. Glasersfeld criticized the use of this concept insofar as it claims to refer to some 'given' that has not been constructed by humans. By taking a more sociological approach to constructivism, however, it is possible to show the relevance of the concept of 'fact' as denoted in general usage. What is being claimed when a person asserts that some event is a fact? What, for example, is meant when someone states that "it is a fact that it rained today?" What is the difference between putting the assertion this way and simply stating that "today, it rained". As concerns the content (that is, the information pertaining to rain), the two statements are equivalent since in any situation where the one is true, the other is, of necessity, too. In the statement referring explicitly to a fact, there is an added emphasis on the relevance of this assertion. To stress that some event is a fact is to proclaim that this interpretation of our history has been met with a consensus such that no one at this time wishes to challenge this assertion. In the statement referring explicitly to 'a fact', the only extra element pertains to social conditions, but does not add anything in relation to the rain. By the same token, it can be said that it is a fact that I am writing with my computer or that the Earth is getting warmer. As long as nobody challenges this way of interpreting the experience at hand, one will speak of a fact. On the other hand, as soon as this interpretation is brought into question, it loses its status as a fact and acquires the status of an interpretation whose relevance has to be proven or subjected to the process by which facts are 'established'. This is what happened with the statement: "It is a fact that the Sun revolves around the Earth each day". For a long time, this statement went unchallenged and nobody considered doing otherwise: it had the status of a fact. Later on, however, this statement ceased to appear self-evident, and those who believed it was correct had then to prove it.

Thus, to characterize something as a 'fact' is to bring into play the existence of a social consensus and not some fundamental given relating to knowledge. In

everyday situations, references to the notion of ‘fact’ continue to be useful. Conversely, to state that “science is based on facts” means only that consensually accepted components are being used to build descriptions or conceptualizations. In everyday practice, moreover, the assertion that science is based on facts has often had the effect of obscuring the role of the subject, as Glasersfeld has so aptly shown.

THE METAPHYSICS OF SCIENTISTS VERSUS THAT OF ENGINEERS

One of the issues often raised *vis-à-vis* radical constructivism occurs in connection with the idea of truth that is plural. Currently in our culture, the idea that there is one and only one truth is firmly implanted; the underlying objective is to reach, or, at the least come close to this truth. Among scientists, it is almost a given that, for every problem, there is one—and only one—explanation. Teachers, moreover, feel themselves to be charged with a doubly impossible mission of, on the one hand, proving that a specific representation is ‘the’ true one, and, on the other, making pupils believe that science has satisfactory answers to all their questions. Thus it is possible to speak of something akin to ‘metaphysics of the single truth’. Those individuals who do not adhere to it are often described as ‘relativist’ and are vilified for this reason. The world of the scientists is a world that requires a faith in scientific truths as single truths. *A priori*, there is a scientific answer—and only one—to every problem.

For an engineer, on the contrary, there is, *a priori*, a plurality of solutions to a problem. Yet no one thinks of accusing engineers of indulging ‘relativism’ for all that. Nevertheless, just as engineers can, *a priori*, devise an infinite number of techniques for solving a problem, so might one, *a priori*, imagine an infinite number of models for representing a situation. These models, like the techniques of an engineer, are multiple but by no means equivalent. Indeed, to pursue the analogy further, while there is presumably an infinite number of ways of imagining a transatlantic crossing, engaging in such an exercise would not entail positing an equivalence between a steamer passage and a canoe crossing for all that.

Why then label as ‘relativistic’ a way of viewing science that resembles the way an engineer views technologies? Might there not be some advantage to comparing the sciences and technologies? To begin with, a technology can be characterized by three main project-related features:

- Material components (e.g., in the case of rail transport: trains, engines, rails, tickets, signals, etc.; of data processing: computers, the Internet, etc; of cooking: food products, pans, etc.);
- A manner of seeing the things (e.g., in the case of rail transport: a physical representation or depiction of what a ‘railroad’ consists of; of data processing: the theories of information and set of notions framing both the discourse and knowledge associated with computer science; of cooking: any number of practices that constitute this art as a discipline);
- A social organization (e.g., in the case of rail transport: the institutional roles related to this technology, such as, train driver, track repairers, etc; of data

processing: research teams, seminars, university departments, trade journal publishers, research centers, etc; of cooking: the entire social organization involved in running a restaurant, or the mode of household organization required to produce family meals; etc.).

It is important to note that none of the above-mentioned project-related features can be considered socially or politically neutral. For example, one has only to examine a map of the French rail system to grasp the way it both reflects and furthers a political, economic and military project of centralization.

Given this definition of technology, a scientific discipline can be looked upon as a technology of representation. Or, in other words, scientific practices can be understood as being part of a technology of representation designed to build representations of ‘our world’, in accordance with our desires and projects. What is more, the three features used to define technologies above may also be enlisted in an effort to analyze technologies of representation. Namely:

1. The material component of technologies of representation consists of sentences, conceptual systems, mathematical structures, laboratories, etc.;
2. The specific way of looking at things and at situations amounts to an understanding of the paradigm of a given discipline and of the handling of its conceptual structures;
3. The social organization of a scientific discipline consists of universities, research centers, laboratory assistants, conferences and congresses, etc.

Seen in this light, the sciences and disciplines can be regarded as developing a type of technology: they produce representations growing out of a project to harness our actions and that have undergone testing and standardization in relation to this same project. It again goes without saying that none of these technologies of representation is neutral socially, politically, economically, militarily or otherwise. Their objective, according to Glaserfeld, is not “the unveiling of the universe but rather the invention of models that allow us to cope with the problems presented by our experience” (this book, chapter 13, p. 142).

Conceiving of the sciences in terms of ‘technologies of representation’ thus seems to me to be a worthwhile extension of radical constructivism. Among other things, such an approach makes it easier to effectively rebut any of the various objections raised in respect of constructivist philosophies.

For one, by revisiting the concept of representation, a basis is laid, in the context of teaching and the classroom, for grappling with a major component of daily living—namely, seeking to identify worthwhile ways of representing a situation with a view to pursuing and completing one’s projects. For example, how might one usefully (or validly) represent the engine of a car in order to determine the cause of a breakdown or how might one represent symptoms in order to perform a medical diagnosis?

Furthermore, viewing the sciences within the mode of technologies of representation helps to clarify the rationales underlying the teaching and learning of scientific disciplines. Namely, it is worth studying scientific disciplines not because the latter are deemed to be ‘true’ but because, as a mode of representing our situations, they have, in the past, proven to be worthwhile and, at this time,

constitute a vital component of our culture. In consideration of the way such usefulness is conditioned by the development of human societies, there is necessarily a societal dimension to this kind of representation. Thus also, and importantly, it is not enough to recognize that there is a subject who structures observation and the scientific object being observed. It is just as important to recognize the social interactions through which the sciences develop and evolve. The upshot, with respect to epistemology, is to go beyond radical constructivism and to adopt a socioepistemological posture that envisions the construction of the sciences in **societal** terms.

In addition, the dogmatism of the sciences begins to melt away once one stops teaching science as a set of truths and instead presents it as a number of worthwhile ways of figuring (that is, representing) our projects. Further, once standardization and legitimization have been more fully accounted for, the construction of representations no longer warrants being conceived of as an individualistic affair. On the contrary, factoring for the standardized components of our figurations opens up new horizons for embracing the social character of the construction of the scientific knowledge. Finally, conceiving of representations as ways of figuring our worlds is to emphasize the inventive and creative character of the production of knowledge.¹

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NOTES

- ¹ The themes I have taken up here are developed at greater length in two of my previous books (2002, 2003). As well, insight into the general context of this chapter is to be had in the following publications: Callon (1989), Collingridge (1989), D sautels and Larochelle (1989), Easlea (1973), Four z (1983, 1994), Ibarra and Mormann (1997), Latour (1983), Roqueplo (1973), Stengers (1993), Thill (1972), Varela (1989).

A CONSTRUCTIVIST ACCOUNT OF KNOWLEDGE PRODUCTION AS A SOCIAL PHENOMENON AND ITS RELATION TO SCIENTIFIC LITERACY

The model of cognition developed by Ernst von Glasersfeld, while generating a considerable amount of heated debate,¹ has also oriented a no less impressive body of research in science education. *The Practice of Constructivism in Science Education*, edited by Tobin (1993), for example, testifies to the fertility of radical constructivism in terms of being both an epistemological position from which to reflect on the status of scientific knowledge and a source of inspiration for renewing classroom strategies conducive to facilitating students' appropriation of these spheres of knowledge. In keeping with this line of thought, and after arguing that it is possible from a constructivist perspective to account for the simultaneously social and cognitive character of the production of scientific knowledge, I will sketch out how radical constructivism also lays the basis for adopting an alternative conception of the contemporary finality pursued through science education—namely, scientific literacy for all students.

RADICAL CONSTRUCTIVISM AND THE SOCIAL CHARACTER OF THE SCIENCES

Radical constructivism has often been reproached with denying the sociality of scientific knowledge. Such criticisms are most certainly the reflection of a misunderstanding, as the texts by Ernst von Glasersfeld gathered in this book contain explicit references to the social character of the production of such knowledge. Thus, in *The Logic of Scientific Fallibility* (chapter 11, this book) he shows, referring to research by Maturana, that in order for the findings obtained by an observer to be considered scientific, they must be corroborated by other independent observers working under well-delimited constraints. Therein resides a condition necessary to the development of a common scientific world: “Corroboration merely establishes an element of consensus, that is, the kind of element that helps to create what Maturana has called a consensual domain among interacting organisms” (p. 122). In *The Radical Constructivist View of Science* (chapter 14), he stresses, following a review of the traditional concept of objectivity, that “it would be preferable (and more accurate) if in all cases we spoke of ‘intersubjective’ and ‘intersubjectivity’” (p. 147), again highlighting the interplay of social actors in the construction and validation of scientific knowledge. In the conclusion to this chapter, moreover, he points out that scientists cannot

merely wave aside the question of ethics attaching to the direction given to scientific research, since, from a constructivist perspective, they bear a responsibility for the knowledge they have had a hand in devising:

The awareness that it is they who are responsible for their theoretical models and thus at least to some extent for actions based on them, might change the widely held belief that the direction of scientific research must not be fettered by ethical considerations. (p. 150)

Such prescriptions for accountability, as urged by their author, are no doubt likely to shape the viewpoint one develops about the direction given to research in our societies—that is, about *science policy*. Stated in those terms, however, this proposition seems, in my view, to leave two major questions hanging: 1) do ethical considerations only come into play downstream from the production of knowledge, or do they directly contribute to the very development of such knowledge? and 2) do scientists alone provide legitimate input into the development of public science policy? I shall return to these questions below. Whatever position one adopts in relation thereto, however, the fact is that adopting the radical constructivist point of view does not in any way amount to denying the social character of the production of scientific knowledge, as is indeed clear from Glaserfeld's statements referred to above. The problem lies, rather, in the absence, among constructivists, of full-fledged theorization of social interaction, as Glaserfeld (2000) has also pointed out elsewhere.

THE KHUNIAN LEGACY

Ernst von Glaserfeld (this book, p. 144) credits Thomas Kuhn with having deconstructed the image of scientists discovering a world-in-itself that is perpetually identical unto itself. According to Kuhn (1983), scientific activity does not consist in decoding ontological reality to ever increasing perfection, but in solving puzzles or problems within the framework of a paradigm (that is, a set of theories, exemplars and norms). This process, which brings into play a number of tacit knowledges, is said to be the basis on which the relevance of the paradigm is founded, but it is also a way to bring into view anomalies that ultimately undermine the paradigm's credibility. At that point, the situation becomes conducive to the emergence of controversies and debates in which alternative theoretical frameworks or paradigms can be entertained. However, the adoption of a new paradigm always generates resistance in the scientific community and cannot be viewed as a strictly rational process. At stake, as Kuhn has mentioned, is a shift in the world view held by scientists, such that commensurability between it and any of the succeeding paradigms will be approximate at best, in view of the changes in meaning to which the concepts are subject. In a way, the difficulty or impasse may be likened to a situation in which two scientists working within the framework of two different paradigms speak, for all intents and purposes, foreign languages. The notion according to which scientific knowledge will gradually bring us closer to the truth thus appears to be on less solid footing.

However, as Kuhn has also shown—somewhat in spite of himself, according to Fuller (2000)—the production of scientific knowledge is a simultaneously social and cognitive process. In other words, a paradigm is constitutive of the very scientific community that constructs it, and the stabilization of this paradigm proceeds through the recruitment, into this community, of new members who in turn learn to work within the framework of the paradigm and pursue the standard task of solving puzzles until such time as new controversies arise. In other words, paradigm and community are mutually constitutive, in a circular manner that recalls the recursive processes seen stabilizing around eigen-behaviours, as Foerster (1997) has theorized. Foerster has also shown how ‘nontrivial machines’—that is, relatively closed information systems that are analytically indeterminate and ‘historically dependent’ and whose behaviour is unpredictable can nevertheless achieve a dynamic equilibrium around fixed points, attractors or eigen-behaviors, whenever such machines operate on their own operations, recursively. Foerster has also generalized this potential for achieving a dynamic equilibrium to entire networks of such machines interacting with one another:

I shall go on to generalise these observations in three steps. The first is to give without proof the essence of a theorem concerning these machines. It says that an arbitrarily large closed network of recursively interacting non-trivial machines can be treated as a single non-trivial machine operating on itself ... This insight entails the second point, namely, that the dynamics between all interacting participants in such a network will converge to a stable dynamics, to the eigen-behavior of this network. The third step, or shall I call it a leap now follows. Let the interacting participants be the participants in a social network, then their eigen-behavior manifests itself in the language spoken, the objects named, the customs maintained, the rituals observed. (p. 24)

The same principle applies, according to the Kuhnian model, to science communities, which develop specialized languages, produce objects of knowledge, and observe academic rituals and local customs.

In a certain way, this model makes it possible to go beyond these veering off into sociologism or psychologism and that, each in their own way, have attempted to produce the defining conception of the sciences. To wit, the sciences were explained as being either a product of social structures, thereby reducing actors to cultural or cognitive puppets, or as the creation of singular geniuses mysteriously endowed with the ability to transcend the social and historical conditions of their existence. Nonetheless, while it may be true that the Kuhnian model was responsible for raising a range of new questions, the research required to answer them was largely the work of sociologists and anthropologists of science. For it was by describing what it is that scientists do in the very places where scientific facts are produced and controversies are resolved that social scientists began to devise responses to what Knorr-Cetina (1993) has called the challenges of a constructivist approach to the production of scientific knowledge and which she has expressed in the following terms:

These new approaches replace the view that observation and experiment play the dominant role in the specification of scientific facts by the view that these processes involve collective negotiations, interests and the infusion of experimental outcomes with contingent features of situations. ... constructivism brings into view social processes, as opposed to methodological and individual processes with which received views of science were concerned. (p. 556)

At this point, it is well worth assessing what constructivist-style studies and analyses have taught us about the production of scientific knowledge. For the purposes of articulating my thoughts on this subject, I will examine an admittedly small sample of the findings from this body of research.

The Scientific Observer as a 'Collective' of Social Actors

The production of scientific knowledge is, by and large, a collective undertaking, most often leading to controversy and over the course of which embodied and culturally situated scientists mobilize a range of symbolic, discursive and material resources to (co)produce knowledges, facts and artifacts. The process of applying the mesh of theoretical and empirical procedures and techniques can be likened to a kind of 'bricolage' or to what Pickering (1995) has termed the 'mangle of practice'. In short, it is a process of resistance and accommodation that entails an ongoing redefinition of the goals governing the work to be accomplished (or the problems to be solved), all depending on whether the effective outcomes do or do not comply with initial expectations. Scientists thus engage in a kind of negotiation about what will be accepted as a valid experiment or as convincing findings (Lenoir, 1993). A good illustration of this model of scientific work is to be found in the analysis performed by Atten and Pestre (2002) concerning Hertz' research in physics. Following Doncel (1995), they have shown that Hertz' experimental work was not intended, in the beginning, to test Maxwell's theory of electromagnetic field but to take up an entirely different problem. In their view, he worked for years on end assembling ('bricoler') experimental designs intended to stabilize the phenomena of interest to him, and it was only somewhat by accident that he ended up producing and detecting electromagnetic waves (what he at the time called 'electrodynamic waves'). Once published, his findings did not meet with a uniform reception amongst the community of physicists of the time. Whereas scientists in England and Ireland quickly recognized that Hertz' findings accorded with Maxwell's theory, his peers in France and Switzerland were decidedly less forthcoming with such conclusions. Whatever their position, none of these contemporaries attempted to reproduce Hertz' experiments, however, but instead concentrated on producing variations of them. It was only several years later that the community of physicists came to an agreement as to what constituted satisfactory proof—that is, what was held to be convincing. Now, if one acknowledges that these transactions amongst scientists do indeed constitute a

social process, what view might one adopt toward a scientist working alone in his or her laboratory?

Actually, it is only once following a break in one of the various different links in the network of human and non-human actors² on which science-making is based that it becomes possible to come by the impression that such activity is a strictly individual affair. For in truth, a researcher must, in order to do his or her job, mobilize the standardized knowledge current in his or her field, use instruments or substances of varying quality that have been supplied to him or her by lab equipment manufacturers, place his or her trust in the know-how of the technicians responsible for setting up experiments, make use of the advice offered by colleagues, negotiate with colleagues over the interpretation to be given to a set of results or the content of articles submitted for publication (a process that necessarily entails, on the one hand, performing a review of the literature and, on the other, obviously, accounting for the potential expectations of the members of the peer review committee responsible for refereeing said article). In short, the presence of others is to be encountered in all these various activities, and the underlying network of social relations is what makes it possible to produce theoretical and materializable entities whose ever-problematic existence cannot be unlinked from the conditions surrounding their production. From this perspective, the observer can be likened to a composite (referred to as a 'collective' in Actor Network Theory) of embodied social actors who are historically and culturally situated and who, through a range of practices (material, symbolic, discursive, theoretical, empirical, etc.), make distinctions which, once integrated into a recursive process of social interactions, contribute to bringing forward ('performing') a world that is held to be real, to the extent that most everybody acts as though it were indeed so. For example, we handle what we call bacteria as though they were entities belonging to a world-in-itself and as though they 'truly' behaved in accordance with the specifications set out in a scientific model—all the while remaining cognizant that our descriptions are indeed model-based representations.

Describing the production of scientific knowledge in such terms is by no means tantamount to denying the fact that every individual develops an interpretation of the situation under question without knowing for certain whether or not it corresponds exactly to that of his or her colleagues. On the other hand, it is equally clear that the resources mobilized in the knowledge-making process, including the discursive processes relied on to formulate propositions, have a social origin. It is only after the fact that it becomes possible to state whether a given individual, as a participating member of a given network of social relations, indeed made an original contribution. Furthermore, it is important to note that this part of the network is also connected to other social actors and that a laboratory will be unable to survive unless it obtains the appropriate financial resources, produces a steady stream of articles and objects, develops a scientific succession, and, ultimately, garners the support of the general public.

The Sciences in Society and Society in the Sciences

For years now, the sociopolitical dimensions of the sciences have been the subject of sociological investigations that ultimately showed contemporary technosciences to be perfectly integrated into the military-industrial complex. Furthermore, as Pestre (2005) has maintained, historically the sciences have always figured prominently in the calculations of the political and economic powers, and it is something of a retrospective illusion to view the sciences as embodying the process involved in producing pure knowledge. This being said, describing the production of scientific knowledge in terms of social practices has major consequences for our manner of envisioning the sociality of this knowledge.

Accordingly, it is no longer possible to conceive of the science/society relationship as though the two entities could be held separately from one another and their respective impacts could be measured. Sociality is not located somewhere outside of the laboratory but is distilled from within, as it were, since the varyingly certain, varyingly controversial entities (e.g., substances, organisms, artifacts, designs/set-ups, concepts, theories, etc.) that are brought into existence help to reconfigure social relationships and interactions and thus to ‘perform’ sociality (*performer*, in French, i.e., to accomplish or to achieve, as with order, coherence, permanence), to borrow from Callon (1999). Thus, an everyday world would not be the same depending on whether or not it was a world having nuclear radiation, electromagnetic waves, the human genome, magnetic cards, and all such entities for which scientists serve as the spokespeople.

An excellent illustration of this notion of the ‘performativity’ action of social interactions can be found in the case of HIV. This virus said to be the culprit behind AIDS, a disease that has assumed epidemic proportions, became a fixture of our world from the time it was isolated and studied in a laboratory setting. Its scientific existence thus duly attested, this entity went on to become an actor in that it has transformed social relations both locally and globally. Since the outbreak of this epidemic, no one makes love quite the same way as before, and one’s current or prospective partner can conceal a potential hazard to one’s existence and well-being. Many HIV-positive individuals have been confronted with forms of social exclusion. Also, in conjunction with the numerous scandals surrounding contaminated blood supplies, steps had to be taken to overhaul the institutional framework and the procedures governing blood donation and processing. The AIDS crisis has also generated a plethora of social (i.e., scientific, economic, legal, organizational, political, etc.) actions worldwide. Following the eruption of this new actor into our daily lives, clearly our world is no longer what it used to be and, for all intents and purposes, it does not occur to anyone to question its existence. For we do indeed behave as though it existed, even if there is no way of knowing whether the way we picture it in our minds actually reflects an ontological reality located outside of our everyday world.

The Laboratory Without Walls and the Risk Society

While not providing foolproof guarantees, the controls performed within the confines of a laboratory are designed to reduce, as much as possible, the risk for civil populations that is posed by the introduction and spread of applications in unconfined settings. The need to grapple with such risks has, on a number of occasions, resulted in the implementation of safety and security measures aimed at keeping the potential for catastrophe to a minimum. For example, in 1975, American scientists attending the Asilomar conference agreed to a set of guidelines governing the confinement of experiments involving recombinant DNA. However, in the time that has elapsed since this ‘Woodstock of molecular biology’, to borrow from Barinaga (2000), the positions of the researchers and groups active in this field appear to have evolved considerably. Thus, full field trialling of transgenic plants does not rigorously control developments of the situation in the field, and little is known as yet about the consequences of the widespread dissemination of these plants for the environment and public health (Berlan & Lewontin, 1998). In addition, the production of transgenic plants has revealed a number of serious shortcomings in the knowledge scientists have brought to play in this endeavour, as is illustrated by the statement of a French researcher at the Institut de Recherche Agronomique de France (INRA) concerning the manipulation of transgenes:

[At this time] there is no way of controlling the moment when integration occurs... The transgene may be present in numerous copies... No one knows what factors govern the integration, or whether the integration site in the genome occurs randomly or in particular regions of choice... There is no control over the locus, the time or the number of copies of the integrated transgene. (In Testart, 2006, p. 22)

This opinion has, as well, been corroborated by other scientists, notes Wynne (2005) who adds that beyond the unknown factors that could be identified at some point in the future, it is also necessary to take into account what is commonly referred to as the ‘fundamental uncertainties’—so named owing to the complete inability to predict the behaviour of so-called complex phenomena. Thus, now that molecular biology’s central dogma (seen in its reductionism and predictive determinism) has been thrown open to question (Atlan, 1999; Kupiec & Sonigo, 2003; Strohman, 1997) in favour of models integrating such notions as emergence, dynamic stability and attractors, certain quarters of the scientific community have begun acknowledging that reading a living organism’s genome no longer suffices for predicting its emerging properties. That, at the very least, is the view expressed by the publishers of the journal *Nature Biotechnology*:

It is the interactions and relationships between the parts—whether protein-protein, protein-gene-metabolite-lipid, protein-cell, cell-cell or brain subsystems—that produce the emergent properties of a living system. These are not evident from an inspection of the parts-list... (In Wynne, 2005, p. 76)

Thus, even after targeting various properties for splicing, it is not possible to predict *all* of a given genetically modified plant's properties, some of which could well place human health at risk of harm or impairment.

In addition, the case of transgenic plants offers a typical example of the phenomena constitutive of what Beck (1997) has qualified as the society of manufactured risk. Among scientists, this society has become a huge laboratory in which they may carry out trials subject to varying degrees of control and whose unexpected effects may negatively impact all citizens.

Everyone is exposed almost without protection to the threats of industrialisation. Hazards are stowaways in normal consumer life. They travel on the wind or in the water, they are concealed everywhere and in everyone, and they are passed on with the necessities of life: air, food, clothing, furnishings, through all of the otherwise monitored protective zones of modernity. (Beck, 1997, p. 21)

Such would appear to be the experience of four members of the Canadian House of Commons who, with a view to raising public awareness on the subject of pollution-related problems, agreed to release the results of an analysis performed on samples of their blood (Francoeur, 2007). The findings showed that these MPs had all been contaminated in varying degrees by various substances—lead, mercury, insecticides, etc.—circulating freely in our local environments unbeknownst to just about everyone. True enough, a toxicologist issued the usual recommendation to not give in to panic, as the levels recorded for the various substances never exceeded the norms determined as being acceptable for human health. It is a well-known fact, however, that uncertainties and controversies shape the process whereby such norms are established—but on such matters, the toxicologist had nary a comment to make.

SCIENCE EDUCATION AS/FOR CITIZENSHIP EDUCATION

The educational system is the political means of maintaining or modifying the appropriation of discourse, with the knowledge and powers that it carries. (Michel Foucault, 1971, p. 46)

The description of production of scientific knowledge that I have outlined to here provides a starting point from which to grapple with the two main questions respecting ethics and science policy with which I began this chapter. As is clear, I believe, collectives of scientists are immersed in a socioculture that simultaneously constrains and empowers their observational activities. At the same time, since there is no way of totally specifying the conditions under which observation is performed, the resulting knowledge necessarily comes bearing blind spots and tacit assumptions (Brier, 2006; Ricoeur, 1986). Feminist studies of the sciences have, in particular, identified sexist biases that are constitutive of, for example, a certain era's prevailing conception of fertilization among the human species (i.e., the active spermatozoid versus the passive ovum) and thus contributed to

'naturalizing' social biases concerning gender inequality (Pestre, 2006). From this point of view, scientists bear responsibility not only for some of the uses to which the knowledge they have produced is put, but also for the fact that the very constitution of such knowledge may work toward legitimizing the existing social order as well as a certain manner of figuring/enacting the world and human beings in the world—in short, toward legitimizing a vision that is unmistakably freighted with ethical questions (Lewontin, 1991). Furthermore, and inasmuch as all citizens are at risk of being harmed or affected by the unexpected, unpredictable consequences of spinoff from scientific research (take the nanotechnologies, for example), science policy can no longer remain the exclusive preserve of scientists. This conclusion is all the more valid in that, as a considerable body of research has vividly shown, the citizens concerned by technoscientific developments are quite able to contribute constructively to any such policy-drafting process (Callon, Lascoumes & Barthe, 2001). What then do such observations portend for the cause of scientific literacy for all students?

One can but agree with Glaserfeld that education should aim to generate amongst students a genuine reflexive comprehension of knowledge rather than to condition them to provide the 'right answers' to questions. Nevertheless, however much the preparation of reflexive individuals constitutes a worthy educational objective, it provides little in the way of an answer to the vital, two-fold question of why science, rather than some other subject matter, should be taught at school and, assuming there is a justification for teaching science, what version of the sciences should be taught? It is on this point that the representation of the production of scientific knowledge I have just outlined—and which makes no claim to state the 'reality' of science—may be of some use for specifying what is meant by 'scientific literacy' in terms of its being a finality of science education.

The notion of scientific literacy is generally associated with the education of citizens who are likely to fully participate in a society that some describe as being shaped extensively by technoscientific developments. However, as soon as it is a question of specifying what is actually meant by scientific literacy and the type of teaching approach and curriculum appropriate to this mission, fundamental disagreements crop up. So much is clear from the highly charged debate that followed the publication of Fensham's (2002)³ proposals for reform on this subject—which should hardly come as a surprise, considering the magnitude of the issues at stake. Thus, with a view to pursuing this discussion, I will add a few comments on the matter here.

We may thank Foucault (1971) for reminding us that education is a political, hence ideological, process. A curriculum—whatever its contents—thus partakes of a struggle for power whose object is to establish or maintain a certain social hierarchy of knowledge, as Biagioli (1993) has aptly shown in his version of the Galileo Affair. In analogous fashion, schools promote the teaching of some forms or types of knowledge and, above all, certain representations of the production of this knowledge over others. For example, it is rather unlikely that students who are taught (as most students continue to be taught, according to Lyons, 2006) that scientific knowledge is the accumulated work of particularly gifted individuals

(that is, individuals who, by faithfully embracing the ascetic regime prescribed by the scientific method, ultimately succeed in becoming disembodied and de-socialized to the extent required to utter pronouncements on the world “as it really is”) will be particularly prone to challenging the social hierarchy of knowledge, of which science and mathematics represents the apex. Or, viewed from the other end of the telescope, it may be added that scientists have little cause for worry from this variety of science education: the ‘school science war’ alleged to be threatening their social power is as yet only a distant possibility.

It would be an entirely different situation, however, if by drawing on the perspective I have sketched out in this text, the goal became one of allowing future scientifically literate citizens to familiarize themselves with technoscience-related policy. If the goal were, that is, to enable the scientifically literate to develop the intellectual instruments and potentialities for action that would enable them, on the one hand, to apprehend the technosciences as social practices and, on the other, to organize themselves and intervene in their capacity as concerned citizens in the sociotechnical controversies confronting our societies and, so doing, share in ‘performing’ (i.e., bringing into existence) science policy. Such a vision obviously presupposes that these subjects have grasped, following Restivo (1988), that “modern science is implicated in the personal troubles and public issues of our time” (p. 209), and that they understand reflexively that the ‘truth’ of scientific knowledge, however robust it may be locally, is relative to the cultural, social and historical conditions surrounding its production.

There is not the slightest thing utopian about making this educational project a reality, as has been shown in an abundance of case studies (Aikenhead, 1992; Roth & Lee, 2002; Simonneaux, 2004). However, it is more than likely that generalizing such an approach across an official curriculum will spark intense opposition, in particular amongst science communities, which have a record of mobilizing to abort secondary science education projects designed from within a Sciences-Technologies-Societies perspective (Blades, 1997). All in all, it would appear that the introduction of sociological and political considerations in science education constitutes as daunting a danger for the knowledge-powers-that-be as the introduction of epistemological considerations in the field of education, as Glaserfeld has emphasized (this book):

To introduce epistemological considerations into a discussion of education has always been dynamite. Socrates did it, and he was promptly given hemlock. Giambattista Vico did it in the 18th century, and the philosophical establishment could not bury him fast enough. ... It seems that to discuss education from an epistemological point of view was a sure way of committing intellectual suicide. (p. 3)

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NOTES

- ¹ Calling into question the traditional attributes of the sciences (objectivity, truth, universality, etc.) has sometimes triggered violent reactions on the part of scientists, as was shown by the Sokal affair, one of the more recent episodes in what has become known as the 'science war'. A similar situation has unfolded in the field of education (see, for example, Steffe & Gale, 1995).
- ² According to Callon (1995), it is possible to conceive of actors as being human or non-human—in other words, “any entity endowed with ability to act. This attribution may be produced by a statement ..., by a technical artifact ..., or by a human being who creates statements and construct artifacts” (pp. 53-54).
- ³ See the two special issues that the *Canadian Journal of Science, Mathematics and Technology Education* devoted to these debates (2002, nos 1, 2).

RADICAL CONSTRUCTIVISM AND “SCHOOL MATHEMATICS”

Talking about radical constructivism as a model of knowing confers a certain status to the model as if it were independent of those engaged in the conversation.¹ I don't object to this way of talking about radical constructivism primarily because I have an externalized concept of radical constructivism in the same way that I have externalized other conceptual entities such as my concept of democracy.² But radical constructivism is much more for me because I use it reflexively in all of my personal and professional activities. This, I believe, is a very important way to consider how radical constructivism can be used in education. So, I urge the readers of the papers in this volume to transform what is being read into a way of knowing that is interwoven throughout their lived experiences.³

Given my stance that radical constructivism is meant to be used reflexively by people as a way of knowing, I have always been somewhat puzzled by the claim that radical constructivism and mathematics teaching are independent (e.g. Janvier, 1996; Simon, 1995). But if I think of radical constructivism as an externalized model of knowing rather than as implicit in my lived experiences, the claim does have meaning for me. As such, one certainly should not expect to find a model of mathematics teaching in Glaserfeld's model of knowing because that was not his purpose when making the model. Similarly, one should not expect to find a model of school mathematics in Glaserfeld's model of knowing because that also was not his purpose in developing radical constructivism. Such models have to be constructed by mathematics educators (Steffe & Thompson, 2000).

My goal in this commentary is to develop elements of a concept of “school mathematics” using my concept of radical constructivism⁴ and distinguish it from the more or less conventional notion of school mathematics. The distinction between the two is similar to Maturana's (1988) distinction between objectivity with parentheses and objectivity without parentheses. Objectivity without parentheses refers to an explanatory path that “necessarily leads the observer to require a single domain of reality—a universe, a transcendental referent—as the ultimate source of validation for the explanations that he or she accepts” (p. 29). This comment fits well with the usual notion of school mathematics where mathematical structures are the transcendental referent.⁵ The alternative, objectivity with parentheses, implies that knowledge is a process in the domain of explanation, which is basic in my concept of “school mathematics”.⁶

The historical use of mathematical structures as the transcendental referent in mathematics education found justification in Cartesian epistemology because of the

classical dualism in the view of mind—an endogenic (mind-centered) *versus* an exogenic (world-centered) view (Konold & Johnson, 1991). In the following text, I provide a brief account of how Cartesian epistemology dominated school mathematics throughout the last century and on into the current century, regardless of the prevailing schools of thought that seemed dominant within particular periods of time. After that, I provide a brief account of the emergence of radical constructivism in mathematics education and document that radical constructivist thought did not eliminate the duality of Cartesian epistemology in official documents on school mathematics. I then point to Glaserfeld's use of Piaget's studies on the construction of children's object concepts to eliminate the duality between mind and reality. Finally, I discuss basic elements of "school mathematics" that do not countermand basic principles of radical constructivism.

CARTESIAN EPISTEMOLOGY AND SCHOOL MATHEMATICS

I have chosen "school mathematics" as the topic of my commentary because of the crisis that we face in mathematics education concerning the difficulties students' face in learning what is essentially a childless curriculum. So, I consider what a mathematical curriculum should consist of and how it should be built as one of the most urgent problems in mathematics education today. But the problem is not a new problem because it was dominant throughout the last century. At the very beginning of the last century, the classic study by Thorndike and Woodworth (1901) ushered in an influential scientific movement in education based on empiricism in which reality is considered to imprint itself on the mind. One might claim that behaviorism has been implicit in the *practice* of mathematics teaching throughout this complete time period. It was certainly implicit in the Post World War II efforts to improve mathematics education, efforts that became known as the modernist movement in mathematics education. In this movement, Piaget's genetic structures ironically served as a rationale for the emphasis on mathematical structures. But there was a conflation in the two types of structures that found expression in Bruner's (1960) famous concept of readiness to learn the fundamental structures of mathematics: "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). Piaget was considered to be an observer rather than a teacher (Goals for School Mathematics, 1963) and the view was that had Piaget observed the mathematical thought of children who participated in the modern mathematics programs, he would have realized the elasticity of the limits of their cognitive processes. The mathematical knowledge of the children that Piaget and his collaborators had documented was essentially discounted and curriculum developers did not regard children's mathematical knowledge as part of the curricula they developed. Rather, the mathematical knowledge of children that Piaget and his collaborators produced was regarded as belonging to the field of developmental psychology; it had to do with school mathematics only to the extent that it served as a rationale that concrete operational children were ready to and could engage in structural thinking (e.g., Dienes, 1964).⁷

THE EMERGENCE OF RADICAL CONSTRUCTIVISM
IN MATHEMATICS EDUCATION

Following the backlash to the modern mathematics movement, empiricism re-emerged as the prevailing school of thought in a back-to-basics movement during the decade of the 1970's and school mathematics changed to emphasize behavioral objectives. But adherence to the basic duality of Cartesian epistemology did not change. During the middle of this decade, I began working with Glasersfeld on a project that we called *Interdisciplinary Research on Number* (IRON). For me, our work was a natural continuation of the emphasis on Piaget's work in the modernist movement, and two of our goals were to produce models of children's mathematical thinking in the context of teaching and learning, and to establish those models as orienting content in “school mathematics”. Also on our agenda was initiating a constructivist revolution in mathematics education to countermand the stranglehold that behaviorism had on the field after the demise of the modern mathematics movement.⁸

That we were somewhat successful in initiating a revolution is indicated by two publications sponsored by the National Council of Teachers of Mathematics (NCTM). The first was an article co-authored by John Richards, a philosopher of mathematics who also worked on IRON, and Glasersfeld (Richards & Glasersfeld, 1980) that was published by *The Journal for Research in Mathematics Education*. This was a landmark publication in mathematics education circles because the authors were able to successfully differentiate the radical aspect of Piaget's genetic epistemology from what Glasersfeld nine years later called trivial constructivism—a form of constructivism that asserts that children gradually build up their cognitive structures while maintaining that the cognitive structures being built up are reflections of an ontological reality. Trivial constructivism in mathematics education began in 1960 with Jerome Bruner's *The Process of Education*, which emphasized active learning, and it took twenty years for it to be challenged in a mathematics education journal.

The second publication was a document published by the NCTM (1989), ‘*Curriculum and Standards for School Mathematics*’, devoted to developing mathematics curriculum in the context of regarding the mathematics learner as an active agent constructing mathematics. Unfortunately, but not unexpectedly, given the influence of Cartesian epistemology in Western thought, mathematics and students were still regarded as two separate and independent entities in this publication as well as in the publication that superseded it (NCTM, 2000).

ELIMINATING THE DUALITY IN CARTESIAN EPISTEMOLOGY

One of the major themes threaded throughout the chapters in this collection concerns explicating a relation between knowledge and reality that can be used to eliminate the duality underpinning the prevailing concept of school mathematics and its teaching and learning. Mathematics educators have been caught in the trap of Cartesian epistemology in a way that is similar to the one in which Stolzenberg

(1984) claims mathematicians fell into in their attempts to establish proper foundations of pure mathematics.⁹ It is, according to Stolzenberg, indisputable that the contemporary mathematician operates within a belief system whose core belief is that mathematics is discovered rather than created or invented by human beings. This belief is similar to the duality between mind and mathematics that has historically served as the rational bridgehead of school mathematics.¹⁰

In the chapter, “Piaget and the Radical Constructivist Epistemology”, Glasersfeld (this book, chapter 7) provides an extensive discussion of research that undermines the belief that, “The knower and the things of which, or about which, he or she comes to know are, from the outset, separate and independent entities” (chapter 1, p. 5). The basic research that he draws from is Piaget’s account of the child’s construction of the concept of an object that “has some kind of permanence in his stream of experience” (chapter 7, p. 77). Rather than attempt to recapitulate Piaget’s research and Glasersfeld’s interpretation of it, I encourage the reader to embark on his or her own journey through their intricate and elegant accounts of how the infant comes to be but one element or entity among others in a universe that he or she has gradually constructed for him-or her-self out of the elementary particles of experience. This powerful insight into the child’s construction of his or her ordinary items of experience serves as a ‘demonstration’ that Cartesian duality is untenable: That is, the belief that from the outset the knower and the things of or about which he or she comes to know are separate and independent entities is not viable.

BASIC ELEMENTS OF “SCHOOL MATHEMATICS”

If indeed it is accepted that the student and mathematics are not two separate and independent entities, what does this mean for school mathematics and for the activities that are involved in specifying such mathematics? Ernest (1996), a social constructivist, has argued that radical constructivism has little to offer for selecting the mathematics that constitutes school mathematics.

What selection from the stock of cultural knowledge is valuable to teach? (Here again, I pause to consider whether radical constructivism is even able to pose this question.). (p. 346)

The implicit assumption in Ernest’s comment is that “cultural knowledge” is the knowledge produced through the mathematical activity of adult mathematicians. This raises a serious issue concerning the commensurability of such knowledge and children’s mathematical knowledge.¹¹ Our goal should not be to transfer cultural knowledge to children in the particular way that we understand it. Rather, our goal should be to learn how to engender children’s productive mathematical thinking and how to build explanatory models of that thinking. This may seem to be a rather ambitious goal, but we have to remember that children spontaneously construct their object concepts and a universe in which they are but one element out of the elementary particles of experience. So, children enter school having already engaged in constructive activity of enormous magnitude; thus, their ability to

construct a universe of mathematical concepts of comparable magnitude should not be an issue. Instead, the main issues are our disposition toward children as productive mathematical thinkers, our ability to engender children’s productive mathematical thinking, and our ability to construct explanatory models of children’s mathematics that portray it as a coherent and internally consistent mathematics.

Conceptual Analysis

How might one operate in order to construct explanatory models of children’s mathematics that portrays it as a coherent and internally consistent mathematics?¹² In chapter 19 (this book), Glasersfeld provides a starting point through his use of conceptual analysis in analyzing experiential foundations of mathematical concepts.¹³ His focus is on the operations that produce object concepts (ordinary items of experience) and the concepts of arithmetical units and number. He cites a particularly important proposition by Husserl that serves in justifying his approach:

Husserl proposed that the mental operation that unites different sense impressions into the concept of ‘thing’ is similar to the operation that unites abstract units into the concept of number. (p. 212)

From here, Glasersfeld introduces the concept of ‘things’ (plural) and differentiates such experiential pluralities from abstract units and the composite wholes made from them. This distinction constitutes a break in the interface between experiential and mathematical reality. The construction of mathematical reality involves a recursive use of the unitizing operation that produces experiential things to produce abstract units and number. In formulating his model of the unitizing operation, Glasersfeld drew on his work with Silvio Ceccato whom he credits as the first to interpret the structure of certain abstract concepts as patterns of attention. So, given that children construct what Glasersfeld calls *experiential* realities out of elementary particles of experience, one can infer that children construct their mathematical realities using their experiential realities. That is, one can infer that a child’s mathematics is abstracted from his or her experiential reality and it is not given from the outset as an entity independent of the child.

First- and Second-Order Analyses and Models

Glasersfeld drew from conceptual constructs outside of mathematics in his account of the conceptual operations that produce the fundamental concept of number, a concept that even Hausdorff (1962) left undefined in his theory of cardinal (and ordinal) number.

This formal explanation says what cardinal numbers are supposed to do, not what they are. ... we must leave the determination of the ‘essence’ of the cardinal number to philosophy. (pp. 28-29)

Rather than leave the essence of cardinal number to philosophy, Glasersfeld provided an epistemological account of operations of the mind that produce number. I refer to his analysis as a first-order analysis. The goal of a first-order analysis concerns specifying the mental operations that produce particular conceptions of the analyst. It is an analysis of first-order models, which are models the analyst has constructed to organize, comprehend, and control his or her experience; that is, the analyst's own knowledge.

The distinction I am making between the mental operations that produce particular conceptions of the analyst and those conceptions is crucial in understanding how the knowledge of adults can be used in constructing a "school mathematics". It is crucial because "school mathematics" is based on the conceptual operations that adults' can infer children use in their production of their mathematical language and actions or interactions. Radical constructivism is a model of knowing, so Glasersfeld's emphasis on operations that produce his knowledge rather than on his knowledge per se is a major reorientation to the constitutive characteristics of "school mathematics".

Second-order models are models an observer constructs of the observed person's knowledge in order to explain his or her observations (Steffe, Glasersfeld, Richards, & Cobb, 1983, p. xvi). Because the goal of the analyst in constructing a second-order model concerns constructing conceptual operations that explain the observed language and actions or interactions of the observed person, I refer to it as a second-order analysis. It involves using the results of first-order analyses in the construction of the conceptual operations attributed to the observed person. First- and second-order analyses differ in what the analyst takes as a subject of analysis. In the former, the analyst focuses on his or her conceptual operations that are within his or her awareness. In the latter, the focus is on constructing explanations of the language and actions or interactions of the observed. Unlike the first-order analyst, the second-order analyst has no access to the conceptual operations of the observed person.¹⁴ The explanations the second-order analyst produces consist of systems of conceptual operations of the analyst that may or may not be a part of the analyst's conceptual operations prior to engaging in the analysis. Both kinds of analyses are involved in producing "school mathematics".

Distinguishing First- and Second-Order Models in "School Mathematics"

Analysis of a first-order model. In Chapter 1 (this book), Glasersfeld commented that, "(T)he experimenter/teacher must not only have a model of the student's present conceptual structures, but also an analytical model of the adult conceptualizations towards which his guidance is to lead" (p. 16). Producing such an analytical model involves conducting a conceptual analysis of the adult conceptualizations. Glasersfeld's conceptual analyses of the experiential foundations of mathematics¹⁵ inspired Thompson and Saldanha (2003) to conduct a similar analysis of fractions and multiplicative reasoning. Rather than consider the mathematical structure that fractions and their operations comprise as given—as a

static conceptual entity—these authors analyzed mental operations that are involved in producing reciprocal relationships of relative size. Of those relationships, the authors commented that:

The system of conceptual operations comprising a fraction scheme is based on conceiving two quantities as being in a reciprocal relationship of relative size: Amount A is $1/n$ the size of amount B means that amount B is n times as large as amount A. Amount A being n times as large as amount B means that amount B is $1/n$ as large as amount A. (p. 107)

Thompson’s and Saldanha’s goal was to bring out aspects of knowing fractions that are important in the design of fraction curricula. In developing their analytical model,¹⁶ it is important to note that these authors used the results of second-order analyses of children’s fractional reasoning that they or others had conducted as conceptions in their first-order analysis. This is important because it illustrates the reciprocal relationship between the two kinds of analyses in the production of “school mathematics”.

A second-order model. Second-order analyses are essentially based in second-order cybernetics (Glaserfeld, this book, chapter 15, p. 160) in that the observer not only plays an active part in the process of observation, but also an active part in the establishment of the observed events. In other words, the curriculum developer is not only an analyst of the mathematical language and actions or interactions of children, but he or she is deeply involved in occasioning the language and actions or interactions being analyzed. In the words of Steier (1995): “Approaches to inquiry ... have centered on the idea of worlds being constructed ... by inquirers who are simultaneously participants in those same worlds” (p. 70).

Second-order models are constructed both in and following social interaction with children, so they unavoidably involve the analytical models of the observer. This is a crucial realization because it opens the way for the analyst’s ways of knowing mathematics to be integrally involved in bringing forth children’s productive thinking as well as in formulating explanations of it.

When working both as a teacher and an analyst, I constructed what I called the *splitting* operation to explain the operations involved in a child making a stick such that a given stick was five times longer than the stick to be made (Steffe, 2002). To make the stick, I inferred that the child posited a stick in visualized imagination such that repeating the visualized stick five times would produce a stick the same length as the given stick. Further, I inferred that the child also posited the visualized stick as one part of the given stick that, when repeated five times, would constitute the given stick. The splitting operation is multiplicative in nature and, in that sense, it is of the same genre as Thompson’s and Saldanha’s reciprocal relation between relative sizes. But it cannot be said to be the same operation. A qualitative distinction in the two operations is that, in splitting, the child seems unaware of a reciprocal relationship between the two sticks prior to actually carrying out splitting activity. The splitting operation might open the possibility of producing a

reciprocal relationship between the two sticks using the results of splitting activity, but reciprocity is yet to be constructed as a generalized operation.

Reciprocity between analytical and second-order models. Reciprocal relationships of relative size and splitting exemplify the reciprocity between the analytical models produced by conceptual analysis of adult conceptualizations and the second-order models produced by analyzing children's mathematical language and actions. When reciprocal relationships of relative size serve as a curriculum goal, it is essential to specify constructive itineraries that are abstracted from actually teaching children in teaching experiments (Hackenberg, 2005, 2007; Steffe & Thompson, 2000). Splitting, like reciprocal reasoning, can also serve as a curriculum goal. In both cases, a teacher can consider them as belonging to her or his ways of mathematical knowing and as mathematics for children. When the teacher experiences children reasoning in such a way that she or he can infer that they are engaging in either way of the operating, she or he can consider that the constructs constitute mathematics of children. Once it has been possible to construct rational ways of thinking like the splitting operation that explain children's language and actions or interactions, these constructs become a part of the ways of knowing of adults that can be used in the production of "school mathematics". The "school mathematics" about which I am speaking is co-constructed by adults and children.¹⁷

The constraints of children's mathematics. So far, I have been concerned with distinguishing between the mathematical knowledge of the observer and the observed in consideration of constitutive characteristics of "school mathematics". But this is only a beginning because differences can be inferred in children's mathematics even within the same chronological age. For example, in the IRON project, we distinguished five stages in the constructive itineraries of children six years of age concerning their counting schemes (Steffe et al., 1983; Steffe, Cobb, & Glasersfeld, 1988). For this and similar reasons, mathematics for particular children is selected on the basis that it has been experienced as mathematics of other children who can be judged as similar to the given children. Even this way of selecting mathematics for children, however, is subject to the judgment of the children's teachers because what children do construct as a result of their interactions, social or otherwise, may not be what an adult expects or desires and it may indeed be a surprise. Another way of saying this is that adults are constrained to and by children's mathematics. We are constrained to children's mathematics in that their mathematical language and actions provide occasions for inferring their concepts and operations. Further, we are constrained by children's mathematics in the sense that it "kicks back" at us in a way similar to how Glasersfeld (1990b) considers the role that constraints play in accommodation.

The constructivist is fully aware of the fact that an organism's conceptual constructions are not fancy-free. On the contrary, the process of constructing is constantly curbed and held in check by the constraints it runs into. (p. 33)

FINAL COMMENTS

“School mathematics” is more about ways and means of operating mathematically than it is about the results of those ways and means. Saying this doesn’t mean that I am not interested in children learning such things as fractional multiplication facts, but Glasersfeld (this book) reminds us that, “‘Facts’, as Vico saw long ago, are made by us and our way of experiencing, rather than given by an independently existing objective world” (chapter 1, p. 9). The conceptual operations that produce one-sixth as the product of one-half and one-third are crucial because they are involved in composing any two fractions multiplicatively. Focusing on these conceptual operations when teaching children, however, can only orient us to consider children’s ways and means of operating in contexts that we adults consider fractional (Steffe, 2003). When we do focus on the productive mathematical thinking of children, we have to learn to listen to children (Confrey, 1991) in order to bring forth their current conceptual operations. Ackermann (1995) made the case for listening to children as well as it can be made.

As a teacher learns to appreciate her students’ views for their own sake, and to understand the deeply organic nature of cognitive development, she can no longer impose outside standards to cover ‘wrong’ answers. She comes to realize that her teaching is not ‘heard’ the way she anticipated, and that the children’s views of the world are more robust than she thought. (p. 342)

In constructing a “school mathematics” it is not enough to listen to children as crucial as it is to do so. Adult’s must also have a viable model of children’s present concepts and operations and “hypothesize pathways and guide the child’s conceptualizations towards adult competence” (Glasersfeld, this book, chapter 1, p. 16). Understanding that adult competence only plays an orienting role in teachers bringing forth, sustaining, and modifying children’s mathematics emphasizes the active role that teachers do play in children’s mathematical education. When children work in mathematical learning environments that emphasize their mathematics and their constructive activity, I have found that their generative power can be impressive (cf. Olive, 1999). But the judgment of ‘impressive’ is always made relative to the second-order model we have of the children’s present concepts and operations and never relative to a single domain of reality.

I can’t emphasize enough that curriculum developers who aim at learning what “school mathematics” might consist of must embed themselves in children’s constructive activity in the context of teaching and in opportunities to engage in analyses of their experiences. As important as the analytical work might be, when interacting with children, adults often interact intuitively and in-the-moment without analysis and reflection, and thus they produce tasks or carry on interchanges with children that were unanticipated prior to teaching. These intuitive, in-the-moment interactions occur as a result of experiencing unanticipated contributions of the children or constraints originating from within the children. These kinds of interactions with children can serve the curriculum developers in understanding that the “school mathematics” they produce can at

most open possibilities for mathematics teachers to construct their own “school mathematics” in conjunction with their children. In fact, “school mathematics” should be regarded as providing possibilities for teachers to explain their students’ mathematical language and actions and for teachers’ goal setting. Perhaps the underlying and most basic message of Glasersfeld’s radical constructivism is that teachers use “school mathematics” and the model of knowing in which it is embedded as personal concepts as they engage their students in mathematical activity (Désautels, 2000). Thinking of a constructivist school mathematics as a dynamic organization of mathematical schemes of operation in the mental life of teachers casts mathematics education as a very exciting field and marks it as an evolving and changing professional practice. Teachers are the chief mathematicians with respect to the mathematics of children and they are by necessity integrally involved in the production, modification, refinement, and elaboration of a “school mathematics” produced by curriculum developers.

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NOTES

- ¹ Confrey and Maloney (2006) as well as di Sessa and Cobb (2004) refer to radical constructivism as a grand theory.
- ² Glasersfeld speaks of an external concept as a concept that is external to the process that produced it.
- ³ See Larochele (2000) for an elegant amplification of embedding constructivism in the educational context.
- ⁴ I have come to believe that my concept of radical constructivism is compatible with aspects of Glasersfeld’s concept, but I do not hold him responsible for what I say in my commentary.
- ⁵ I use ‘mathematical structure’ generically to refer to the body of a mathematical theory including the axioms, definitions, basic concepts, theorems, proofs, and such other matters that may be relevant in the particular theory.
- ⁶ School mathematics without quotation marks refers to school mathematics in Cartesian epistemology and the phrase with quotes refers to school mathematics within radical constructivism.
- ⁷ The curriculum developers apparently did not understand that Piaget’s genetic structures were his formalizations of what he observed children do. He made no claim that children were aware of the structure that he saw in their mathematical behavior.
- ⁸ See Steffe & Kieren (1994) for an historical account of the rise of constructivism in mathematics education and the *zeitgeist* in which we operated.
- ⁹ Stolzenberg (1984) characterizes a trap as “a closed system of attitudes, beliefs, and habits of thought for which one can give an objective demonstration that certain of the beliefs are incorrect and that certain of the attitudes and habits of thought prevent this from being recognized” (p. 260).

- ¹⁰ As Bernstein explains the rational bridgehead, “Either there is some support for our being, a fixed foundation for our knowledge, or we cannot escape the forces of darkness that envelop us with madness, with intellectual and moral chaos” (p. 18) (in Konold & Johnson, 1991, p. 2).
- ¹¹ The issue is analogous to the separation between mathematical structures and genetic structures that plagued the modernist movement during the last century.
- ¹² Given that Glasersfeld does not deny reality (this book, chapter 14, p. 150), neither should we adults deny that children have a mathematical reality that is independent of us. I refer to such a conceptual construct as ‘children’s mathematics’.
- ¹³ Conceptual analysis was imported by Glasersfeld through his work with Ceccato and widely used in the IRON project. A conceptual analysis is an analysis of mental operations.
- ¹⁴ Even in a first-order analysis, the analyst constructs the involved conceptual operations and, of course, even the concept of a conceptual operation (I thank M. Larochelle for emphasizing this point).
- ¹⁵ Also see Glasersfeld (1995).
- ¹⁶ I use ‘analytical model’ to refer to the results of a first-order analysis of adult conceptualizations. An analytical model is indeed a first-order model, but it is a model of knowing mathematics and, in that sense, it is epistemological.
- ¹⁷ See Steier (1995, p. 78) for a cyberneticist’s view of co-construction.

POSTSCRIPT

THE REVOLUTION THAT WAS CONSTRUCTIVISM

A look at the literature published in high impact journals identifies more than three thousand publications that used the keywords constructivism or constructivist from the mid 1970s to the present. In the database the first reference to constructivism is in 1967, increasing to 12 sources by 1976. From 1993 onwards the trajectory increased to more than 100 sources a year and then, since 1999 more than 200 sources a year refer to constructivism. Of course this is the tip of an iceberg since the vast majority of the literature is not included in the *Web of Science* database and many articles probably address constructivism without including the term either in the title, abstract or keywords. Nonetheless, the trend provides insights into a revolution involving constructivism. Perhaps it is too soon to judge whether this revolution made a difference and possibly I am like the fish that is unable to know about the water in which it lives—I used constructivism to improve science education and had a stake in it being successful. From where I stand as an urban science educator, changes occurred because constructivism is a subversive process that got people thinking about the purposes of education and the nature of teaching and learning. Although there are times when it seems as if nothing has changed at all in education research, the wheels of change still turn and do not seem to be easily reversed. Furthermore, from my experience, constructivism itself was not immune from the change process because it too changed and then was subsumed in a new wave of sociocultural theory that is just as subversive as constructivism.

THE DOMINANCE OF BEHAVIORISM

Trends within a field are productively viewed in a historical context. In the 1960s when the post-Sputnik fiscal floodgates opened to entice scientists and science teachers into science education, positivism provided a sturdy framework for the natural sciences, as it had for as long as science existed, and behaviorism was a foundation for the social sciences. Within that context, scientists and science teachers, with support from federal agencies in the United States, became science educators, getting involved in science teacher education, curriculum development, and research. The incorporation of behaviorism into their practices may have been tacit, just a matter of common sense for this new breed of science educators.

However, there were problems in science education and differences of opinion concerning how best to think about teaching and learning. Notwithstanding the emphasis on behaviorism, curriculum reforms also embraced the theories of Piaget, Bruner and Ausubel. In the 1960s debates between competing paradigms appeared

in the leading journals and changes began to occur in science curriculum development, exhortations about how best to teach science, and the questions researchers sought to answer. Even so, the center of mass remained with behaviorism. I was aware of Piaget's theory and had applied it in my research in Australia prior to coming to the University of Georgia (UGA) to study for my doctorate in science education. However, the practices of science education faculty and students were strongly behaviorist and embraced Gagné's theory of hierarchical learning. In fact, in my first course in science education at UGA a book on radical behaviorism, written by B. F. Skinner's daughter, Julie Vargas was used as a text.

I first heard the term constructivism, used during a heated conversation I had with some fellow doctoral students from the mathematics education program at UGA. I was about to graduate with a doctorate in science education and though the programs were in the same building they were as unlike as could be imagined. Both were highly ranked programs in the United States, arguably the top ranked programs in their respective fields. What a contradiction. Whereas the science education program was strongly behaviorist, the mathematics education program focused on learners constructing meaning and pioneered methods to explore learning in ways that reflected emerging constructivist orientations. To be sure there were differences within the mathematics education group, but there was an upwelling of enthusiasm for all that constructivism brought to the table and, faculty like Leslie P. Steffe, were energetic and effusive about their research. In contrast, my classroom oriented research reflected a standard methodology of positivism and incorporated multivariate analyses of complex data sets. However, I was searching for improved ways to theorize teaching, learning to teach, and learning. Also, I was acutely aware of the reductionist nature of my research and frustrated at its inability to be responsive to what I experienced when I visited classes and to explore macrostructures such as the impacts of high stakes testing.

Ernst von Glasersfeld's (hereafter EvG) work in mathematics education was gaining traction—especially his collaboration with Leslie P. Steffe. Also, a steady stream of mathematics educators graduated from UGA and immediately began to assert themselves internationally as researchers and teacher educators. Simultaneously, graduate student networks encouraged more science education students to take courses with EvG and, as Russell Yeany tells it, when they returned from EvG's classes to science education, they were different, questioning the status quo and seemingly empowered with respect to what they were to learn and how they were to learn it. These signs of change in science education at UGA were just a sample of what was to come; and changes were not confined to the United States. The adoption of constructivism in education was to be global in extent.

METHODOLOGICAL CONSTRUCTIVISM

"Constructivism! Ah. That's what it's called. I needed a name for it. This is what I've been doing for years". My colleague smiled at me and continued to describe

the impact of his curriculum projects on global science education. He meant well. It was 1987 at Florida State University—site for major curriculum developments in the 1960s, mainly based on Gagné’s behaviorist oriented theories. The hands-on fraternity was appropriating constructivism to legitimize business as usual. Inwardly I groaned because I was attracted to the subversive aspects of constructivism—what it meant for the nature of science, what would count as science, and how science curricula would be planned and enacted. Colleagues using constructivism as a label to legitimize continuing with the status quo was a worst-case scenario. Already I had experienced the joy of visiting revolutionary mathematics classes in Indiana where Grayson Wheatley and Paul Cobb had collaborated with teachers and students to create problem-oriented mathematics. I marveled at the way the students in those grade 1 classes were curious and emboldened in solving problems and convincing others about their answers. Classroom environments were transformed by the use of constructivism as a way of thinking about classroom structures and the possible roles for teachers and students. I was struck by the emancipatory potential of using constructivism to think about the creation of learner centered classroom environments.

Yet people are opportunistic in how they appropriate schema such as constructivism and many scholars seemed to use the label of constructivism to describe whatever it was they were doing. Some scholars thought about learning through one lens of constructivism, that is the need to take into account what students knew and could do and build fresh understandings accordingly. These were the conceptual change researchers. Missing were epistemological and ontological perspectives relating to what was learned. EvG distanced himself from such scholars, referring to this way of thinking as trivial constructivism. The distinction drew the attention of many science educators to consider the ontological status of science—as a social truth or as a God’s eye view of reality?

Just as my colleague had done, some scholars created learning environments on the basis of thinking about teacher and student roles from a constructivist perspective. These were called constructivist classrooms, which detailed preferred ways to organize teachers and students to optimize learning. Typically this involved students organized in small work groups, having chances for direct experiences with phenomena, discussions about sensory experiences, generation of alternative ideas through interpretation of data, and then among peers, negotiating a consensus that could then be put to the test. All of this usually was done with the teacher taking a role of supporting the active participation of students. I refer to this as methodological constructivism because it refers to a way to teach and learn—a best way to organize classrooms to optimize learning. The approach ignores the fact that any social activity at all can be interpreted from a constructivist perspective, including a two-hour lecture with 700 students in a lecture hall. Having thought about such a scenario, it is presumed that teacher and learner roles could be formulated to ensure that learning was optimized. Perhaps it is not surprising that methodological constructivism spread like wildfire in teaching universities and the teachers who passed through them. It seems that science teachers like prescriptions and teacher educators like prescribing methods.

BEYOND CONTROVERSIES AND CONTRADICTIONS

By the time I returned from Australia to UGA in 1984, a revolution in science education was well underway. The top science education students were taking courses with EvG and meeting well into the night to discuss ways to change school science to focus more on learning. The graduate student group at the time also was looking quite international and included scholars from Taiwan (Hsiung and Tuan), Spain (Espinete) and Portugal (Bettencourt). The US doctoral students also included several who would later make their marks internationally, including Swanson, Cronin-Jones and Davis. These students rejected methodological behaviorism and tried qualitative research and evaluation methods.

Three keynote addresses by EvG not only created major international perturbations but also sowed the seeds for the field to move on. The first of these, in 1989, was delivered at the first meeting of the *History and Philosophy of Science and Science Teaching* group in Tallahassee, Florida. His other keynote addresses were to the *American Association for the Advancement of Science* and the *National Association for Research in Science Teaching* in the 1990s. As one of the organizers of all three events, I was startled to experience a small amount of high-energy dissension to radical constructivism among philosophers and scientists. There appeared to be a base need for science to retain its position as the ultimate voice, a discourse that produced truths or knowledge claims that over time iterated toward the truth. Following the conference several of the philosophers of education launched attacks on constructivism in books and journals—often misrepresenting EvG's claims and never taking the time to check with him on the viability of how they represented his perspectives. I found the attacks to be polarizing and not useful. Whereas EvG collaborated with Steffe and others in research on learning and teaching, especially in mathematics, the opponents "tore planks from the boat" and rarely offered alternatives.

As a theory I assumed radical constructivism illuminated some aspects of social life while obscuring others. As a classroom researcher I found constructivism useful as a referent to make sense of teaching, learning and learning to teach. Just how teachers and students organized knowledge about science and their roles as teachers and students were central tenets of my scholarly interests and I became focused on the role of metaphors, images and beliefs in learning to teach and teaching. I was attracted to contradictions that arose between what people say and how they participate, how schema are translated into actions, and how the presence of others mediate learning and teaching. As I encountered blind spots I added new theories to my frameworks and as necessary I adapted constructivism based on what I learned from my research—notably with respect to social processes, power and emotions. I did not need or benefit from the line-by-line philosophers' critiques of a single EvG manuscript taken in isolation from a lifetime of scholarship. The critiques were a distraction, a storm in a teacup that for a short time diverted precious resources from good scholarly practice.

EvG made such a difference in science education because his theoretical work with radical constructivism could be applied in our research to make sense of

teaching and learning sciences, curriculum development, and the professional development of teachers. Similarly, in research radical constructivism provided a foundation for building new research methods. Because radical constructivism was never presented as a truth, a master narrative to be used in all places and times, science educators were not compelled to use it at all let alone use it without adaptation. Many faces to constructivism emerged based on its applications by scholars in many different fields. Not surprisingly, this was an Achilles heel for those who regarded theories as master narratives that had to apply to all of social life. However, to those of us who adopted a bricolage view of theory and research, I understood that different theoretical perspectives illuminate while obscuring. The use of radical constructivism was very useful in many contexts but could not be a complete master theory to account for all of social life.

EvG never viewed radical constructivism as a master narrative, nor did he seek to indoctrinate others to his perspective. I vividly remember a colleague decline to attend a presentation that EvG was giving at my university. Since we were a relatively small faculty I wanted her to participate and see the potential of radical constructivism. After his talk I spoke to EvG about how to resolve the problem as I perceived it—a colleague who appeared unable to separate her religious beliefs from her beliefs about learning and teaching. “Leave her alone. Let her think however she wants to think”. These words remained with me for two decades. At that time this was not my style as I enjoyed the joust—pushing and poking at other’s ideas, hoping to see changes in the ways in which they considered knowledge and coming to know. His approach was to speak with eloquence, lay it out for others to consider, and then leave them to make up their own minds. Radical constructivism is a subversive way of thinking that might change a person’s ways of being in the world—but never a truth for all to adopt and apply to all circumstances, and especially not an instrument for the oppression of non-believers.

Constructivism and Research Methodology

Methodologists like Guba and Lincoln (1989) employed constructivism to create a brand new line of ethnography that acknowledged the epistemological and ontological underpinnings of social life. For a significant number of years I adopted Guba and Lincoln’s authenticity criteria as benchmarks for assessing the quality of our research. I employed a methodology that acknowledged that individuals and collectives are positioned in social space in ways that provide them with different experiences and ways of making sense of what they experience. The polysemic nature of social life led me to a methodology that looked for and preserved differences as well as patterns of coherence. I rejected the dichotomy of researcher and researched and embraced methods that necessitated that, as a result of participating in research, all participants would change their ontologies. Through the research different categories of participant would become aware of one another’s perspectives, understand their rationale, and respect the differences. Also, as a result of participating in research positive changes would be catalyzed to

benefit all participants. If necessary efforts would be made to help those who had difficulty in using what I learned from research to improve the quality of their social life. Interestingly, the use of constructivism as a theory for my research methods forced me to move beyond constructivism. I needed to address power differences, how to resolve differences in perspective, and how to ensure that research overcame social injustices. Just as the students at UGA found constructivism to be a subversive referent that undermined the status quo—so did I.

SHAPING A FIELD

Radical constructivism has had a profound impact on the field of science education, especially in the past three decades. Initially introduced by EvG in mathematics education the ideas were picked up and widely applied in many scholarly ways as science educators considered the nature of science, science curricula, science teaching, science learning, how learners could show what they know, and assessment of science achievement. A focus on the learner and the role of language in negotiating meaning and arriving at consensus were critical steps in improving the quality of science education through scholarly activities such as research, teacher education, and curriculum design. Gradually researchers developed constructivist theory to address issues that went beyond individuals making sense of science constructs. Constructivism emerged with “multiple faces” as science educators incorporated social, political, and power relationships into the constructivist framework (e.g., critical constructivism), allowing a plethora of problems to be explored through constructivist lenses. In conjunction with the use of constructivism to identify, articulate and solve research and practice problems, many science educators who use qualitative genres of inquiry, use methods that are consistent with constructivism, with full regard to epistemology and ontology.

This book is important as it brings together in one volume a set of scholarly perspectives that can continue to serve educational scholars, including researchers for some time to come. The claims I make about changing a field are not global across the field. Just as there is evidence that constructivism has a foothold in science education there also are contradictions in the form of evidence that positivism also is deeply rooted. Unfortunately, everyday we experience macrostructures framed in terms of positivism that shape education in the form of national standards, policy and legislation such as *No Child Left Behind*, and accountability mandates that place the responsibility for achievement with teachers and school administrators. Today, as yesterday, there is a need to think about education through the lenses of constructivism and reject the ongoing harms of positivism. The chapters in this book are a resource for developing the tools needed for taking the critical stance needed to make the long overdue changes to an educational system that appears to reproduce inequities.

Within science education there are just a few scholars that have shaped the field profoundly and in ways that warrant their work being cited for years after its initial publication. John Dewey, Jean Piaget, and Lev Vygotsky are three field shapers. Perhaps EvG is another. The revolution that occurred when radical constructivism

seeped into the science education community suggests that EvG can continue to sow the seeds to hold science educators accountable for what they do and exhort others to do. It is clear that the work of improving science education is not done and more educators could benefit from considering the viability of radical constructivism for today's problems—not as the only solution, but as part of a bricolage that sheds light to illuminate the learner and his or her learning. As it was when I first encountered it, radical constructivism is subversive, calling into question how we are in the world, how we relate to others, what we regard as worth knowing, and how we can come to know and assist others to know. Having read the chapters of this volume the rest will follow as present practices are subject to radical doubt.

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Key Works in Radical Constructivism

(edited by Marie Larochele)

Ernst von Glasersfeld

Key works on radical constructivism brings together a number of essays by Ernst von Glasersfeld that illustrate the application of a radical constructivist way of thinking in the areas of education, language, theory of knowledge, and the analysis of a few concepts that are indispensable in almost everything we think and do. Ernst von Glasersfeld's work opens a window on how we know what we know. The present work grew out of a desire to make more accessible this line of thought, to highlight its originality and consistency, and to illustrate its fecundity in the domains of cognition and learning.

The first three parts of this book contain texts by Glasersfeld that outline the constructivist approach and explicate the frequently drastic reconceptualizations he has suggested. Both the last part and the postscript consist of commentaries by Edith Ackermann, Jacques Désautels, Gérard Fourez, Leslie P. Steffe and Kenneth Tobin, scholars in the fields that Glasersfeld has been concerned with. They examine a number of critical aspects pertaining to (radical) constructivism's current and future development, often tracing out paths that warrant further exploration and reflection, in particular concerning the sociopolitical dimension of knowledge.

Key works on radical constructivism is intended as a reference book for researchers, educators, and students of education – and for anyone interested in grasping, or deepening their grasp of, radical constructivism's tenets, ambitions and concerns. Readers will discover in this collection of firsthand contributions the contours of a bold, contemporary debate about a most compelling current of thought.

Ernst von Glasersfeld was brought up with more than one language from the very beginning. This taught him early on that the realities people think and talk about are noticeably different. He was much influenced by Silvio Ceccato, the founder of the Operational School in Italy, and then by Jean Piaget's Genetic epistemology, to which, he believes, he was able to add some details. He worked as a language analyst at the Center for Cybernetics in Milan, directed a language research project for the Us Air Force from 1962 to 1970, and then taught as professor of cognitive psychology at the University of Georgia, USA. In 1987 he retired at the age of 75 and became Research Associate at the Scientific Reasoning Research Institute of the University of Massachusetts. Throughout, his main interest was how we come to know what we know and how thought and language are linked. He has published several books in English, German, and Italian.

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